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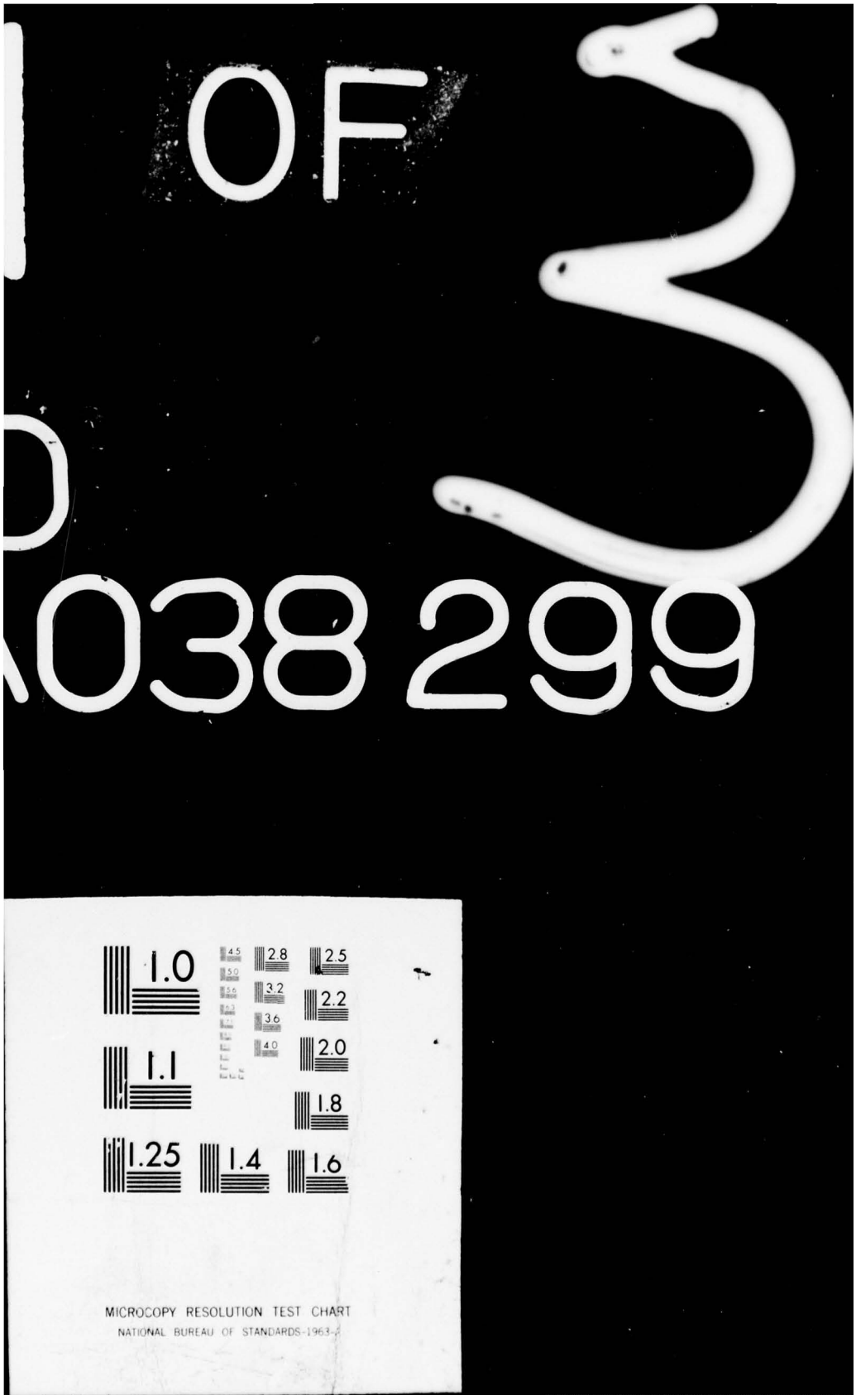
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TECHNIQUES OF DETERMINING IONOSPHERIC STRUCTURE FROM OBLIQUE RADIO PROPAGATION MEASUREMENTS

Ionosphere Radio Laboratory
Department of Electrical Engineering
University of Illinois at Urbana-Champaign

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Computer techniques are developed to home the ray at a specified location with specified accuracy. Several methods have been investigated whereby oblique radio propagation data can be used to obtain ionospheric profiles. In one method the optimum quasi-parabolic layer can be found that is supposed to fit the given data within a tolerable error. Another method makes use of the Backus-Gilbert inversion technique.

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EVALUATION

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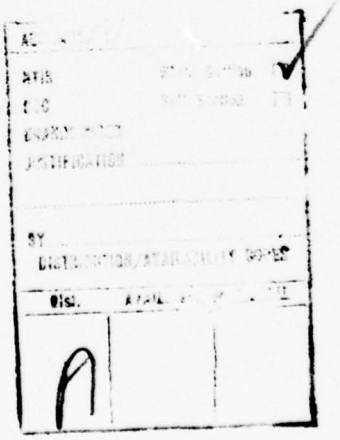
1. This report is the Final Report on this contract, covering two fiscal years work on the development of mathematical techniques for inversion of oblique ionograms.

2. The work reported is significant from both scientific and applied viewpoints and represents a major contribution to the subject. The techniques developed can and will be applied to the propagation management of operational OTH radars and have other potential applications in HF radio propagation research and technology.

TERENCE J. ELKINS
Contract Monitor
Ionospheric Radio Physics Branch
Electromagnetic Sciences Division



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1. INTRODUCTION

The use of pulse sounding as an experimental technique in the investigation of the ionosphere can be traced back to Breit and Tuve (1926). These authors invented such a technique to prove the existence of the ionosphere. Modern reviews of these early endeavors can be found in Waynick (1974) and Booker (1974). The improved electronic technology after the Second World War stimulated interest in the possibility of computing the electron density profiles from pulse sounding data whose carrier frequency was swept. While the major emphasis has been towards inverting the vertical sounding data for electron density profiles, there have also been sporadic attempts to devise techniques of obtaining ionospheric structure from oblique sounding data. Our work, reported here, is concerned with this later problem.

Mathematically, the group delay measured experimentally is a nonlinear functional of the electron density profile. As such this is very difficult to solve. Fortunately, for vertical incidence case with the geomagnetic field ignored, the integral can be cast in the form of a convolution integral known as Abel's integral equation which can then be solved, for example, by Laplace transform (Manning, 1947). Once the geomagnetic field effect is introduced or when the propagation path becomes oblique, the convolution integral property is destroyed. The problem becomes much more difficult. The methods that have been devised in the literature are almost all numerical. Ours is no exception. In this connection, the capability of homing the ray at a specified

location with specified tolerance must be established first. This is the subject of Chapter 2. In ray homing we are interested in three configurations: ground-to-ground homing via a reflection from the ionosphere, ground-to-satellite homing via transmission through the ionosphere and satellite-to-ground homing also via transmission through the ionosphere. In each case, the homing is achieved through an iterative procedure whereby the ray lands at a point successively closer to the given location. Several examples are also given to illustrate the method used. The inversion of backscatter ionograms utilizes the values of the minimum group path computed for a given ionospheric profile. It is therefore desirable to have the capability of computing the minimum group path. A chopping method is used to achieve this and two examples are given to illustrate the procedure involved. In order to achieve these capabilities, the Air Force supplied three-dimensional ray tracing program and the associated electron density model are augmented by seven subroutines. These subroutines are fully described and documented with computer listings.

The major objective of this research is to investigate methods whereby oblique radio propagation data can be used to obtain ionospheric profiles. This is contained in Chapter 3. There exist in practice two kinds of oblique propagation data: point-to-point oblique ionograms and backscatter leading edges. The nature of these data and their intuitive limitations are discussed in section 3.1. In many ionospheric propagation studies, the quasi-parabolic layer has been found to give adequate accuracy. Further, such a layer has a relatively small number of parameters (3, consisting of critical frequency, base

height of the ionosphere and the semi-thickness) and, in many cases, the desired expressions can be evaluated analytically. For this reason our inversion is based on finding the optimum quasi-parabolic layer that fits the given data within a tolerable error. After some discussion of the nature and properties of the quasi-parabolic layer in section 3.2, we go directly to the inversion problem. The inversion of a point-to-point oblique ionogram is discussed first (section 3.3), which is then followed by a discussion of backscatter leading edge (section 3.4). The results are given in terms of graphs and tables. In addition to approaches discussed in 3.3 and 3.4, we have also looked into the inversion problem from general considerations (section 3.5 and Appendix 4). This inversion method makes use of the Backus-Gilbert technique and seems to be very promising. Finally, the report is concluded in Chapter 4 with recommendations for future research.

2. HOMING OF THE RAY

2.1 Introduction

Homing of the ray is the main objective of this chapter. Our study deals with the modification of the three dimensional ray tracing program to incorporate the homing features for ground-to-ground, ground-to-satellite, and satellite-to-ground propagation.

Homing of the ray to the required location can be obtained through an iterative procedure by homing first to the ground distance for a fixed value of the azimuthal angle. If the ground distance homing is successful the azimuthal angle of the transmitted ray is then corrected next. This type of iteration requires back and forth transfer from homing in ground distance to homing in azimuthal angle. The iteration stops when the errors are smaller than the prescribed values.

For each of the three configurations studied we present a detailed development of the technique applied to achieve homing of the ray together with some examples. Difficulties encountered during the course of the study are also pointed out. In section 2.2 we discuss the ground-to-ground homing procedure for a single and multiple layer ionosphere. The ground-to-satellite and satellite-to-ground configurations together with examples to illustrate the homing technique are presented in section 2.3. In section 2.4 we discuss the chopping method utilized in finding the minimum group path for a given transmitter location, azimuthal angle, and oblique frequency. A listing of the computer programs developed under this study together with their description is included in section 2.5. Finally, in section 2.6 we discuss the current

accomplishments and outline suggestions towards future improvement and possible additions to the three dimensional ray tracing program.

2.2 Ground-to-Ground Homing

The ground-to-ground homing is the most involved configuration of the homing study. Given the transmitter and receiver coordinates on the Earth's surface, the electron density profile, the oblique transmission frequency we find an initial approximate elevation angle that will home the ray to the receiver via reflection from the ionosphere. This approximate elevation angle is then used as an initial input for the final homing of the ray through the three dimensional ray tracing computer program.

2.2.1 General Description

It is well known that for a given single-layer ionosphere and a given radio frequency the one-hop ground distance, D, as a function of elevation angle, β, has the dependence sketched in Figure 2.1. Several features of this figure are worth noting. Due to spherical geometry, the ground distance has some finite value, D_o, even when the elevation angle is zero. As the elevation angle increases, the ground distance decreases to a minimum value D_s at β_s. This is the skip distance and consequently the use of subscript s on all quantities related to the skip ray. For $\beta > \beta_s$ the ground distance increases again and approaches to infinity at $\beta = \beta_t$ at which the ray becomes trapped. A figure such as Figure 2.1 suggests the following three possibilities:

i) For $D<D_s$ the observer is in the shadow region and no ray can reach the observer via reflection from the ionosphere.

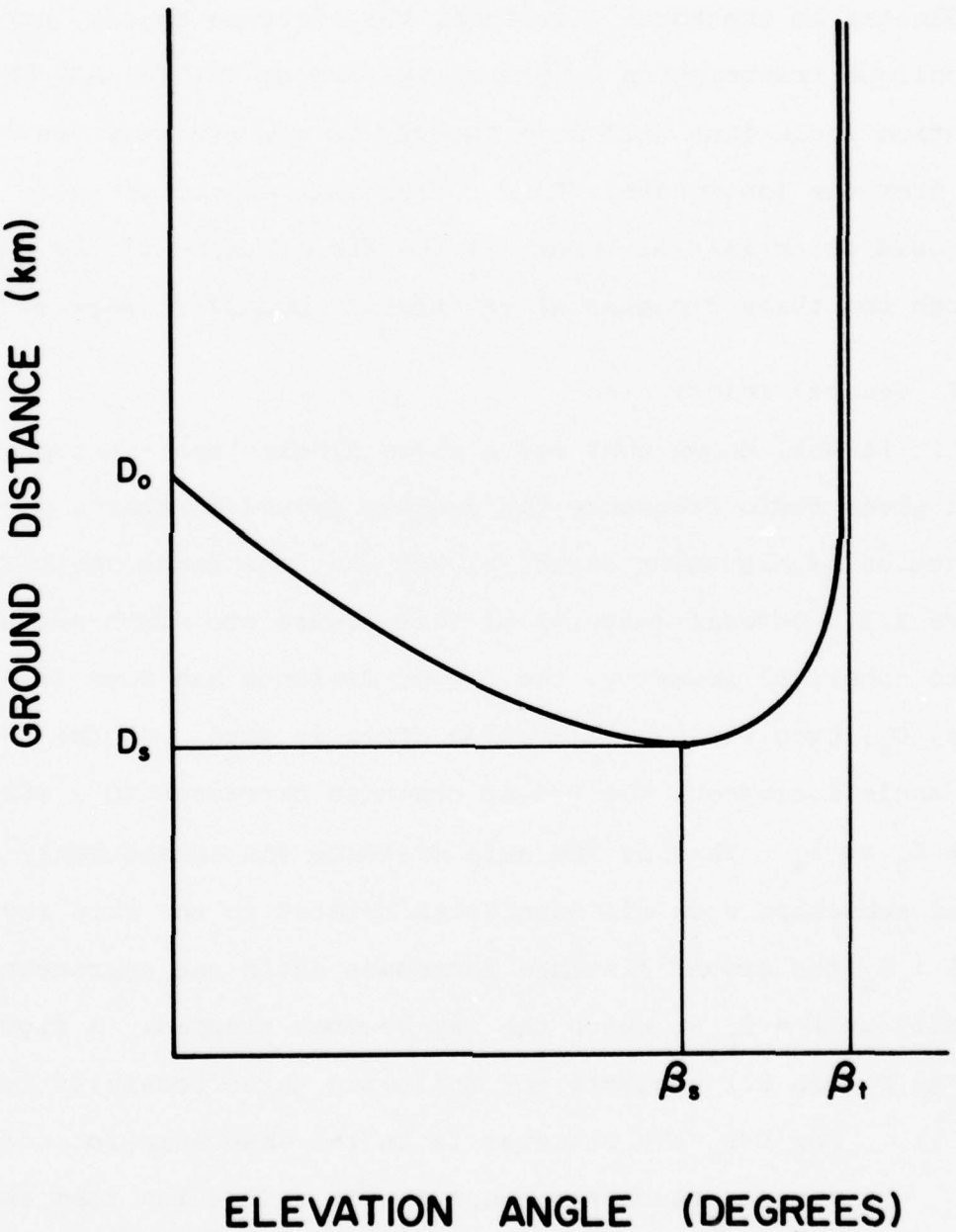

Figure 2.1. Idealized sketch showing the nature of ground distance dependence on the elevation angle of once reflected ray from a single layer ionosphere.

ii) For $D_o<D<D_s$ the observer is in the illuminated region within which a low ray and a high ray (Pedersen ray) can reach the observer.

iii) For $D>D_o$ the low ray is cut off by the earth shadow and only the high ray can reach the observer. Generally, the high ray with an elevation angle β near β_t is very much de-focussed and is very sensitive to elevation angles and small changes in the ionospheric model.

Based upon the above observations, we outline the procedure to obtain an approximate homing elevation angle. This approximate angle is then refined through utilization of the three dimensional ray tracing program.

The given ionosphere at the midpoint of the transmitter and receiver is approximated by segments of a second degree polynomial which for the i^{th} segment has the form

$$N_i(r) = a_i + \frac{b_i}{r} + \frac{g_i}{r^2} \qquad r_{i-1} \leq r \leq r_i \qquad (2.1)$$

The coefficients are chosen by fitting to the given ionospheric model and making sure that both the density and its derivatives are continuous in going from segment to segment. Using the electron density profile given by (2.1) we find analytic expressions for the ground distance (range) D and the virtual path (group path) P'. Since the trapped rays are known to have ray apogee slightly below the peak of the layer and since the high rays are very sensitive near β_t we let the ray apogee occur at the ionospheric peak as an approximation and find the penetration angle β_p. In general β_p so found is slightly less than β_t. Then, from the range

of elevation angles we calculate the corresponding ground distances and group paths through the analytical expressions, thus generating a table of elevation angles versus ground distance and group path. The ground distance between the transmitter and receiver is compared with the tabulated ground distances. If $D<D_s$ there is no solution; if $D=D_s$ there is possibly one solution; if $D_s<D<D_o$ there are two solutions; and $D>D_o$ there is one solution. The solutions, if they exist, are found by linear interpolation. Once the solutions are found, the approximate elevation angles serve as the initial input in the ray tracing program to eventually home the ray.

2.2.2 Approximate Elevation Angle-Single Layer Ionosphere

The geometry of the ground-to-ground homing problem is shown in Figure 2.2. We assume that the transmitter and receiver coordinates are given in geographic east longitude and north latitude. The ground distance, D_{TR}, between the transmitter and receiver is given by

$$D_{TR} = r_o \gamma \tag{2.2}$$

where r_o is the earth radius in km and γ is the angle subtended at the earth center between the transmitter and receiver. Applying the cosine law of spherical trigonometry to Figure 2.3, angle γ is

$$\gamma = \cos^{-1} [\cos \theta_T \cos \theta_R + \sin \theta_T \sin \theta_R \cos (\phi_T - \phi_R)] \tag{2.3}$$

where θ_T, θ_R are the colatitudes, ϕ_T, ϕ_R are the longitudes of

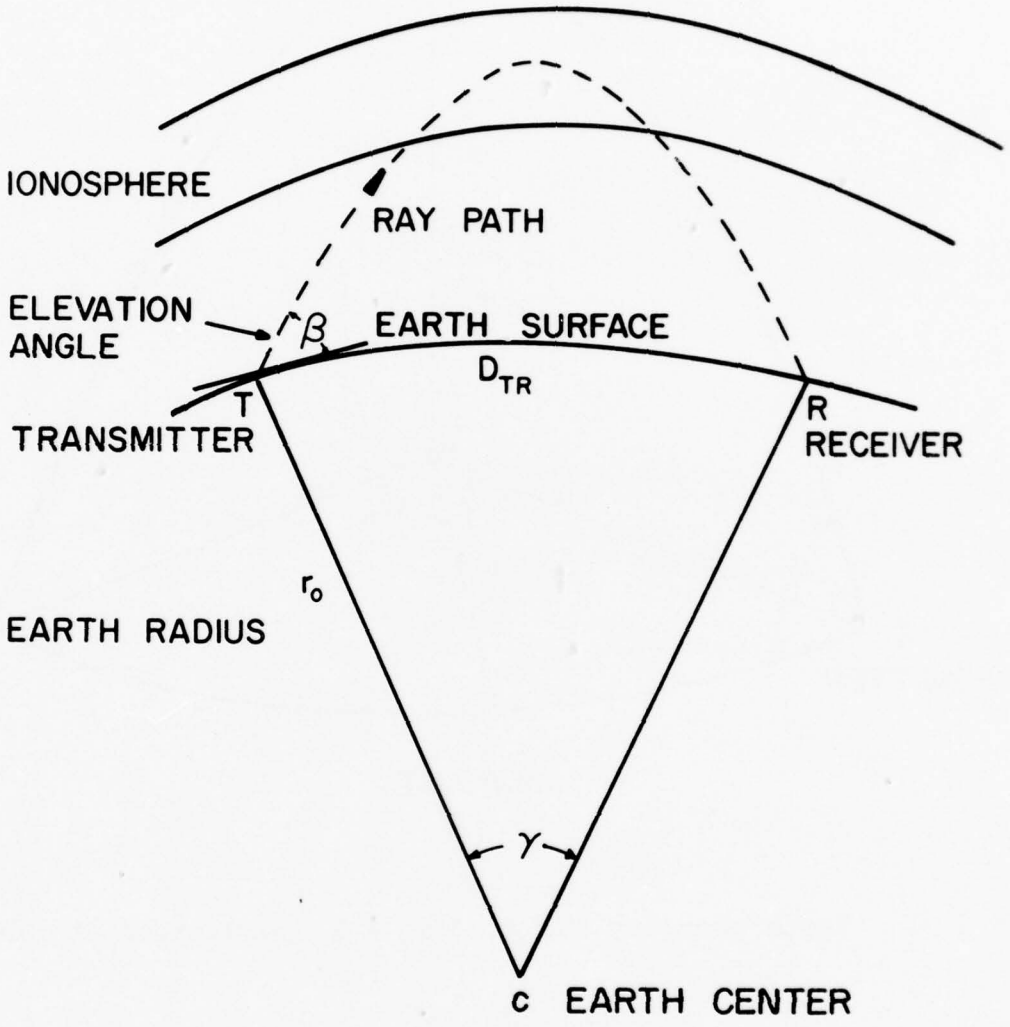

Figure 2.2. Sketch showing the geometry of the reflected ray.

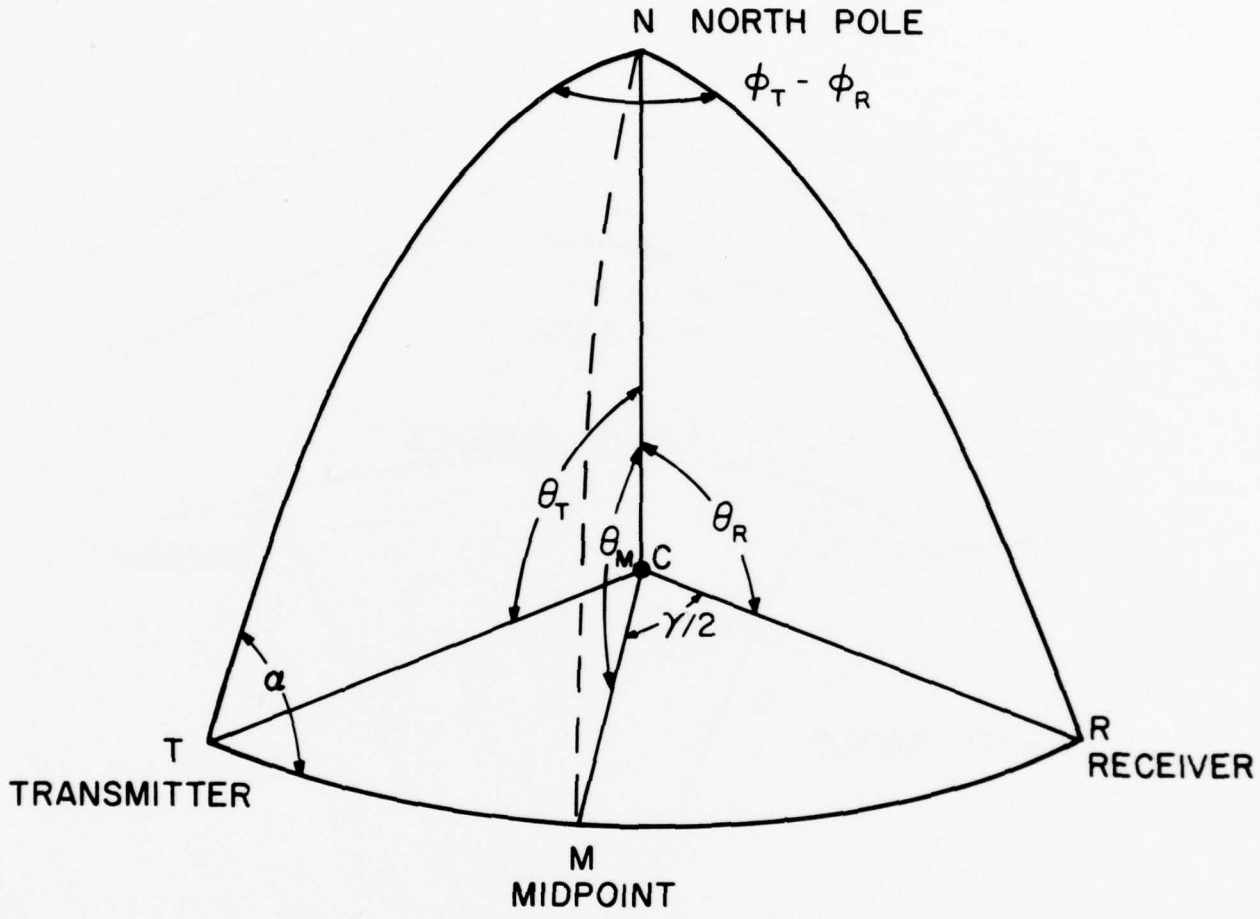

Figure 2.3. Earth geometry for finding the coordinates of the midpoint.

the transmitter and receiver respectively. The coordinates of the midpoint between the transmitter and receiver are then found through the application of the sine and cosine laws of spherical trigonometry to Figure 2.3. From Figure 2.3, the azimuthal angle of the receiver as viewed from the transmitter is

$$\alpha = \sin^{-1} [\sin (\phi_T - \phi_R) \sin \theta_R / \sin \gamma] \qquad (2.4)$$

The colatitude of the midpoint, M, is

$$\theta_M = \cos^{-1} [\cos \theta_T \cos \frac{\gamma}{2} + \sin \theta_T \sin \frac{\gamma}{2} \cos \alpha] \qquad (2.5)$$

The relative longitude of the midpoint M with respect to the transmitter longitude, $\delta\phi_M$, is given by

$$\delta\phi_M = \sin^{-1} [\sin \alpha \sin \frac{\gamma}{2} / \sin \theta_M] \qquad (2.6)$$

The exact longitude of the midpoint M is found by adding or subtracting the relative longitude $\delta\phi_M$ from the transmitter longitude ϕ_T, i.e.

$$\phi_M = \phi_T \pm \delta\phi_M \qquad (2.7)$$

If the receiver is located east of the transmitter the positive sign is used otherwise the negative sign is applied. The coordinates of the midpoint thus found are compared with the coordinates of the grid points at which ionospheric models are given. The profile corresponding to the grid point having the smallest difference in coordinates is chosen to represent the profile at the midpoint of the path.

The midpoint profile selected in this manner is approximated by segments of a second degree polynomial which for the i^{th} segment has the form given by (2.1). The coefficients in equation (2.1) are found by making sure that both the density and its derivatives are continuous in going from one segment to another. Each segment value corresponds to the height increment dictated by the composite ionospheric profile. Utilizing equation (2.1) we find analytical expressions for the ground distance D and the group path P'. Neglecting the effects of the Earth's magnetic field and collisions, the ground distance and the group path can be expressed in the form

$$D = 2 \int_{r_o}^{r_t} \frac{r_o^2 \cos \beta_o \, dr}{r[r^2\mu^2 - r_o^2 \cos^2 \beta_o]^{\frac{1}{2}}} \tag{2.8}$$

and

$$P' = 2 \int_{r_o}^{r_t} \frac{rdr}{[r^2\mu^2 - r_o^2 \cos^2 \beta_o]^{\frac{1}{2}}} \tag{2.9}$$

where r_t is the radial distance from the Earth center to ray apogee, β_o is the ray elevation angle, and μ is the refractive index of the medium. The refractive index μ and the plasma frequency f_p are related through

$$\mu^2 = 1 - (f_p/f)^2 \tag{2.10}$$

Since the density N is proportional to the square of the plasma frequency, we can substitute (2.1) in (2.10) to obtain

$$\mu_i^2 = 1 - \frac{1}{f^2} \left(a_i + \frac{b_i}{r} + \frac{g_i}{r^2}\right) \tag{2.11}$$

Through substitution of (11) in (8) and (9), the ground distance and the group path can be written in the form

$$D = D_o + \sum_{i=1}^{n} D_i + D_f \tag{2.12a}$$

and

$$P' = P'_o + \sum_{i=1}^{n} P'_i + P'_f \tag{2.12b}$$

The terms D_o and P'_o in (2.12) are evaluated from the earth surface up to the base of the composite ionospheric profile. The terms $\sum_{i=1}^{n} D_i$ and $\sum_{i=1}^{n} P'_i$ are evaluated from the base of the composite profile up to layer n just below the layer containing the reflection height. The last terms D_f and P'_f are the contributions from the top of the n^{th} layer up to the reflection height of the ray. Explicitly, the terms in (2.12) are

$$D_o = 2r_o \left[\cos^{-1}\left(\frac{r_o \cos \beta_o}{r_b}\right) - \beta_o\right] \tag{2.13a}$$

$$P'_o = 2r_o \left[\sqrt{(r_b/r_o)^2 - \cos^2\beta_o} - \sin\beta_o\right] \tag{2.13b}$$

$$D_i = 2 \int_{r_i}^{r_{i+1}} \frac{r_o^2 \cos \beta_o \, dr}{[A_i r^2 + B_i r + G_i]^{\frac{1}{2}}} \quad i=1,2,\ldots, n \tag{2.14a}$$

$$P'_i = 2 \int_{r_i}^{r_{i+1}} \frac{r \, dr}{[A_i r^2 + B_i r + G_i]^{\frac{1}{2}}} \quad i=1,2,\ldots,n \tag{2.14b}$$

and

$$D_f = 2 \int_{r_n}^{r_t} \frac{r_o^2 \cos\beta_o \, dr}{[A_n r^2 + B_n r + G_n]^{\frac{1}{2}}} \tag{2.15a}$$

$$P'_f = 2 \int_{r_n}^{r_t} \frac{r \, dr}{[A_n r^2 + B_n r + G_n]^{\frac{1}{2}}} \tag{2.15b}$$

where r_b is the base radius of the composite ionospheric profile, and

$$\begin{aligned} A_i &= 1 - a_i/f^2 \\ B_i &= b_i/f^2 \\ G_i &= g_i/f^2 - r_o^2 \cos^2\beta_o \end{aligned} \tag{2.16}$$

Analytical expressions for the integrals in (2.14) and (2.15) are readily available from standard integral tables. The resulting analytical expressions together with (2.13) are programmed on the computer under subroutine FITT. Given the elevation angle β_o, the oblique frequency f, and the density profile N, the subroutine FITT calculates the corresponding ground distance and group path.

Since we are seeking all possible homing solutions, we need to generate a table of ground distances versus elevation angles corresponding to the midpoint density profile. Due to the sensitivity of the trapping angle to changes in the ionospheric model we take the penetration angle β_p as the maximum elevation angle possible. The penetration angle β_p is the solution of the equation

$$r_{max}^2 \mu^2 - r_o^2 \cos^2 \beta_p = 0 \tag{2.17}$$

Therefore, β_p is given by

$$\beta_p = \cos^{-1}\left\{ \frac{r_{max}}{r_o} \left[1 - (^f c/f)^2\right]^{\frac{1}{2}} \right\} \tag{2.18}$$

and the range of elevation angles is

$$0 \leq \beta \leq \beta_p \tag{2.19}$$

The interval in (2.19) is subdivided into 50 subintervals of elevation angles. For each of the resulting 51 elevation angles the ground distance and the group path are computed through subroutine FITT. The generated values of ground distance and group path together with their corresponding elevation angles are assembled in a table. The table is then scanned to locate the two successive entries within which the ground distance between the transmitter and receiver is located. For each pair of entries thus found we perform linear interpolation to locate a possible homing solution within a specified tolerance. We assume that the ground distance and the elevation angle within the two entries are linearly related through

$$D = a + b\beta \tag{2.20}$$

Starting with the two entries of ground distances and their corresponding elevation angles the coefficients a and b are found and a new elevation angle and ground distance are computed. A comparison test is then carried out between the new ground distance and the exact ground distance, D_{TR}. If the difference is

larger than the specified ground distance tolerance, δD, then the procedure of calculating the coefficients in (2.20) utilizing the two closest values of ground distances to D_{TR} and their corresponding elevation angles is repeated until $| D-D_{TR} | \leq \delta D$. The number of interpolations is limited to ten (10) trials for finding the approximate elevation angle, otherwise homing of the ray is said to be not possible. The specified ground distance tolerance, δD, is an input parameter in W(387) of the ray tracing program.

Every possible homing solution of elevation angles is assembled in a table. Then, through the three-dimensional ray tracing program these elevation angles are refined to obtain the corresponding true elevation angles that will home the ray. In some cases true elevation angles are not possible. In either situation a message is printed to show whether or not a homing solution has been achieved. The complete procedure described in this section is simulated on the computer under subroutine HOME.

2.2.3 Approximate Elevation Angle - Multiple Layer Ionosphere

In a single layer ionosphere there are at most two approximate initial elevation angles, one for the low ray and the other for the high ray. In a multiple layer ionosphere, the number of approximate initial elevation angles depends upon the number of layers in the given ionospheric profile. The approximate elevation angle solutions are designated by first layer low or high ray, second layer low or high ray, third layer low or high ray, and fourth layer low or high ray. In our study we selected four as the highest number of layers. This restriction could be removed

with minor modifications on the computer code in subroutine HOME.

Whether the given ionospheric profile is single or a multi-layered one, the procedure of finding the approximate elevation angle is the same as the one described in section 2.2.2. The computer code does not differentiate between single and multi-layered profiles and it is written for a four-layered ionosphere. A sample of the ground distance versus elevation angle curve for a four-layered ionosphere is shown in Figure 2.4.

2.2.4 Refinement of the Elevation Angle

Thus far we have discussed the computation of the approximate elevation angles. In this section we focus our attention on the procedure to refine these approximate elevation angles through actual ray tracing.

Let the exact ground distance between the transmitter and receiver be D_{TR}. Then starting with one of the tabulated approximate elevation angles, β_1, we trace one ray through the given ionospheric model utilizing the three dimensional ray tracing program. The ground distance corresponding to β_1 is D_1. Utilizing the table of ground distances and elevation angles generated in the previous section we increment or decrement elevation angle β_1. The value and sign of the increment depends on the relative location of D_1 with respect to D_{TR} in the ground distance versus elevation angle table. With the new elevation angle β_2 we compute D_2 through the ray tracing program. The ground distance, D_{TR}, must then be located either above D_1 and D_2, below D_1 and D_2, or in between D_1 and D_2. For the cases where D_{TR} is above or below D_1 and D_2 the elevation angles and their corresponding ground

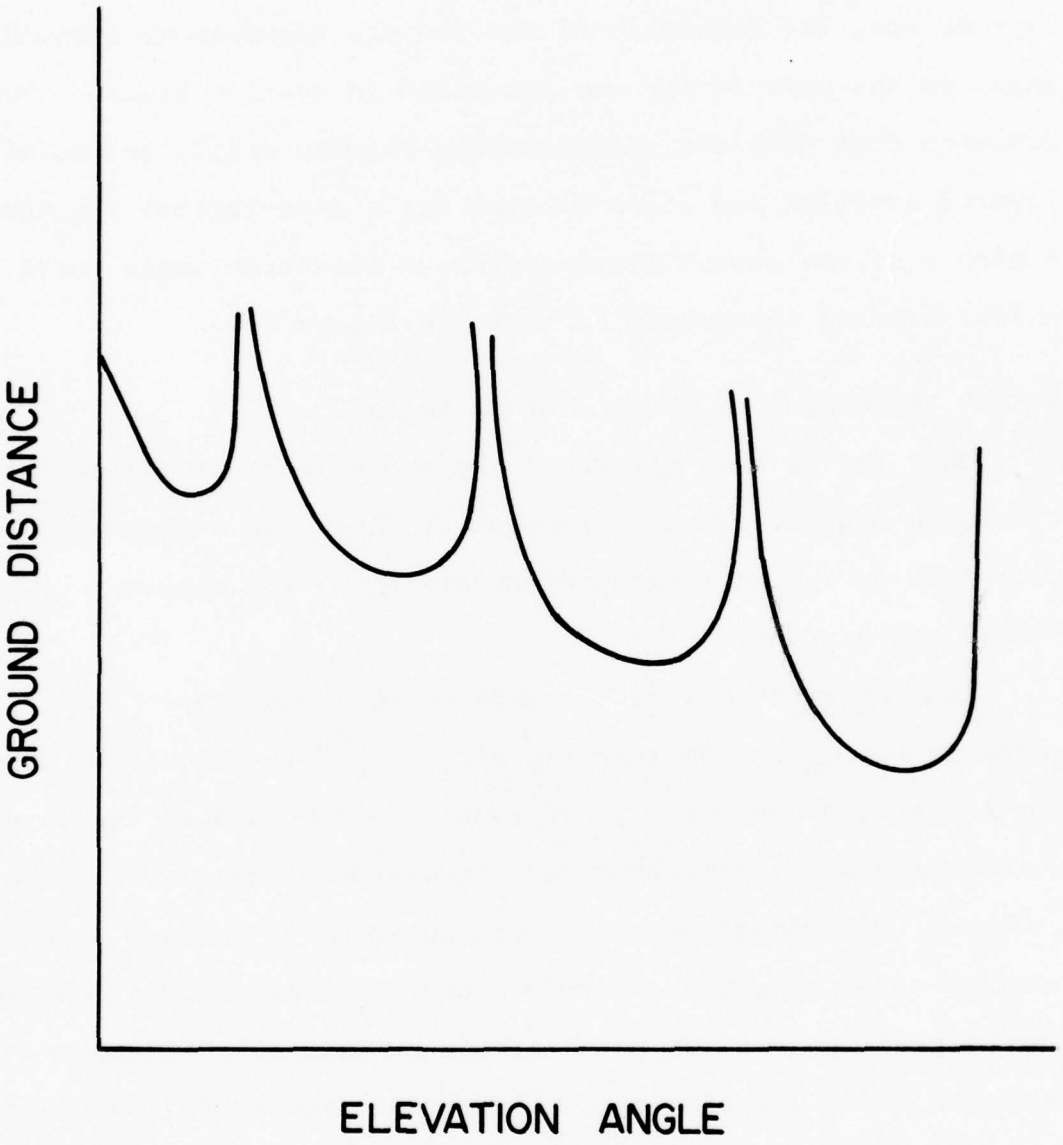

Figure 2.4. Sketch showing the ground distance - elevation angle curve for a four-layer ionosphere.

distances are adjusted through linear interpolation in accordance with equation (2.20) until D_{TR} is either located between D_1 and D_2 or such a condition cannot be achieved. In the latter situation the elevation angle found through (2.20) lies outside the range of elevation angles set for the ray in a multiple layer ionosphere. In the event that D_{TR} cannot be located between D_1 and D_2, homing of the ray cannot be accomplished and one of the statements 'High angle ray' or 'Discontinuity in the ray traced range - Beta curve or range too close to the skip distance' is printed from subroutine ADJUST.

Let us assume that we can locate D_{TR} between D_1 and D_2 then we can find β_3 through linear interpolation on β_1, D_1, β_2 and D_2. With β_3 we calculate a new ground distance D_3. If the difference between D_{TR} and D_3 is less than a specified ground distance tolerance, then β_3 is the desired homing elevation angle except for azimuthal angle corrections. The ground distance tolerance, δD, is an input parameter and is specified in W(387). However, if the difference between D_{TR} and D_3 is greater than δD then we need to correct for the ground distance discrepancy. Since the dependence of the ground distance on the elevation angle is known to be nonlinear, we let

$$D_i = a_o + a_1\beta_i + a_2\beta_i^2 \qquad i=1,2,3 \qquad (2.21)$$

The coefficients a_o, a_1, and a_2 are computed by inverting the matrix resulting from the substitution of ground distances D_1, D_2, D_3 and their corresponding elevation angles β_1, β_2, β_3. The new elevation angle is then given by

$$\beta_{\pm} = \frac{1}{2a_2} \left[-a_1 \pm \sqrt{a_1^2 - 4a_2(a_0 - D_{TR})}\right] \qquad (2.22)$$

If the quantity under the radical sign in (2.22) is negative, then we cannot find an elevation angle and an appropriate message is printed. Equation (2.22) yields two solutions, β_+ and β_-. In order to decide whether β_+ or β_- is the desired elevation angle we set up the following criterion. First, we take the absolute differences between D_{TR} and D_1, D_2, D_3 and the resulting three difference values are compared with each other. The elevation angle β_1, β_2, or β_3 corresponding to the smallest difference is then selected. The elevation angles β_+ and β_- are then compared with the selected elevation angle. The one yielding the smaller difference is chosen as the new elevation angle i.e. $\beta_4=\beta_+$ or $\beta_4=\beta_-$. With β_4 we trace another ray and find D_4. If the difference between D_{TR} and D_4 is within the specified tolerance δD, then β_4 is the homing angle except for possible azimuthal corrections. However, if the difference between D_{TR} and D_4 is the largest difference then we cannot home the ray and the message 'Discontinuity in the ray traced range-beta curve or range too close to the skip distance' or 'High angle ray' is printed. On the other hand, if D_4 is closer to D_{TR} than any or all of D_1, D_2, D_3 then the ground distance D_1, D_2, or D_3 yielding the largest absolute difference with D_{TR} is replaced by D_4 and its corresponding elevation angle by β_4. The procedure starting with equation (2.21) is repeated to find a new elevation angle and a new ground distance. The comparison steps are carried out until the ground distance tolerance criterion is met or a message is printed. The number of allowable tries to find the homing angle is controlled

by an input parameter in W(386).

Once the ground distance tolerance criterion is satisfied a first order elevation angle that will home the ray to the receiver is said to be found except for possible azimuthal deviation correction. The azimuthal deviation is defined as the difference between azimuth of the computed receiving point from the transmitter and the azimuth of the intended receiving point from the transmitter. If the azimuthal deviation of the ray is within the calculated azimuthal tolerance, δA, the homing of the ray is completed. Otherwise, the azimuth of transmission has to be corrected in order to satisfy the azimuthal tolerance condition. The azimuthal tolerance, δA, is computed with the help of Figure 2.5. In Figure 2.5, we show the transmitter T, the receiver R, the computed receiver R', and the angles γ_1, γ_2, γ_3 subtended at the earth center opposite arcs TR, TR', and RR' or δD respectively. Applying the cosine law to the spherical triangle TRR' we obtain

$$\cos \gamma_3 = \cos \gamma_1 \cos \gamma_2 + \sin \gamma_1 \sin \gamma_2 \cos \delta A \tag{2.23}$$

If we let $\gamma_1=\gamma_2=D_{TR}/r_o$ and $\gamma_3=\delta D/r_o$, then

$$\cos \delta A = [\cos (\delta D/r_o) - \cos^2 (D_{TR}/r_o)] / \sin^2 (D_{TR}/r_o) \tag{2.24}$$

Since δD is much smaller than r_o, the argument of the cosine function, $\delta D/r_o$ is very small. Expanding this term and retaining two terms of the expansion, (2.24) takes the form

$$\cos \delta A \simeq 1 - \frac{1}{2} \left[\frac{\delta D}{r_o \sin (D_{TR}/r_o)} \right]^2 \tag{2.25}$$

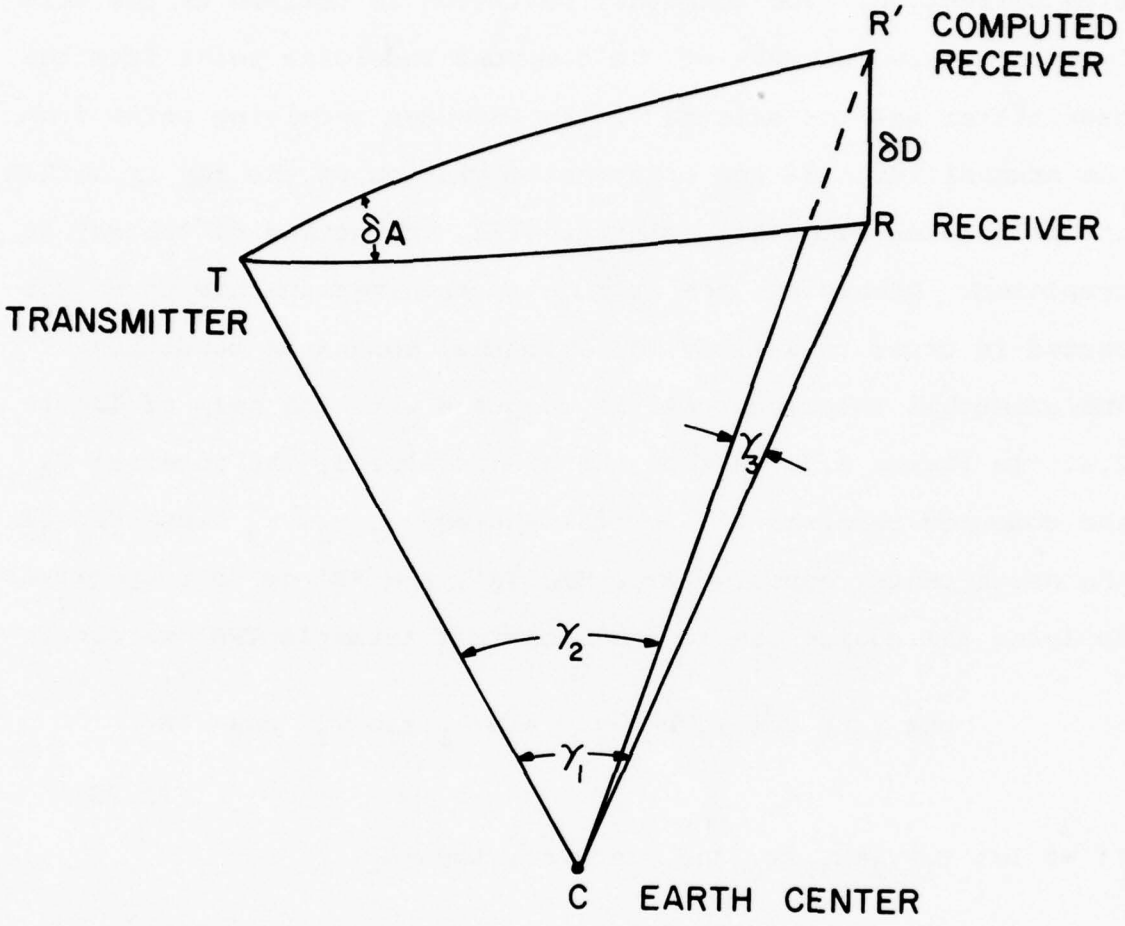

Figure 2.5. Sketch illustrating the homing in azimuth.

Since the azimuthal tolerance δA is also small, we expand the left hand side of (2.25) retaining two terms of the expansion. The resulting expression for δA is

$$\delta A \simeq \frac{\delta D}{r_o \sin (D_{TR}/r_o)} \tag{2.26}$$

For each ray homed in ground distance a comparison test is performed between the ray azimuth and the computed transmitter-receiver azimuth. If the ray azimuthal deviation is within the azimuthal tolerance δA then homing is achieved and control is transferred to another activity within the ray tracing program. However, if the ray azimuthal deviation is larger than δA we adjust the azimuth of transmission by the amount of the ray azimuthal deviation. With a new azimuth of transmission and the same elevation angle we repeat the complete procedure of ground distance homing and find a new homing angle. The ray azimuthal deviation corresponding to this new homing angle is then compared to the azimuthal tolerance, δA. The above steps are repeated until both ground distance tolerance and azimuthal tolerance criteria are satisfied or a message is printed.

2.2.5 Group Path Homing

The capabilities of the three-dimensional ray tracing program was extended to include homing of the group path. The procedure described in the ground-to-ground homing case is also used for the group path homing. The only difference being in the parameters supplied to the ray tracing program through the W-array data input.

The parameters needed to perform group path homing are the transmitter coordinates, the azimuthal angle of transmission, the oblique frequency of transmission and the value of the homed in group path. The homed in group path is supplied through W(394). In the event that group path homing is not desired, then W(394) should be set to zero. An example of group path homing is discussed in the next section.

2.2.6 Examples and Discussion

In the preceding section we discussed in detail the technique through which the ground-to-ground homing of the ray is achieved. In order to illustrate the homing procedure we present three examples. The first example is for a single layer ionosphere, the second example is for a two layered ionosphere, and the third example is for group path homing. In all cases the ray is traced through the Air Force supplied ionospheric model. This model gives the electron density profile on a geomagnetic longitude-geomagnetic colatitude grid spanning from -130°E to -110°E in geomagnetic longitude and 0° to 54° in geomagnetic colatitude. The increments in longitude, colatitude and height are 5°, 3° and 10 km respectively. The base height in all cases is 90 km extending upward to 600 km. Note that these increments are very coarse for accurate ray tracing even with careful interpolation routines. This causes some difficulty as discussed later.

Example 1. Single Layer Ionosphere

Let the transmitter and receiver geographic coordinates be located at (125°W, 78°N) and (185°W, 56°N) respectively. For this

configuration the ground distance D_{TR} = 3296.55 km and the azimuthal angle of transmission is 258.2 degrees. Since the ionospheric profile grid points are given in geomagnetic coordinates we transform the transmitter and receiver geographic coordinates to obtain colatitudes of 10.81658 and 40.06548 degrees and longitudes of 114.47052 and 128.20226 degrees east. Utilizing equations (2.4), (2.5), and (2.7) the geographic and geomagnetic coordinates of the midpoint are found to be (-169.81, 69.18) longitude and latitude, (-125.12, 25.33) longitude and colatitude respectively. A comparison between the geomagnetic coordinates of the midpoint and the given ionospheric profile results in the selection of the density profile located at (-125.0, 24.0). For an oblique frequency of transmission of 20 MHz equation (2.18) yields the penetration angle of 14.236 degrees. Then, the interval of elevation angles from zero to 14.236 degrees is divided into 50 intervals. For each of the 51 resulting elevation angles the ground distance and group path are calculated through equations (2.13), (2.14), (2.15) and (2.16) utilizing the density profile of the midpoint located at (-125.0, 24.0). The result of this calculation is shown in Figure 2.6. For the specified ground distance tolerance of 5 km, figure 6 shows that the approximate initial elevation angle was found to be 3.312 degrees.

Figure 2.7 shows the steps taken in the refinement procedure and the eventual homing of the ray. The top portion of Figure 2.7 displays the ground distance versus elevation angle homing steps. The figure shows that the ray is homed in ground distance in the 5th step. However, the bottom portion of the figure also shows

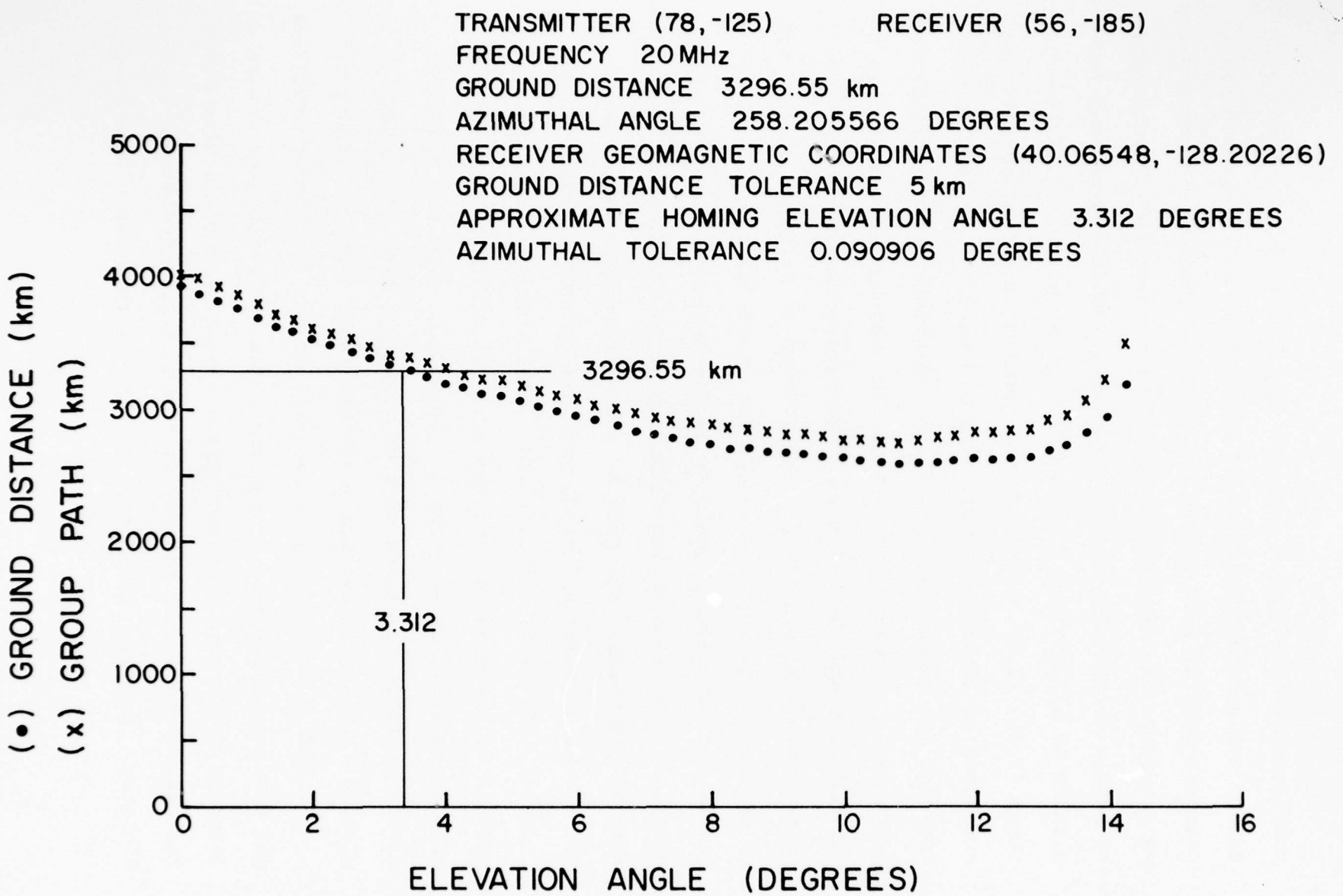

Figure 2.6. Example 1 illustrating the ground-to-ground homing using the Air Force supplied model ionosphere. The example shows how the initial elevation is determined.

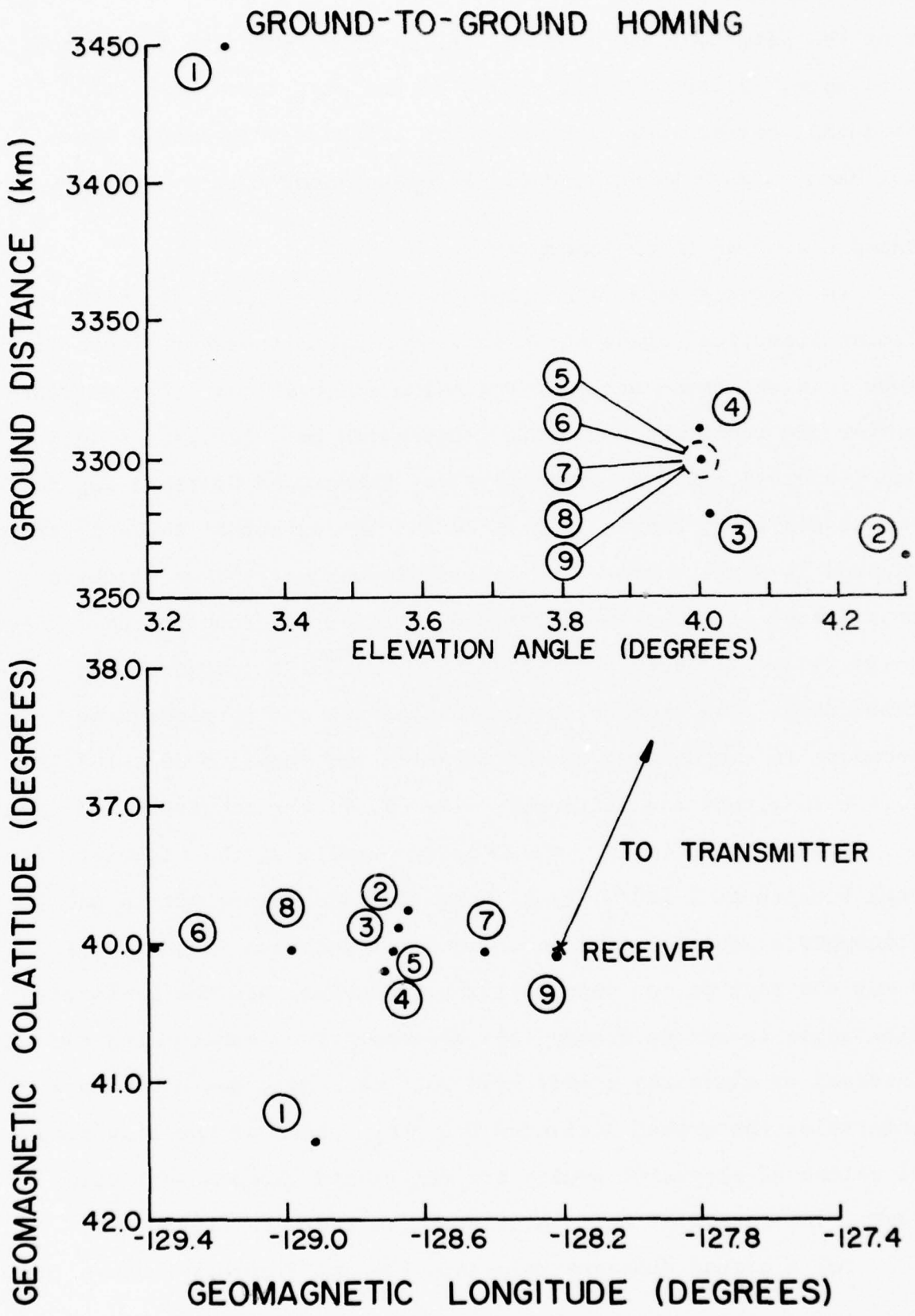

Figure 2.7. Continuation of the example shown in Fig. 2.6. This shows the refinement procedure in ground distance and azimuth homing using 3D ray tracing.

that the azimuthal angle of the ray is not within the calculated tolerance. Prior to final homing of the ray, there were four azimuthal corrections with no ground distance adjustments since for each azimuth the ground distance was homed in.

Example 2. Two Layer Ionosphere

In a single layer ionosphere there are at most two initial homing elevation angles while in a two layer ionosphere there are four initial homing angles. Following example 1 we let the transmitter and receiver geographic coordinates be (-175.0°E, 68.0°N) and (-150.0°E, 72.0°N) respectively. The ground distance D_{TR} for such a configuration is 1039.36 km and the azimuthal angle of transmission is 53.51 degrees. Upon transformation, the geomagnetic coordinates of the transmitter and receiver are found to be (-127.7972, 27.21986) and (-114.25255, 19.5317) longitude and colatitude. The geographic coordinates and the corresponding geomagnetic coordinates of the midpoint are found to be (-163.72, 70.43) longitude and latitude, (-122.09, 23.22) longitude and colatitude respectively. The density profile at the midpoint is then located at (-120.0, 24.0) longitude and colatitude in the ionospheric profile. For an oblique transmission frequency of 7 MHz the rays do not penetrate the ionosphere and the penetration angle is set to ninety (90) degrees. Upon subdividing the interval of elevation angles from zero to ninety degrees into 50 intervals, the ground distances and group paths of the resulting 51 values of elevation angles are calculated through subroutine FITT. The result of this calculation is shown in Figure 2.8.

For a ground distance tolerance δD=2 km Figure 2.8 shows that

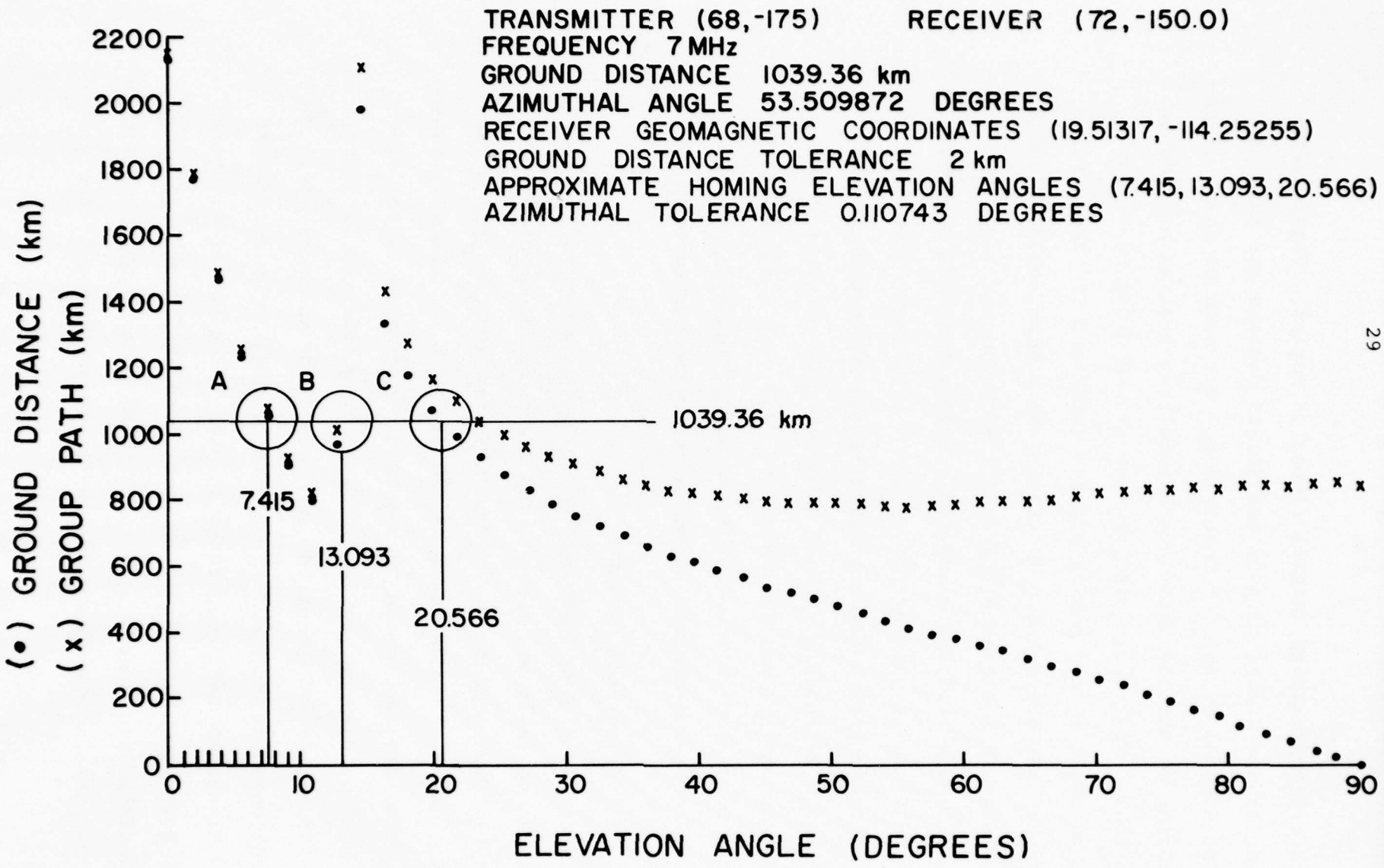

Figure 2.8. Example 2 illustrating the ground-to-ground homing for rays reflected from a two-layer ionosphere. Note that for a ground range of 1039.36 km, three approximate solutions (indicated by circle A, circle B and circle C) in the elevation angle exist in this case.

there are 3 approximate initial elevation angles. These elevation angles are 7.415, 13.093, and 20.566 degrees and they are designated by A, B, and C respectively. For each of these elevation angles we use the three dimensional ray tracing program to refine them. Figure 2.9 displays the ground distance versus elevation angle in the homing process for A and C. The circled numbers are the steps taken to obtain the homing angle. Homing for B cannot be achieved since it corresponds to a high angle ray. In Figure 2.10 the geomagnetic colatitude versus geomagnetic longitude is plotted for A and C. For case A the azimuthal angle of transmission does not require any correction and the homing angle is found by linear interpolation on steps 2 and 3 without recourse to matrix inversion. For case C the azimuthal angle is corrected twice in order to meet azimuthal tolerance criterion corresponding to ground distance tolerance of 2 km.

Example 3. Group Path Homing

Using the transmitter geographic coordinates (-150.0, 78.0) longitude and latitude, the oblique transmission frequency of 12 MHz, and the azimuthal angle of transmission of 209.4 degrees we calculated the coordinates of the midpoint and the penetration angle. The geographic and geomagnetic coordinates of the midpoint are found to be (-164.97, 66.86) longitude and latitude, and (-119.04, 26.59) longitude and colatitude respectively. The penetration angle is found to be 38.9 degrees. The calculated ground distances and group paths for the 51 values of elevation angles from zero to 38.9 degrees are shown in Figure 2.11. For the specified group path value of 2654.08 km in W(394) the figure shows two initial elevation angles for which homing of the group

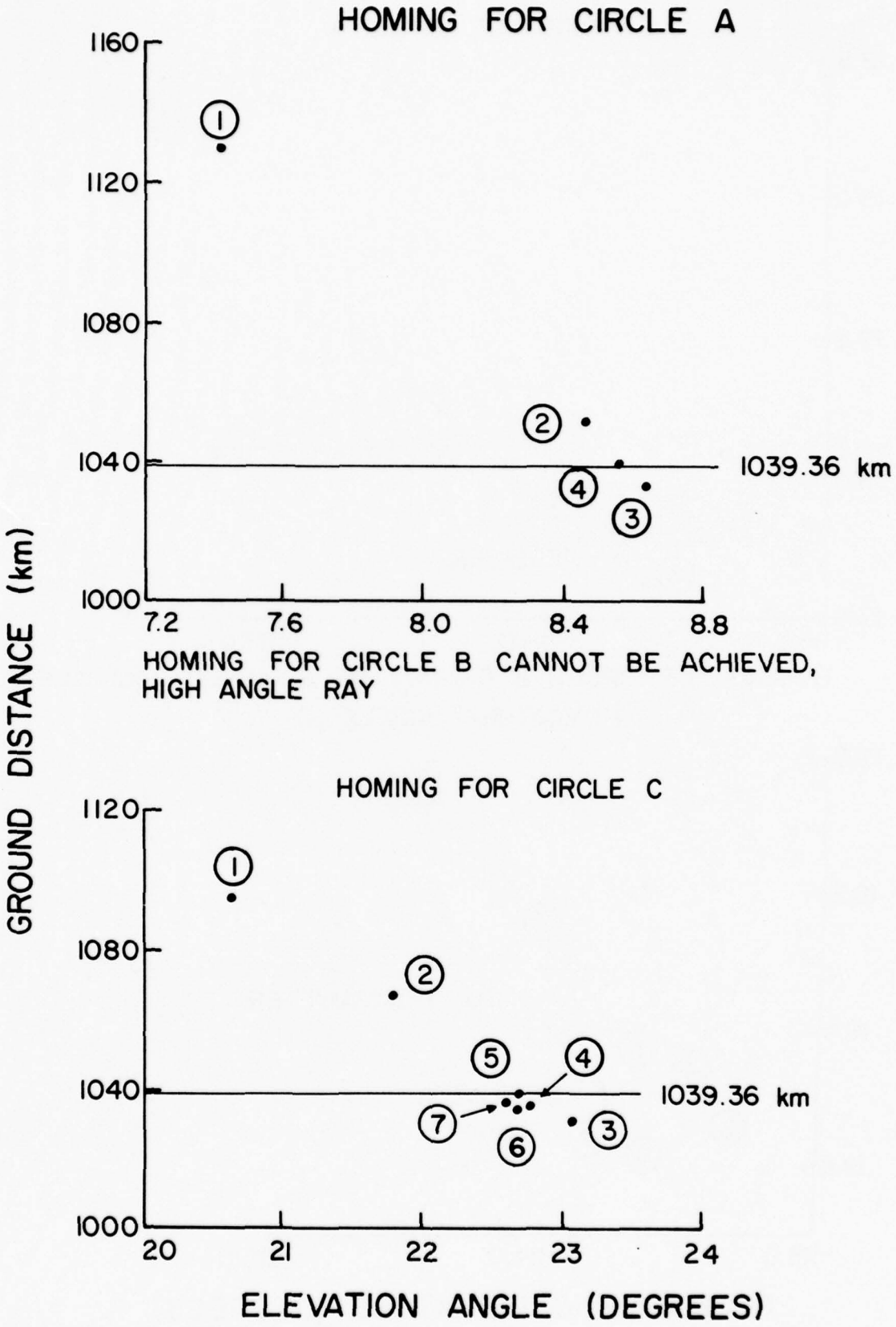

Figure 2.9. Continuation of Example 2 shown in Fig. 2.8 illustrating the ground distance homing for circle A and circle C using 3D ray tracing.

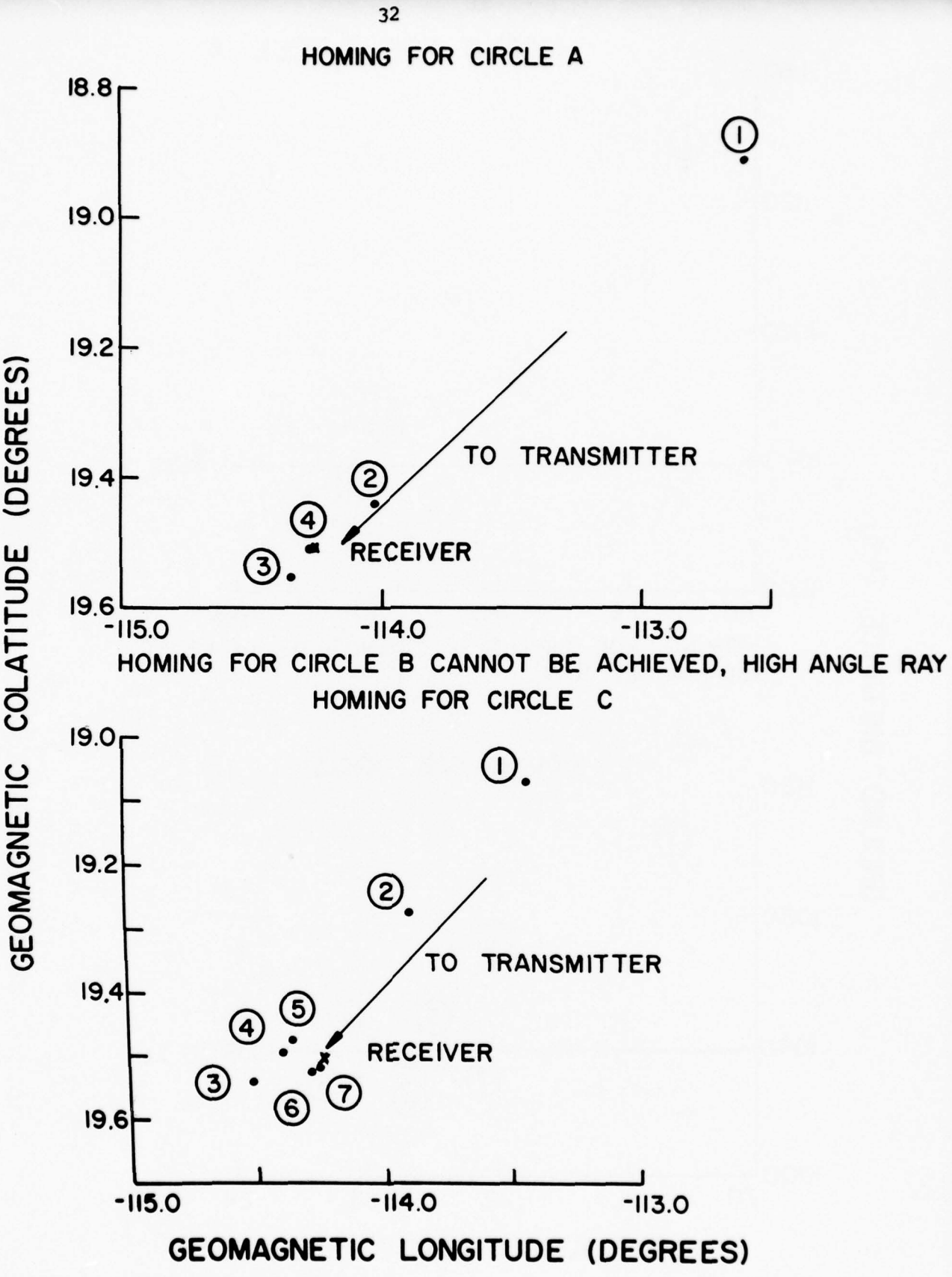

Figure 2.10. Continuation of Example 2 illustrating azimuthal correction for final homing of the ray on the receiver.

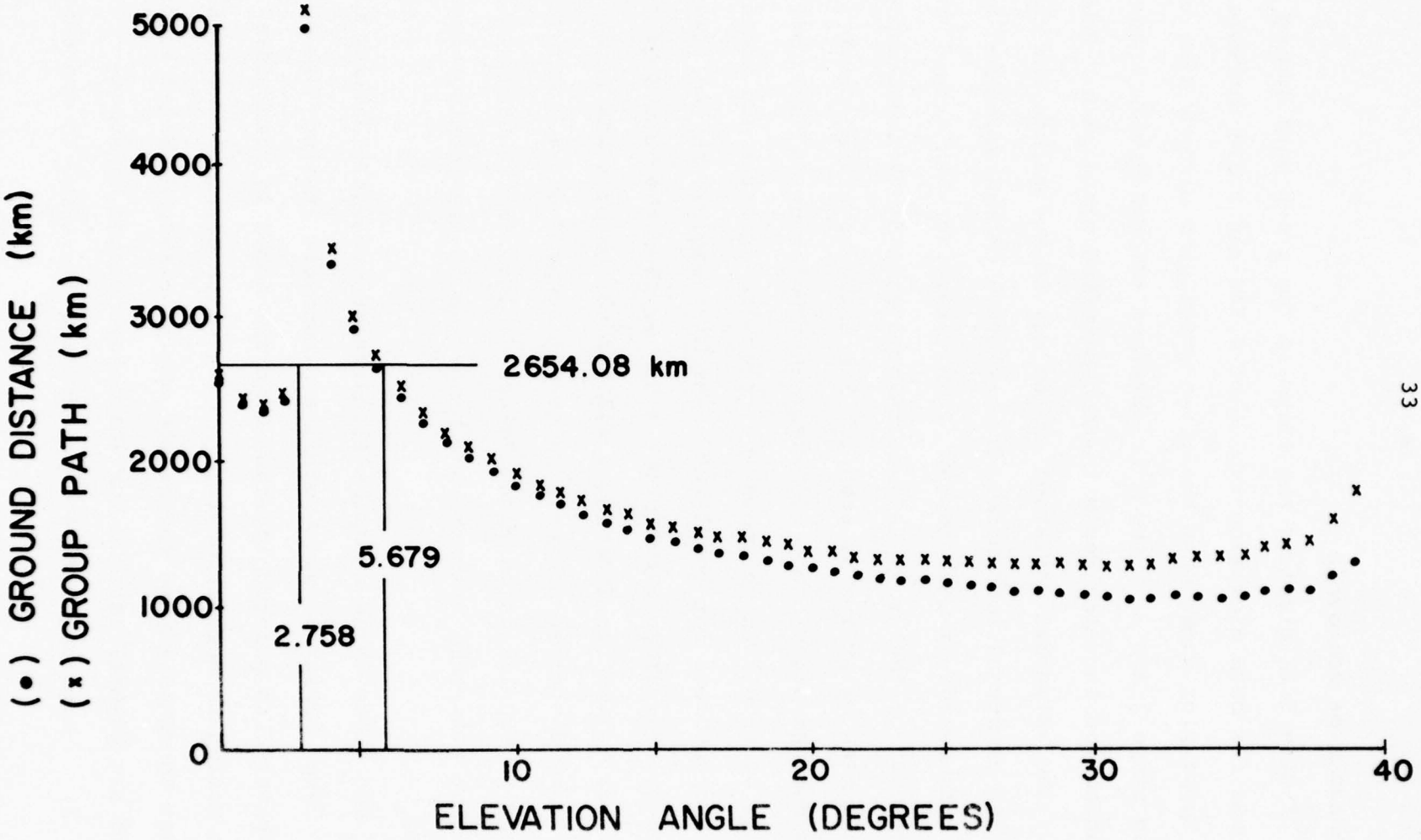

Figure 2.11. Computed initial group path versus elevation angle curve for Example 3. For a specified group path of 2654.08 km two solutions are indicated.

path could be achieved.

Figure 2.12 displays the steps in the group path homing process for both elevation angles of 2.758 and 5.679 degrees. The upper plot shows that the given group path already lies between steps 2 and 3. Step 4 is obtained through linear interpolation and it coincides with the specified group path. However, the azimuthal deviation of the ray is not within the calculated azimuthal tolerance. A correction on the azimuthal angle of transmission yields step 5. Then, starting with step 5 and using increments of the previously calculated ground distances and elevation angles we arrive at step 6. Since the specified group path lies between steps 5 and 6, linear interpolation on them yields values of step 7 which meets the tolerance criteria and homing is thus complete for the first layer high angle. In the lower graph the specified group path is higher than the values of steps 1, 2, 3, 4, and 5. Therefore, all the values of steps 1, 2, 4, and 5 are discarded and only values of steps 3 and 6 are retained. Linear interpolation on these values yields step 7 which satisfies the tolerance criteria and hence the desired homing elevation angle.

The above three examples illustrated successful homing of the ray. When homing is successful, a diagnostic stating that homing is achieved is printed together with the homed in parameters. When homing is not successful, a diagnostic stating that homing cannot be achieved and the reason why it cannot be homed is printed. There are several reasons for which rays cannot be homed.

i) high angle rays which are too sensitive to ionospheric

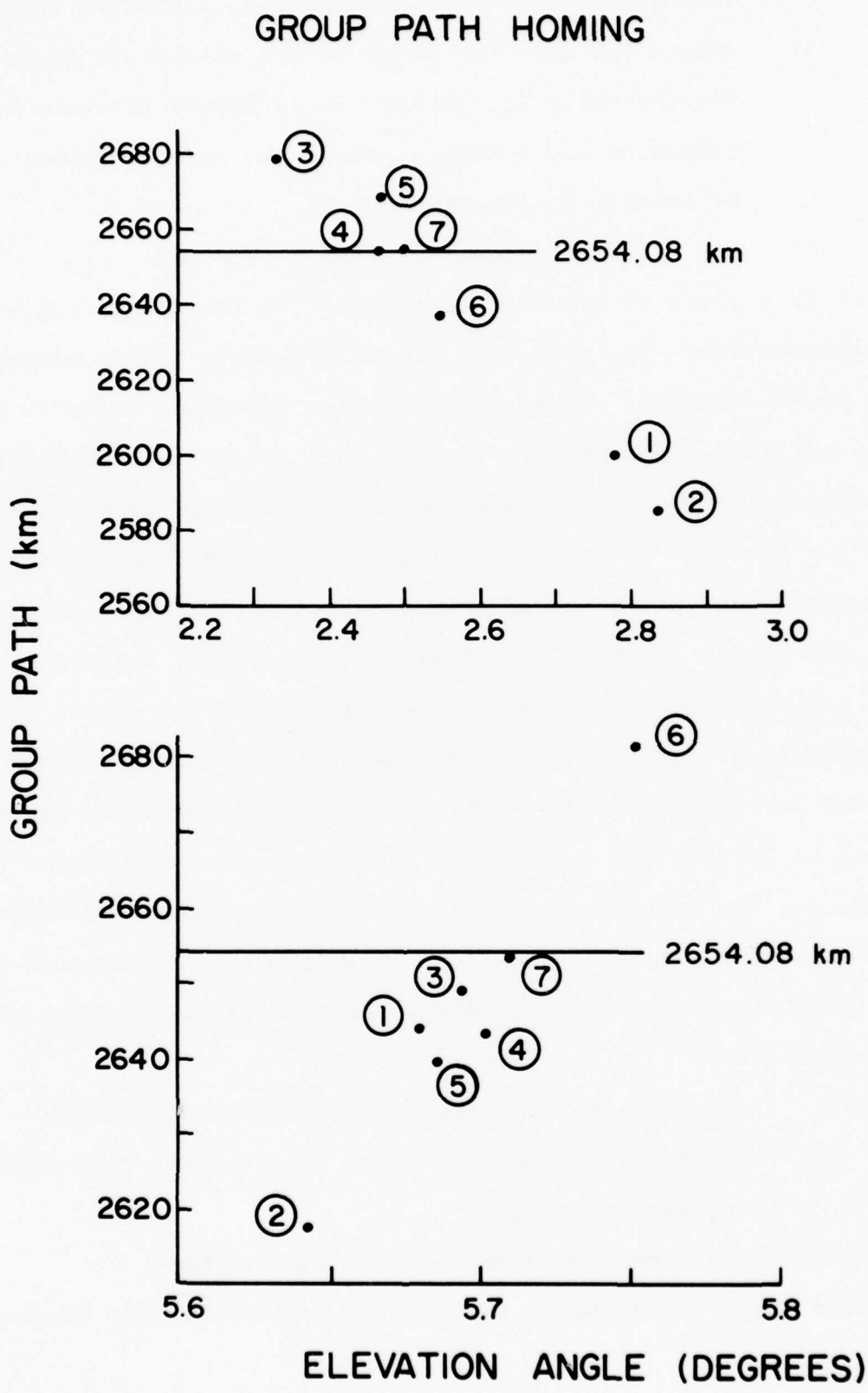

Figure 2.12. Continuation of Example 3 illustrating group path homing for two rays.

parameters and small increments in elevation angles.

ii) ground distance too close to the skip distance or discontinuity in the ray traced ground distance versus elevation angle curve. These two reasons cannot be separately distinguished.

iii) the specified number of trials exceeded.

Up to this point we are only concerned with the single hop mode of propagation. The extension of the ground-to-ground homing to the two-hop mode of propagation has been tried and found to be only partially successful. The two hops are assumed to be similar or mixed. Initially, the two hops are considered to be similar and the two sets of ground distance versus elevation angle curves corresponding to the ionospheric profiles at the two midpoints are generated. Then, the initial elevation angle required in the homing procedure is found by searching the two curves for an elevation angle for which the sum of the two ground distances is the desired ground distance. This initial elevation angle if found is used in the same manner as for the single hop case to carry out the homing. Utilizing the above technique we had success in homing for a few cases while we encountered difficulties in the majority of the cases studied. These difficulties may be attributed to

i) the coarseness of the given ionospheric profile

ii) the interpolation procedure in the three-dimensional ray tracing program

iii) the technique developed for the two hop.

Further study is necessary in order to achieve two-hop homing.

2.3 Ground-Satellite Homing

When the transmission frequency is much higher than the critical frequency of the given ionospheric profile the ray penetrates the ionosphere. The geometries of the two possible configurations are shown in Figures 2.13 and 2.14. In Figure 2.13 the transmitter is below the receiver and the configuration is called ground-to-satellite homing. The satellite-to-ground homing configuration is shown in Figure 2.14. In either case our objective is to home the ray to the receiver within a specified ground distance tolerance and a computed azimuthal tolerance. The following discussion is applicable to either configuration.

2.3.1 Approximate Elevation Angle

We assume that the transmitter and receiver coordinates are given in east longitude and latitude north in degrees and their heights above the earth surface in km. Then, to a first approximation we calculate the initial or approximate elevation angle for a straight line path between the transmitter and receiver.

Let θ_T, θ_R, ϕ_T, ϕ_R be the transmitter and receiver colatitudes and longitudes respectively. Compute the cartesian coordinates of the transmitter and receiver through the transformation

$$\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta \end{aligned} \qquad (2.27)$$

where $r=r_o+h$ and h is the height of the transmitter or receiver above the earth surface. From Figures 2.13 and 2.14, the vectors

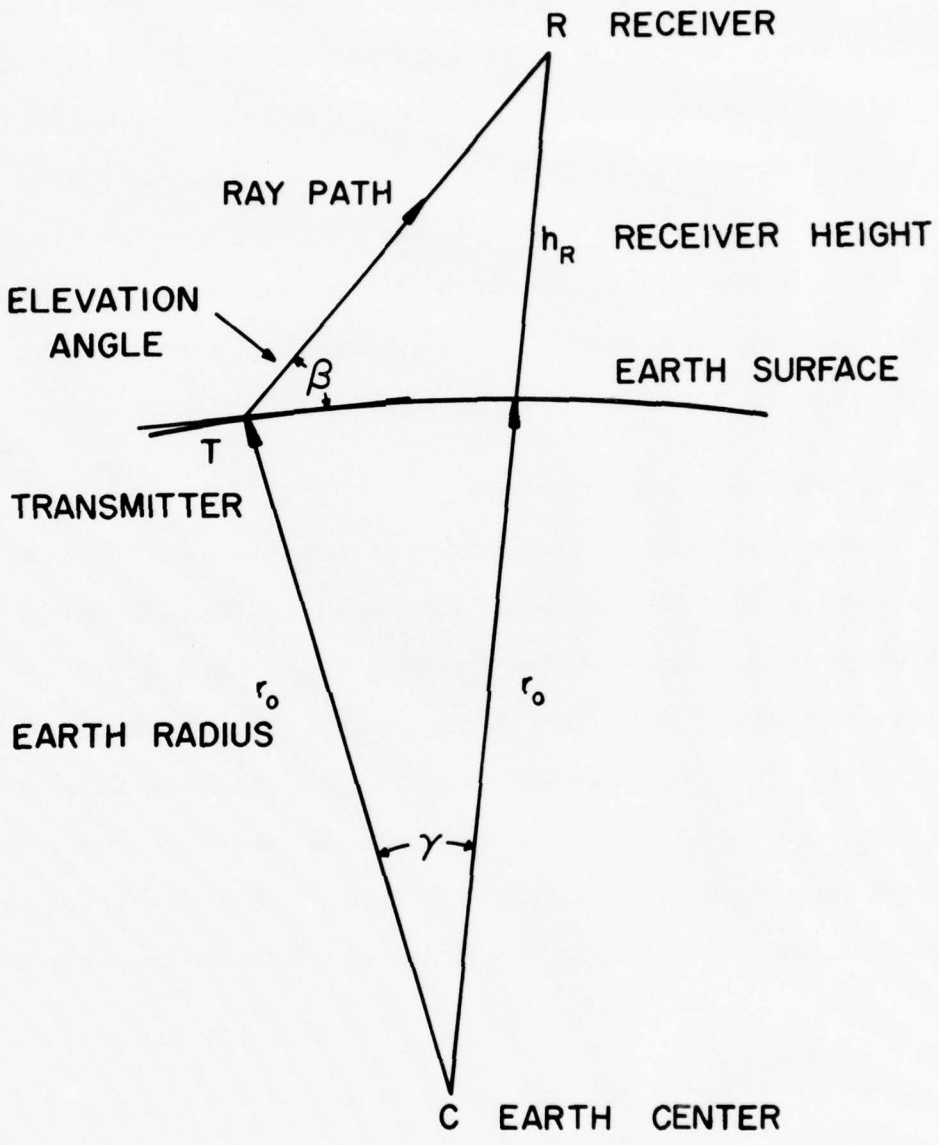

Figure 2.13. Sketch showing the geometry of ground-satellite homing.

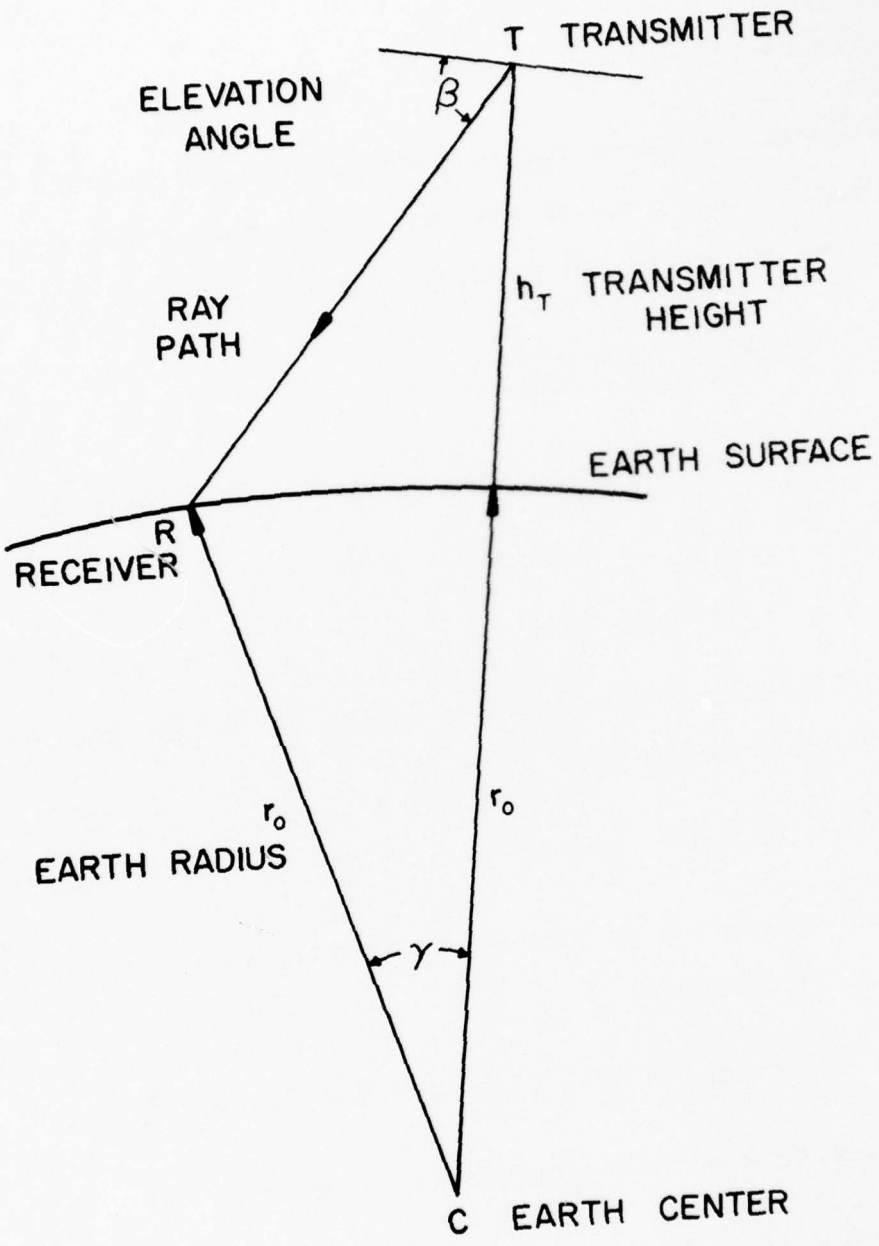

Figure 2.14. Sketch showing the geometry of satellite-ground homing.

$\vec{CT}$ and $\vec{CR}$ are given by

$$\vec{CT} = x_T\hat{i}+y_T\hat{j}+z_T\hat{k} \tag{2.28}$$

$$\vec{CR} = x_R\hat{i}+y_R\hat{j}+z_R\hat{k}$$

Then the ground distance D_{TR} between the transmitter and receiver can be found either through the use of the cosine law equation (2.3) or by taking the dot product of the vectors in (2.28), i.e.

$$\gamma = \cos^{-1}\left[\frac{\vec{CT}\cdot\vec{CR}}{|\vec{CT}|\ |\vec{CR}|}\right] \tag{2.29}$$

The ground distance D_{TR} projected at the transmitter radius is given by

$$D_{TR} = (r_o+h_T)\gamma \tag{2.30}$$

In order to find the elevation angle, we define the straight line path by the vector $\vec{TR}$,

$$\vec{TR} = \vec{CR} - \vec{CT} \tag{2.31}$$

Applying the cosine law of plane trigonometry to triangle CTR, angle CTR is found to be

$$\measuredangle\ CTR = \cos^{-1}[(|\vec{TR}|^2+|\vec{CT}|^2)-|\vec{CR}|^2)/2|\vec{TR}||\vec{CT}|] \tag{2.32}$$

and the approximate elevation angle is given by

$$\beta = \measuredangle\ CTR - \pi/2 \tag{2.33}$$

In the ground-to-satellite case β is positive while in the satellite-to-ground case β is negative. The above procedure is pro-

grammed in subroutine HOMES.

2.3.2 Refinement of the Elevation Angle

In section 2.2.3 we presented in detail the steps used to refine the approximate elevation angle for the ground-to-ground configuration. The refinement of the elevation angle for both the ground-to-satellite and satellite-to-ground is basically the same as that of the ground-to-ground case except for the initial elevation angle increment. The computer code of ADJUST is used for all three configurations. The initial elevation angle increment is only used once upon entry into routine ADJUST.

Starting with the known value $\beta_1=\beta$ of the approximate straight line path between the transmitter and receiver we trace one ray and obtain the ground distance D_1. In order to find a new elevation angle β_2 we assume a plane earth geometry and consider the equivalent triangle TRA in Figure 2.15. In this figure, T is the transmitter, R is the equivalent receiver, which, when a perpendicular line is dropped on RC from T, is a distance D km away and which is h km above the foot of the perpendicular point A as shown in Figure 2.15. From triangle TRA, the approximate elevation angle $\tilde{\beta}$ is given by

$$\tan \tilde{\beta} = h/D \tag{2.34}$$

Differentiating both sides of the equation and solving for the approximate initial increment in $\tilde{\beta}$ we obtain the relation

$$\delta\tilde{\beta} \simeq \sin \tilde{\beta} \cos \tilde{\beta} \, \delta D/D \tag{2.35}$$

where $\beta=\beta_1$, $\delta D=D_{TR}-D_1$, and $D=D_{TR}$. Then, with an elevation angle

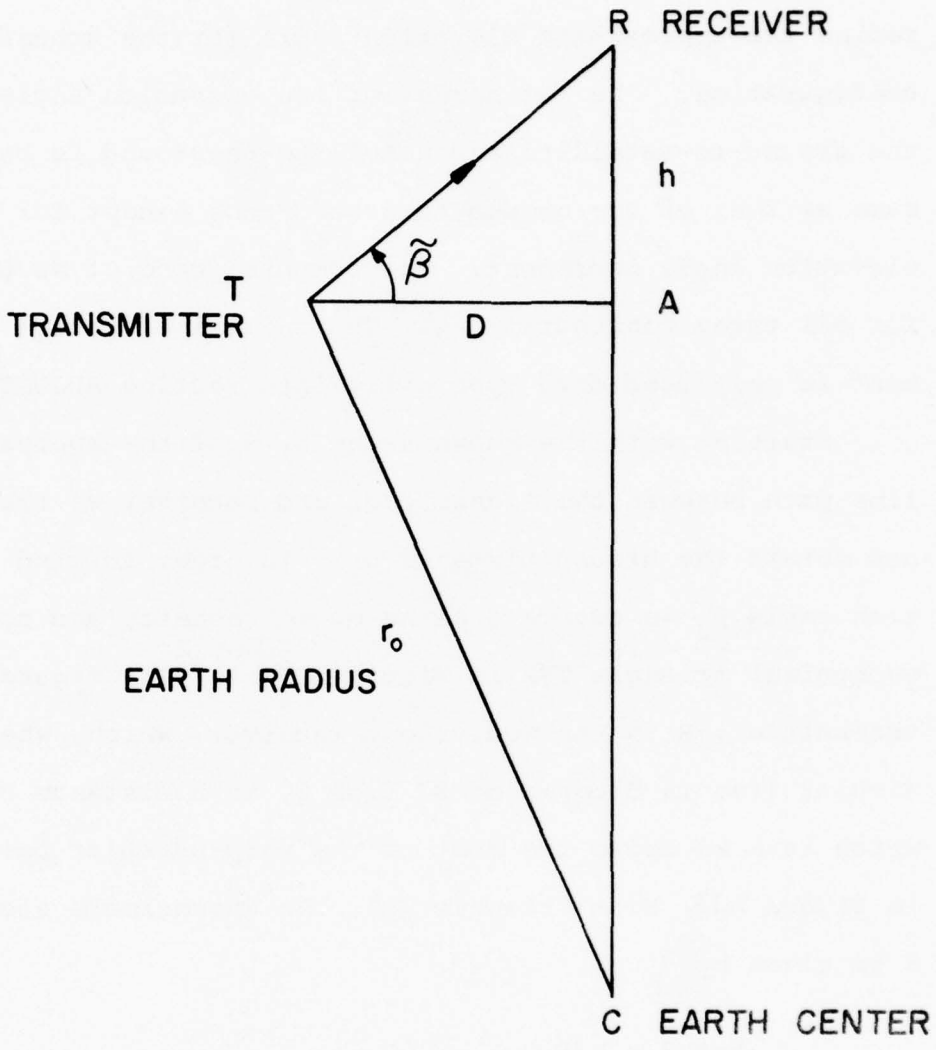

Figure 2.15. Earth geometry for finding the optical elevation angle.

$\beta_2=\beta_1+\tilde{\delta\beta}$ we proceed in the same manner as that described in section 2.2.3 to find the true homing elevation angle.

2.3.3 Examples and Discussion

In order to illustrate our homing technique we present two examples, one for ground-to-satellite and the other for the satellite-to-ground case. In both examples we studied the homing technique for the different oblique frequencies. The initial elevation angle is independent of the frequency of transmission. Since the straight line path approximation approaches the true path for high frequencies we expect the initial elevation angle to be very close to the true homing elevation angle.

Example 1. Ground-to-satellite.

Let the transmitter and receiver geographic coordinates be (-165.0, 72.0) and (-170.0, 56.0) longitude and latitude respectively. Also, let the transmitter and receiver heights above the earth surface be 0.0 and 560.0 km respectively. Then, for such a configuration the ground distance at the transmitter radius is 1793.97 km, the azimuthal angle of transmission is 190.1 degrees, and the initial approximate elevation angle is 8.475. The oblique transmission frequencies considered for this example were 90, 170, and 250 MHz. For each of these frequencies and starting with an elevation angle of 8.475 degrees we utilize the ray tracing program through subroutine ADJUST. The result of the calculation is presented in Figure 2.16. From Figure 2.16 we note that as the frequency gets higher the number of steps needed to achieve homing decreases. We also note that neither matrix

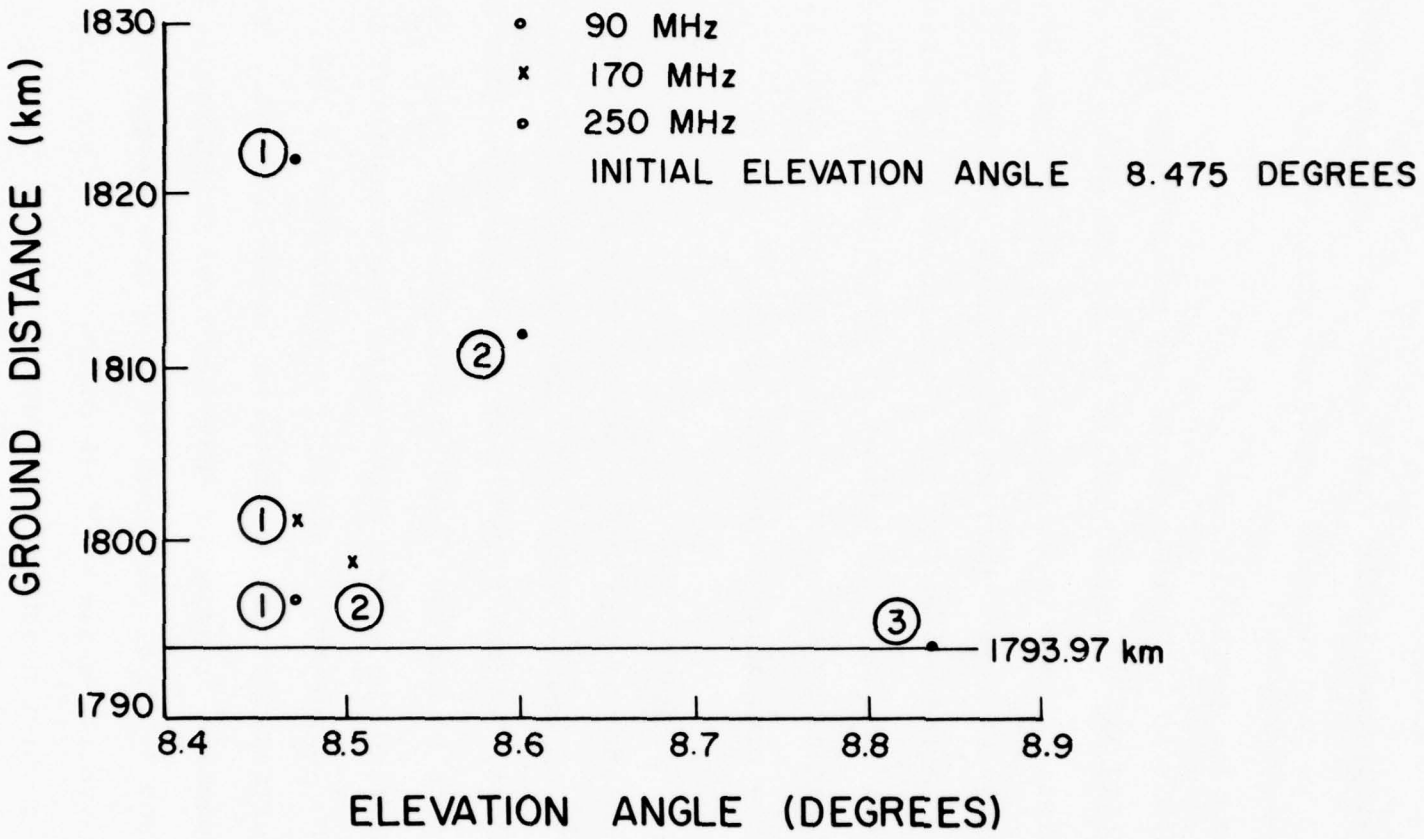

Figure 2.16. Example illustrating ground-to-satellite homing. The plot shows ground distance between the transmitter and the sub-satellite point versus the elevation angle for successive iterations using 3D ray tracing at three frequencies 90 MHz, 170 MHz and 250 MHz.

inversion nor linear interpolation is needed to achieve homing for all three frequencies. In Figure 2.17 the geomagnetic co-latitude is plotted versus the geomagnetic longitude to show the homing steps for the three frequencies. For all frequencies the figure shows that no azimuthal correction is needed.

Example 2. Satellite-to-ground

For this example we let the transmitter and receiver be located at (-150.0, 78.0) and (-165.0, 60.0) longitude and latitude respectively. We also let the transmitter and receiver heights above the earth surface be at 550 and 0.0 km. For these geographic coordinates the ground distance at the transmitter radius is 2251.96 km, the azimuthal angle of transmission is 203.87 degrees, and the initial approximate elevation angle is -23.472 degrees. For each of the oblique frequencies 70, 140, and 210 MHz we used routine ADJUST to find the homing elevation angle. Figure 2.18 shows the ground distance versus the elevation for all frequencies together with the number of steps needed to achieve homing. For the 70 and 140 MHz it takes 4 steps while for 210 MHz only 3 steps are needed. One linear interpolation and one matrix inversion are required for the 70 and 140 MHz while one linear interpolation is carried out for 210 MHz. Figure 2.19 displays the geomagnetic colatitude versus geomagnetic longitude for the three frequencies. This figure shows the number of steps along the azimuth of transmission for each frequency. Homing for all frequencies is achieved without azimuthal corrections.

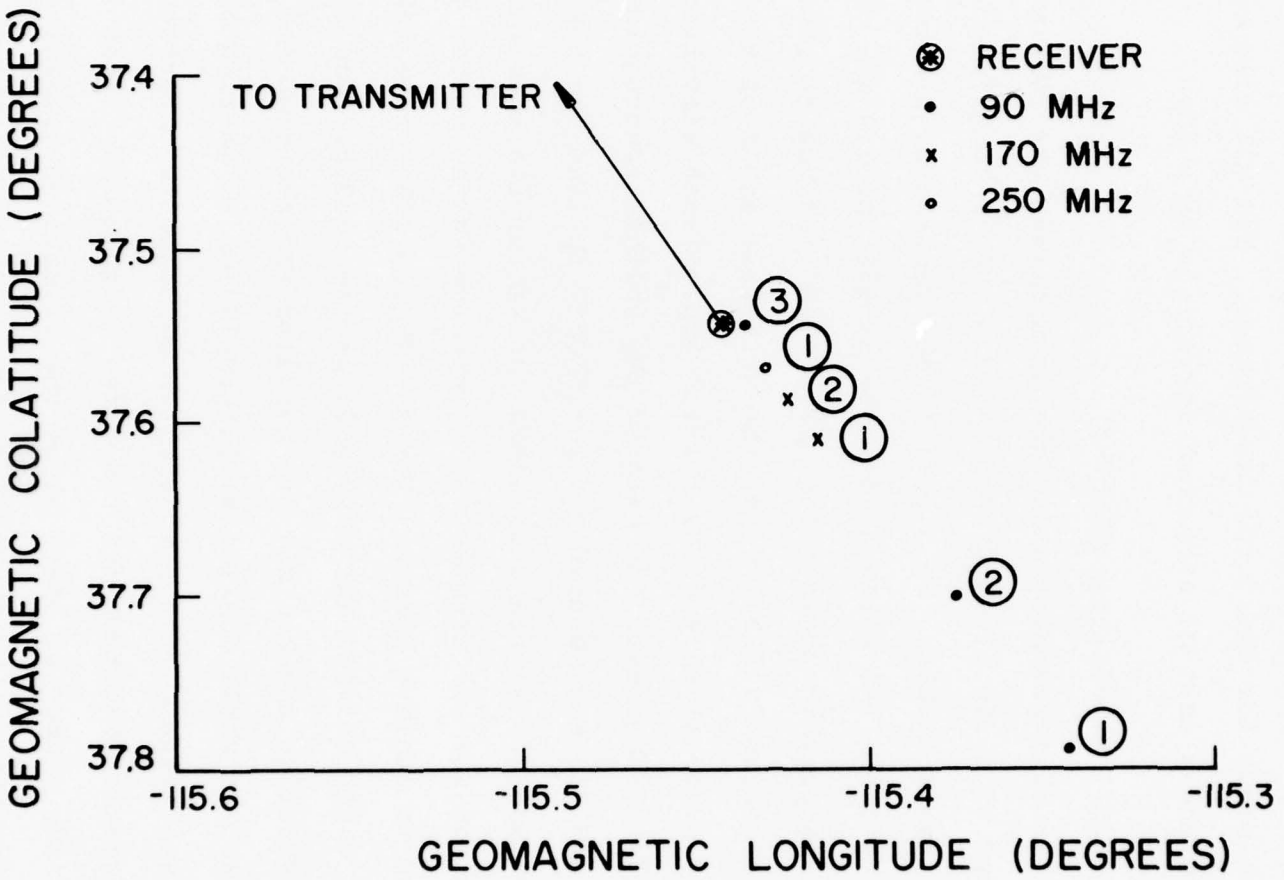

Figure 2.17. Continuation of example shown in Fig. 2.16 illustrating that no azimuthal correction is needed for homing at all three frequencies.

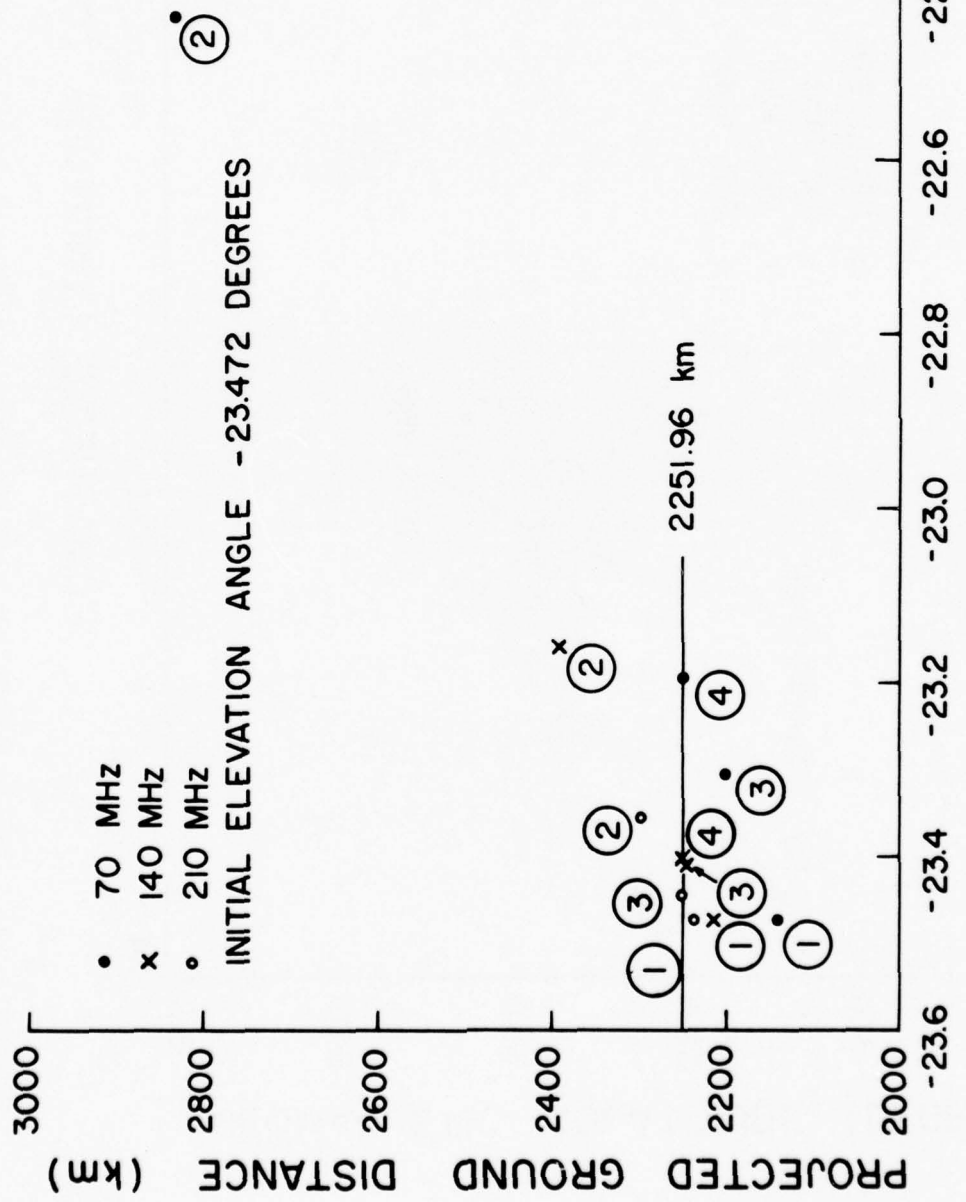

Figure 2.18. Example illustrating satellite-to-ground homing for three radio frequencies 70 MHz, 140 MHz, and 210 MHz.

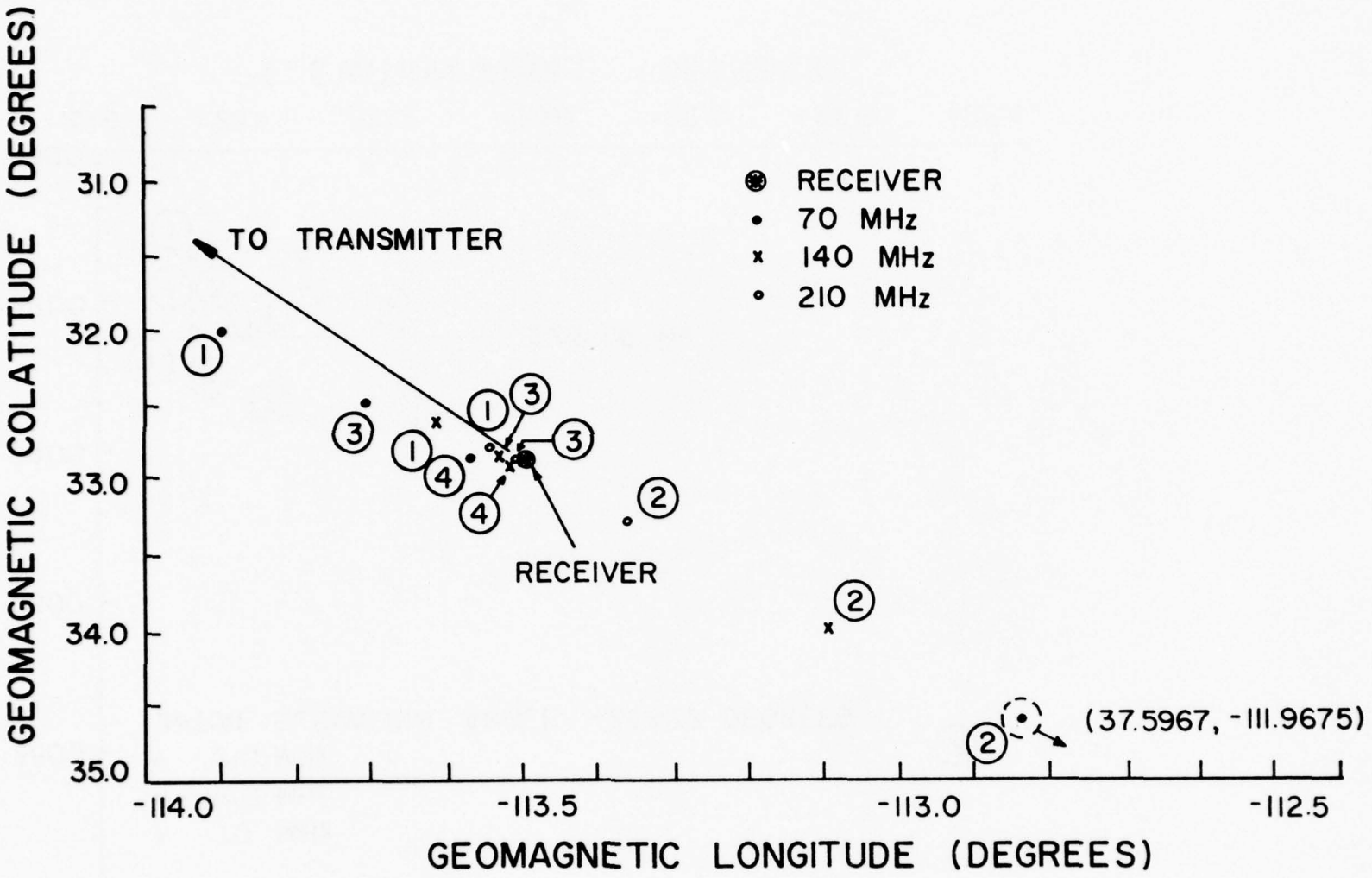

Figure 2.19. Continuation of example shows in Fig. 2.18 illustrating that no azimuthal correction is needed for homing at all three frequencies.

2.4 Minimum Group Path

The technique of backscatter ionogram inversion utilizes the values of the minimum group path computed for the given ionospheric profile. Therefore, it is necessary to extend the ray tracing program to include the numerical computation of the minimum group path. The complete method is documented in subroutine GROUP M.

2.4.1 The Chopping Method

We assume that the transmitter location, the ionospheric model, the oblique transmission frequency, and the azimuthal angle are given. In addition, the minimum group path tolerance $\delta P'$ is also specified. At the present, the minimum group path tolerance is internally specified with a value of 1 km. With the known transmitter coordinates we can select the closest ionospheric model and find an approximate penetration angle β_p. This approximate angle is found through equation (18). Utilizing the ray tracing program and the given ionosphere we correct the approximate penetration angle until β_1 is just below the penetration condition. Since β_p is approximate we could encounter two main possibilities; either β_p is larger or smaller than the true penetration angle. Very seldom β_p will be equal to the true penetration angle and this situation is included in the cases where β_p is larger or smaller. For the case where β_p is larger than the true penetration angle we successively decrement β_p by one percent i.e. $\beta_{p_i}=0.99\beta_{p_{i-1}}$, i=1,2,3,... , and obtain β_1 just below β_p. While in the case where β_p is smaller we successively increment β_p by five percent i.e. $\beta_{p_i}=1.05\beta_{p_{i-1}}$, i=1,2,3,... until the

ray penetrates the ionosphere and then we successively decrement β_p by one percent and obtain β_1 just below β_p.

Associated with β_1 is a group path P_1'. In order to locate the minimum group path we need three points along the group path versus elevation angle curve. The general behavior of this curve is shown in Figure 2.20. Define β_2 and β_3 as ninety five (95) and ninety (90) percent of β_1 and generate their corresponding group path values P_2' and P_3' through ray tracing. A comparison test is then performed on P_1', P_2', and P_3'. If $P_1'>P_2'>P_3'$ as in Figure 2.21a we discard β_1 and P_1', shift indices backward and calculate a new β_3 equal to ninety five (95) percent of the old β_3. With the new β_3 we generate a new group path P_3'. On the other hand, if $P_1'<P_2'<P_3'$ as in Figure 2.21b we discard β_3 and P_3', shift indices forward and calculate a new β_1 equal to 105 percent of the old β_1. With this new β_1 we generate a new group path P_1'. In either case we repeat the comparison tests on P_1', P_2', and P_3' until the condition $P_1'>P_2'$ and $P_3'>P_2'$ is satisfied.

Let us assume that we finally arrive at the situation depicted in Figure 2.21c where the condition $P'>P_2'$ and $P_3'>P_2'$ is satisfied. Then, in order to decide whether the minimum group path has been found or not we consider the following steps;

Step 1. Utilizing the chopping technique we take β_4 and β_5 at the mid point of the intervals β_1, β_2 and β_2, β_3 respectively. Tracing two rays with β_4 and β_5 we obtain two group paths P_4' and P_5'.

Step 2. If $|P_5'-P_2'|>\delta P'$ then we transfer to actions taken in step 3; otherwise, we have arrived at the minimum group path and a decision is made to select the

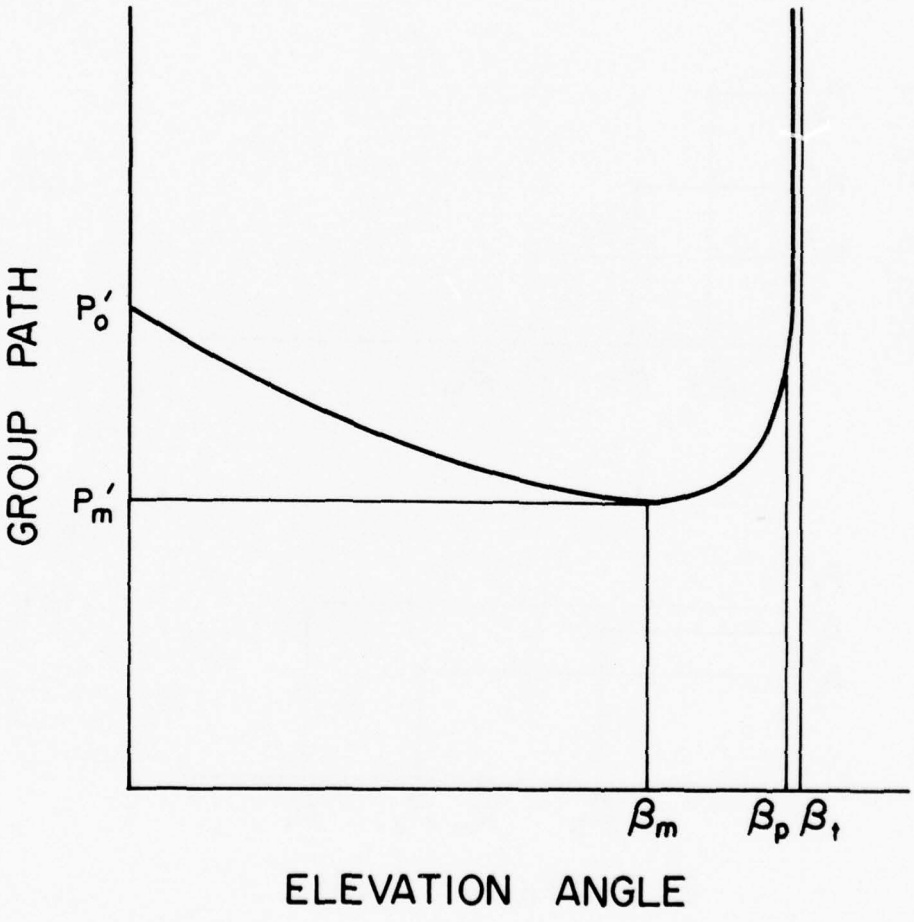

Figure 2.20. Sketch showing the general behavior of group path versus elevation angle curve.

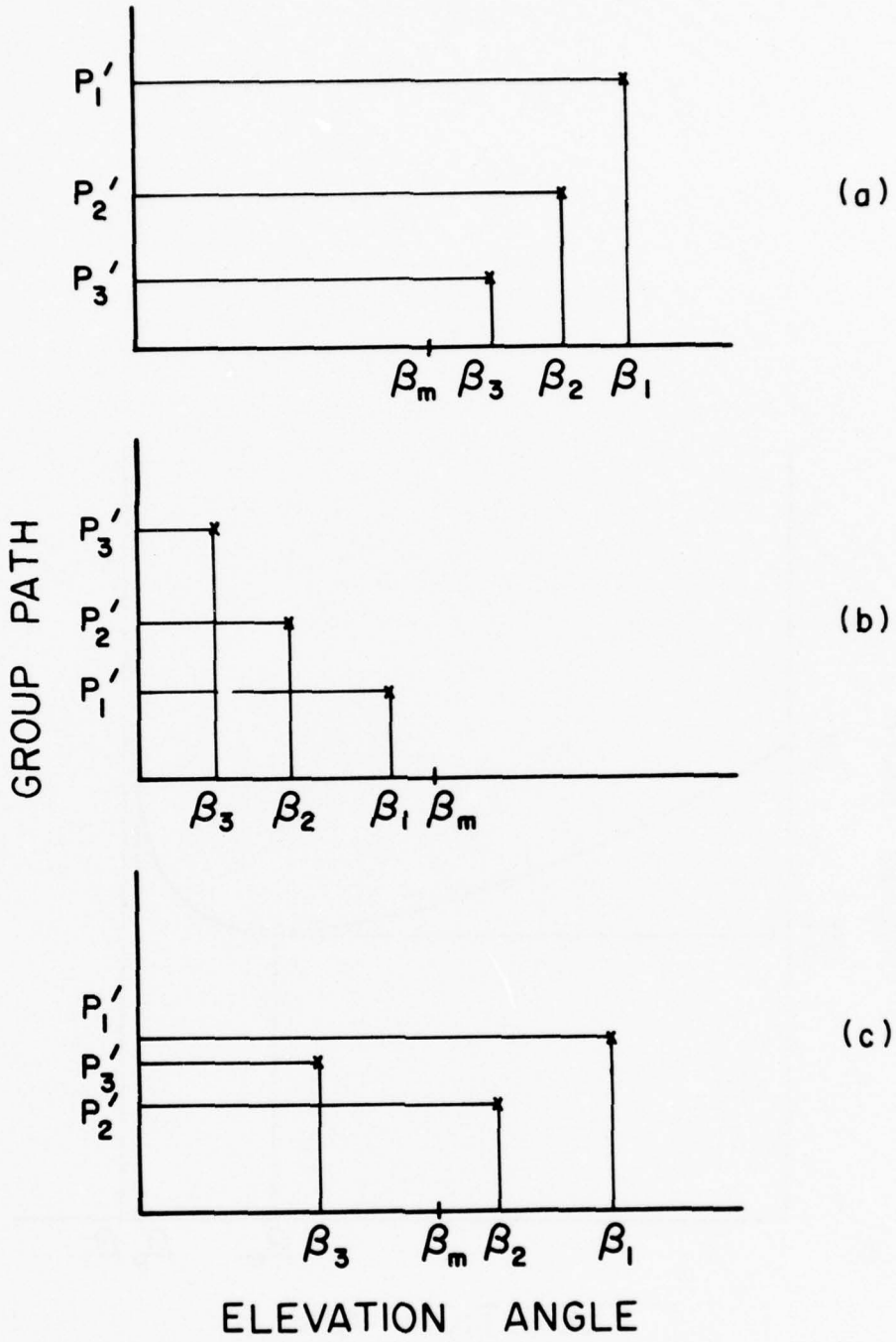

Figure 2.21. Sketch illustrating three possibilities for purpose of finding the minimum group path ray.

smallest of P_2', P_4', and P_5'.

Step 3. If $P_5'>P_2'$ then we concentrate on actions taken in step 4. For $P_5'<P_2'$, the interval given by $P_3'>P_5'$ and $P_2'>P_5'$ is our main concern. In this interval we replace the values of β_1, β_2, P_1', P_2' by the values β_2, β_5, P_2', P_5' respectively and start over from step 1.

Step 4. If $|P_4'-P_2'|>\delta P'$ then we transfer to actions taken in step 5. Otherwise, we have arrived at the minimum group path and a decision is made to select the smaller of P_2' and P_4'.

Step 5. If $P_4'>P_2'$ then we concentrate on the interval given by $P_5'>P_2'$ and $P_4'>P_2'$ and replace the values of β_1, β_3, P_1', P_3' by the values β_5, β_4, P_5', P_4' respectively. Otherwise, our main concern is the interval given by $P_2'>P_4'$ and $P_1'>P_4'$. In this interval we replace the values β_2, β_3, P_2', P_3' by the values β_4, β_2, P_4', P_2' respectively. In either case we transfer to step 1 and start over again.

Once the minimum group path is found, control is transferred to the main program to perform a prescheduled acitivity in accordance with the input data through the W array.

2.4.2 Examples

In order to illustrate the technique of finding the minimum group path we present two examples. Both examples are derived from the same transmitter location at -175 and 40 degrees geographic longitude and latitude respectively. In the first example the azimuthal angle of transmission is 330 degrees and the oblique

frequency of transmission is 12 MHz, while in the second example the azimuthal angle and the oblique frequency are 308 degrees and 10 MHz respectively.

Example 1.

For an oblique frequency of 12 MHz the penetration angle from equation (2.18) is found to be 52.91 degrees. Starting with β_1=52.9 we generate the group path P'_1 through ray tracing. Following the discussion in section 2.4.1 we take 95 and 90 percent of β_1 and define them as β_2 and β_3. The values of β_2 and β_3 then are given by 50.26 and 47.62 degrees. The corresponding group path values P'_2 and P'_3 are found to be 1077.37 and 1022.81 km. Since $P'_1>P'_2>P'_3$ we discard β_1 and P'_1 and shift the indices backward to obtain β_1=50.26, P'_1=1077.37, β_2=47.62 and P'_2=1022.81. Taking 95% of 47.62 degrees we generate β_3=45.24 and its corresponding group path P'_3=995.86. From the values thus known we still have $P'_1>P'_2>P'_3$. Discarding β_1 and P'_1, taking 95% of 45.24 degrees we obtain the following three sets of values, β_1=47.62, P'_1=1022.81, β_2=45.24, P'_2=995.86, β_3=42.98, P'_3=981.11. Since the condition $P'_1>P'_2>P'_3$ is still satisifed we carry out the same operations as above and arrive at the values β_1=45.25, P'_1=995.86, β_2=42.98, P'_2=981.11, β_3=40.83, P'_3=973.56. Comparison of the group path values reveal that $P'_1>P'_2>P'_3$ is the prevailing condition. Therefore, the next set of values are β_1=42.98, P'_2=981.11, β_2=40.83, P'_2=973.56, β_3=38.79, P'_3=973.17. Since the condition $P'_1>P'_2>P'_3$ is satisifed we carry out the calculations once more and arrive at the situation given by the values β_1=40.83, P'_1=973.56, β_2=38.79, P'_2=973.17, β_3=36.85, P'_3=976.87. An examination

of the group path values shows that the condition $P'_1 > P'_2$ and $P'_3 > P'_2$ is satisfied. Following the chopping method steps we find β_4 and β_5 at the midpoint of the intervals β_1, β_2, and β_2, β_3, respectively. With the values of $\beta_4 = 39.81$ and $\beta_5 = 37.82$ we find the group path values $P'_4 = 972.60$ and $P'_5 = 974.07$. Going to step 2 of the chopping method we find that $|P'_5 - P'_2| > \delta P' = 1$. Therefore, we find the minimum group path, it is either P'_2, P'_4 or P'_5 whichever is the smallest. In this case the minimum group path is given by $P'_4 = 972.60$ and the elevation angle is $\beta_4 = 39.81$. Figure 2.22 shows the group path versus elevation angle for the values generated under this example. The test sequence and indices shifting is also displayed on the graph.

Example 2.

In this example we use an azimuthal angle of 308 degrees and an oblique frequency of 10 MHz. The result of the calculation, testing and the value of the minimum group path are displayed in Figure 2.23. The circled numbers beside each point is the sequence calculation number. The table in the figure presents the sequence of operations performed before converging on the minimum group path value. In this example also the chopping technique is applied only once to achieve the final result.

2.5 Computer Programs

During the course of the study we augmented the 3D ray tracing program with six subroutines. They are discussed in the following.

2.5.1 General Description

For each of these subprograms we will be presenting

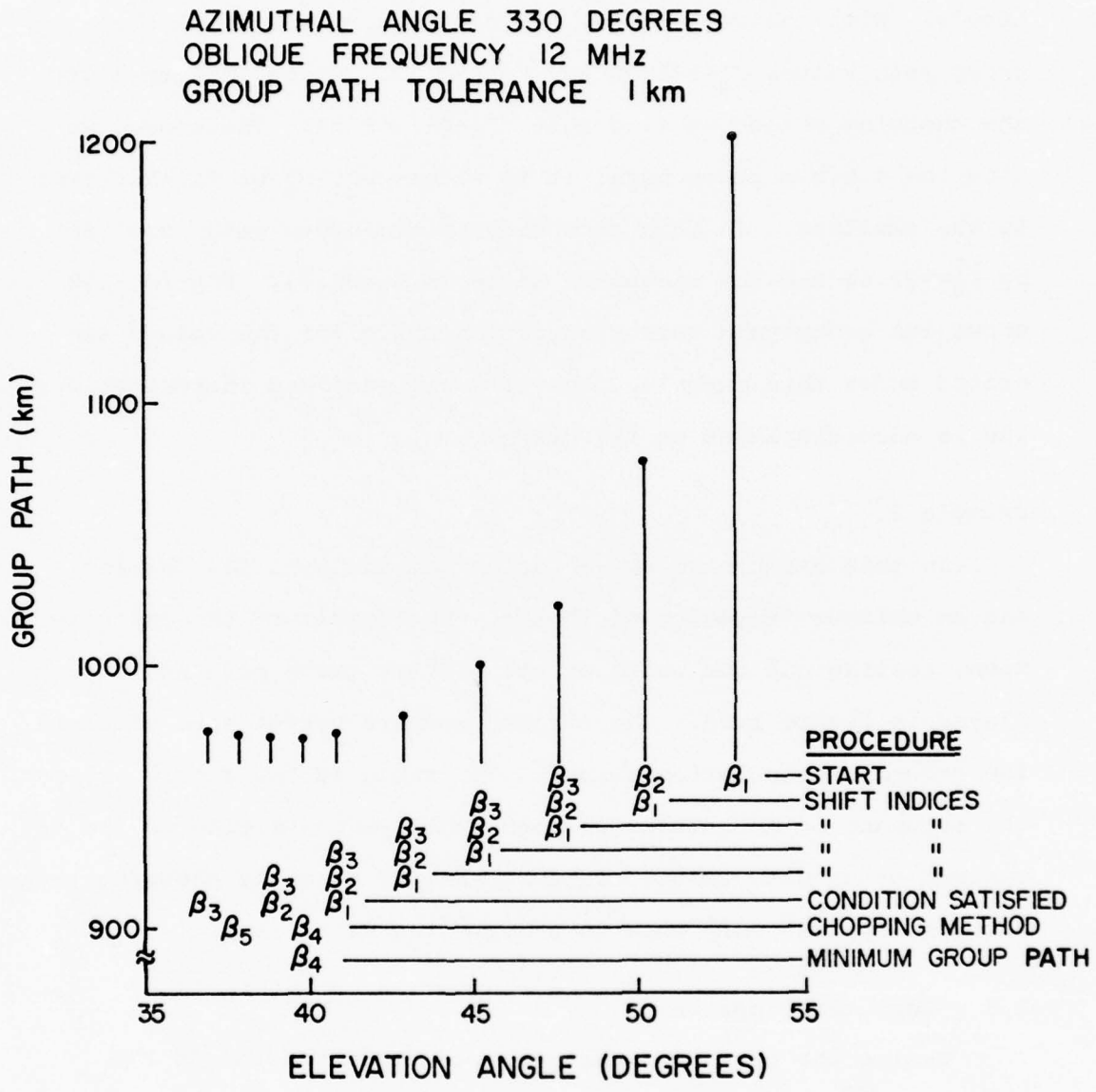

Figure 2.22. Example illustrating the steps in finding the minimum group path ray in a group path versus elevation angle plot. Note the index shifting, equal chopping and final location of the minimum group path to within the tolerance.

AZIMUTHAL ANGLE 308 DEGREES
OBLIQUE FREQUENCY 10 MHz
GROUP PATH TOLERANCE 1 km

#	β_1	β_2	β_3	P_1'	P_2'	P_3'	β_4	β_5	P_4'	P_5'
1	78.28	74.36	70.45	948.38	899.78	862.87	-	-	-	-
2	74.36	70.45	66.93	899.78	862.87	837.53	-	-	-	-
3	70.45	66.93	63.58	862.87	837.53	819.36	-	-	-	-
4	66.93	63.58	60.40	837.53	819.36	806.67	-	-	-	-
5	63.58	60.40	57.38	819.36	806.67	798.30	-	-	-	-
6	60.40	57.38	54.51	806.67	798.30	793.28	-	-	-	-
7	57.38	54.51	51.79	798.30	793.28	791.73	-	-	-	-
8	54.51	51.79	49.20	793.28	791.73	793.19	53.15	50.49	792.50	791.83
9	-	51.79	-	-	791.73	-	-	-	-	-

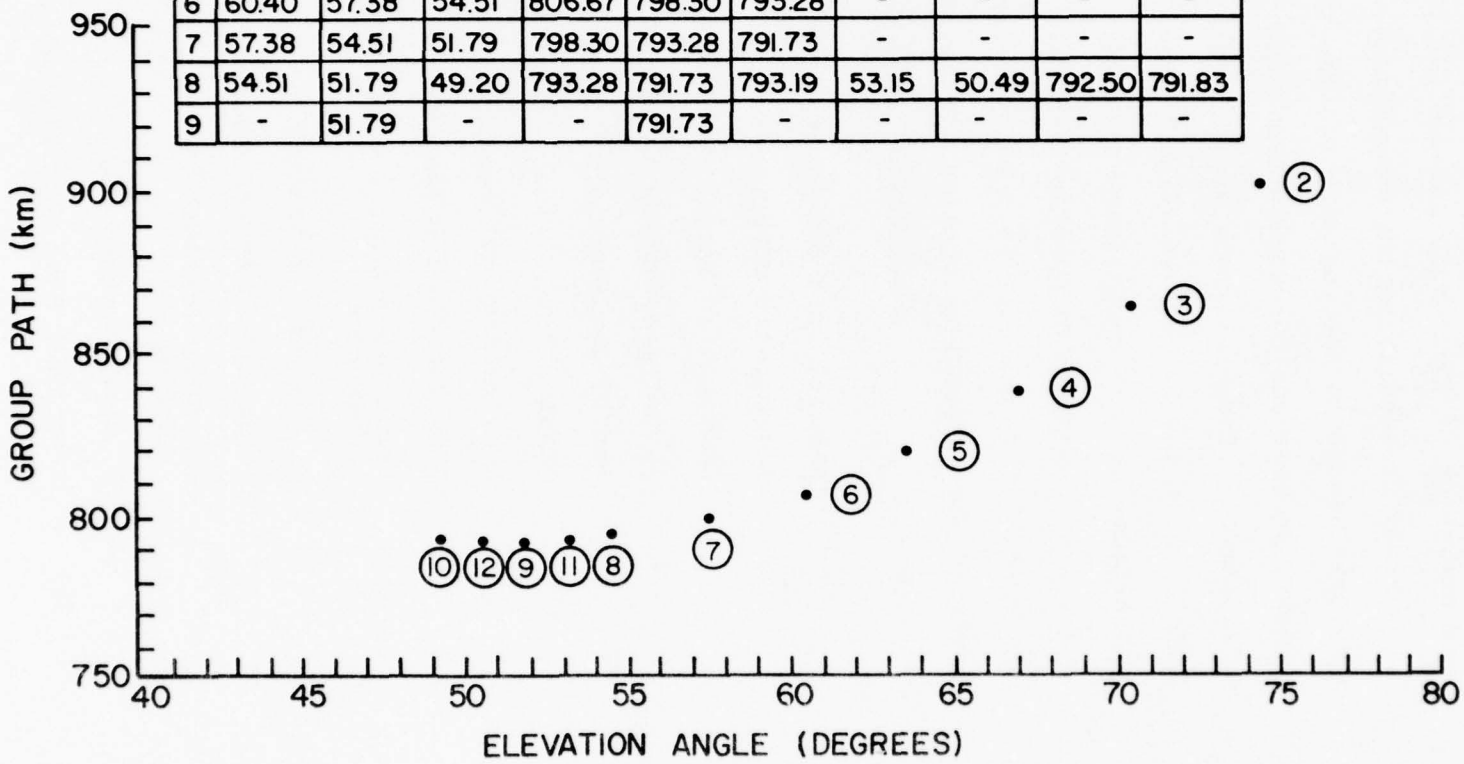

Figure 2.23. Another example illustrating the steps in finding the minimum group path ray.

a description of their main purpose and the functions they perform.

Prior to describing the subprograms we present a list of the controls needed to successfully run the three dimensional ray tracing program for the homing cases and the minimum group path calculation. These controls are either internal or external allocations in the already existing W-array input data. The internal allocations are equivalence statements in the subprograms and there is no need to specify them. The external allocations are specified by the user in the input data of the W-array. The internal and external allocations of the W-array are:

W	Description	Internal (In) External (Ex)
371	Control used for penetration conditions	(In)
372	Geomagnetic colatitude of the ray	(In)
373	Geomagnetic longitude of the ray	(In)
374	Local azimuthal deviation of the ray	(In)
375	Radial distance to the transmitter	(In)
376	ALPHA	(In)
377	THO, geomagnetic colatitude of transmitter	(In)
378	PHO, goemagnetic longitude of transmitter	(In)
379	Azimuthal deviation of the ray at the transmitter	(In)
380	0 regular ray tracing 1 ground-to-ground homing 2 ground-to-satellite or satellite-to-ground homing 3 minimum group path	(Ex)
381	Geographic East longitude of receiver in degrees	(Ex)
382	Geographic latitude north of receiver in degrees	(Ex)

383	Height of the ray above the earth surface	(In)
384	Ground distance computed in PRINTR	(In)
385	Ground distance computed from the geometry of the problem	(In)
386	Maximum number of iterations for finding the approximate elevation angle	(Ex)
387	The tolerance allowed between the ground distance in W(385) and calculated ground distance	(Ex)
389	Penetration angle used in homing	(In)
390	Height of the bottom of the ionosphere above earth	(Ex)
391	Maximum height of the ionosphere above earth	(Ex)
393	0 utilizes the given ionospheric profile 1 generates a new ionospheric profile based upon the given quasi-parabolic parameters	(Ex)
394	Value of the group path used in the group path homing	(Ex)

Subroutine HOME

Purpose:

The HOME subroutine calculates the initial approximate elevation angle in the ground-to-ground homing for the ground distance and the group path.

Description:

The HOME routine calculates several parameters and generates tables prior to obtaining the final result of initial approximate elevation angles. The azimuthal angle of transmission and the exact ground distance are computed from the given transmitter and receiver coordinates. The midpoint between the transmitter and receiver is found and the electron density profile at this midpoint is extracted from the given ionospheric profile. With

the critical frequency of the midpoint profile and the given oblique frequency of transmission the penetration angle is computed from equation (2.18). Then, the elevation angle interval from zero to this penetration angle is subdivided into 50 intervals thus generating 51 values of elevation angles. For each value of elevation angle in the interval a ground distance and a group path value are calculated from the analytical expressions for a quasi-parabolic density profile. These analytical expressions are programmed in subroutine FITT. Through this procedure a table of elevation angles, ground distances, and group paths is assembled.

The exact ground distance is then compared to the ground distance values in the table. If the exact ground distance is lower or higher than the ground distances in the table, then no approximate elevation angles can be found. However, if the exact ground distance is located between two ground distances in the table, we interpolate linearly to find an approximate elevation angle. Since there might be more than one homing solution we scan through the table and locate all possible elevation angles.

The HOME subroutine is capable of storing approximate elevation angles for a four layered ionosphere. In addition, this subroutine has been mainly written for the single hop mode of propagation. The two hop mixed mode of propagation is already included as described in section 2.2.6.

Subroutine HOMES

Purpose:

The HOMES subroutine calculates the initial approximate ele-

vation angle for a straight line path in the ground-to-satellite and the satellite-to-ground configurations.

Description:

Given the transmitter and receiver coordinates and the oblique transmission frequency subroutine HOMES computes the azimuthal angle of transmission and the ground distance projected at the transmitter radius. Assuming a straight line path between the transmitter and receiver, the approximate elevation angle is found from the triangle in the plane containing the transmitter, the receiver, and the earth center.

Subroutine FITT

Purpose:

Given an elevation angle routine FITT calculates the corresponding ground distance and group path from analytical expressions for a quasi-parabolic density profile.

Description:

Utilizing the analytical expression given by equations (2.13)-(2.16), the electron density profile at the midpoint of the path between the transmitter and receiver, and the oblique frequency of transmission, subroutine FITT computes the ground distance and the group path for a specified elevation angle. The contributions of ground distances and group paths are added up from the transmitter height up to the ray reflection height. The integration step corresponds to the height increment of the electron density profile at the midpoint.

Subroutine RAYINT

Purpose:

Routine RAYINT initializes the ray parameters before any actual ray tracing is performed.

Description:

The code in subroutine RAYINT is extracted from the main subprogram of the ray tracing program to enable the homing and minimum group path procedures to function independently from the main program. With a specified elevation and azimuthal angles this subroutine initializes the ray parameters prior to tracing the ray via subroutine TRACE.

Subroutine ADJUST

Purpose:

The ADJUST subroutine takes the approximate elevation angles for any of the three configurations, ground-to-ground, ground-to-satellite, or satellite-to-ground and modifies them through the ray tracing program to achieve homing.

Description:

Utilizing the initial elevation angle a ray is traced in the ionospheric profile to obtain a ground distance value. Then, the initial elevation angle is incremented or decremented by a small amount depending upon the configuration under consideration. In the ground-to-ground case the initial increment or decrement is found from the table assembled in subroutine HOME, while in the other two cases the initial increment or decrement is found from equation (2.35). Adjusting the initial elevation angle by the initial increment or decrement we obtain a new elevation angle

and a corresponding ground distance through ray tracing. The objective is to locate the exact ground distance between the ground distances of the traced rays. Sometimes, the rays are homed without further calculations and at other times we cannot locate the exact ground distance between the ground distances obtained through ray tracing at which point a message is printed. Let us assume that the exact ground distance lies between the two ground distances obtained from ray tracing. With the two values of elevation angles and their corresponding ground distances we interpolate linearly and find a third elevation angle. Utilizing this third elevation angle we trace another ray to obtain the ground distance. If the ray is not homed in yet, we have three elevation angles and their ground distances. Since the variation between ground distance and elevation angle is nonlinear we fit a second degree polynominal in elevation angle and invert the resulting matrix to find a new elevation angle that will eventually home the ray.

With this elevation angle we trace a ray and find the ground distance. A comparison test on the ground distances will reveal whether or not homing is achieved. In the event that homing is not achieved, the procedure of fitting the second degree polynomial is repeated utilizing the closest three ground distances to the exact ground distance and their corresponding elevation angles. Then, either homing is achieved and no further calculations are needed, or the above procedure is repeated to attain homing. Homing may not be achieved due to discontinuities in the ray-traced ground distance versus elevation angle curve, high angle ray, or that the maximum number of specified trials to find

an elevation angle have been exceeded.

Subroutine GROUPM

Purpose:

Subroutine GROUPM calculates the minimum group path for given transmitter coordinates, azimuthal angle of transmission and an oblique transmission frequency.

Description:

With the given transmitter coordinates, the closest density profile is selected from the composite ionospheric profile. From the knowledge of the critical frequency of the selected profile and the oblique frequency of transmission, the penetration angle is computed through equation (2.18). Then, starting with this penetration angle we initialize the ray parameters and trace a ray through the given ionosphere. If the ray penetrates the ionosphere we decrement the elevation angle (penetration angle) by one percent and trace another ray. This procedure is repeated until the ray is reflected and a value for the group path is obtained. There, we take 95 or 90 percent of the resulting elevation angle thus generating two new elevation angles. With these two new elevation angles we trace two rays to obtain their corresponding group path values. The three values of elevation angles and their corresponding group paths describe a curve. This curve may assume three shapes, sloping to the left (most common), sloping to the right (rare) or concave upwards. In the cases where the curve is sloping to the right or left we increment or decrement the elevation angle and trace rays until the shape of the curve is concave upward. For this shape the values

of the group paths are compared with each other to test for the minimum value within a specified 1 km tolerance.

Let us assume that we did not attain the minimum group path value, then we subdivide the two intervals in the curve and obtain two elevation angles, one in each interval, thus generating two additional intervals. Through ray tracing we obtain two values of group paths corresponding to the two new elevation angles. A test is performed between the five group path values and the largest two are discarded retaining only three values and their corresponding elevation angles. Through repeated application of this procedure we converge on the minimum group path value with the specified tolerance of 1 km. This tolerance is imbeded in the subroutine and could be changed to any desired value.

2.5.2 Program Listings

A complete listing of all the programs discussed in this section is given in Appendix 1.

2.6 Discussion

We have modified the three dimensional ray tracing program to include the homing features for ground-to-ground, ground-to-satellite, and satellite-to-ground configurations. In addition, the ray tracing program has been augmented by a chopping technique to find the minimum group path. In the preceding sections we presented in detail the approach and methods utilized in the homing of the ray together with representative examples. We also outlined the purpose and description of each computer subprogram added to the originally supplied AFCRL three dimensional ray tracing program.

The results of our homing approach for the one hop mode of propagation show that homing is almost always achieved provided the input parameters are correctly entered through the input data. We say almost since there are situations where homing of the ray may not be possible. During the course of our study we came across three situations in the ground-to-ground case whereby homing might not be possible. These three situations are

i) high angle ray

ii) ground distance too close to the skip distance or discontinuity in the ray traced ground distance versus elevation angle curve.

iii) number of tries to find an elevation angle is exceeded.

A preliminary investigation of the two mixed hop mode of propagation has been initiated and the technique has been implemented in the HOME subroutine. The application of our technique to achieve homing is only partially successful; the ray was homed in for a few cases while for the majority of the cases tested we

encountered difficulties in homing of the ray. These difficulti may be attributed to the following reasons:

i) the coarseness of the given ionospheric profile

ii) the interpolation procedure used in the ray tracing program

iii) the technique applied to the two hop mode

Through further research, the true nature of the problem could b pinpointed and if need be a sophisticated homing technique for the multi-hop mode of propagation can be developed.

Aside from the homing probelms, we have also incorporated a chopping method to find the minimum group path. The results of such a method are quite evident from the examples presented in section 2.4.2. For all the cases studied we have been success ful in finding the minimum group path within the specified tolerance. The number of steps needed to obtain the minimum group path decreases as the frequency of transmission gets higher. Once we arrive at the condition whereby the variation of group path versus elevation angle curve is concave upward, the convergence of the chopping method is very fast.

From the results presented in this report and other examples derived from the techniques used in homing of the ray and findin the minimum group path it is quite evident that we have been successful in achieving our objective. The difficulties encounte during the course of this study could be overcome through modification and refinement of the ray tracing program and the ionosphe profile. Such modification and refinement can only be achieved through further research.

3. ANALYSIS OF OBLIQUE PROPAGATION DATA

3.1 Introduction

The goal of this investigation is to devise techniques for inverting h.f. oblique radio propagation data for ionospheric models with horizontal gradients. There are two kinds of oblique propagation data: (a) Point-to-point oblique ionograms, and (b) Backscatter leading edge. In this section, we shall consider briefly the nature of these two types of data and discuss qualitatively their capabilities and limitations in providing information concerning ionospheric models with horizontal gradients.

3.1.1 Point-to-Point Oblique Ionograms

These are traces of group path (P') versus frequency (f) for oblique propagation between two fixed points separated by some distance. Since the end points of the propagation path are fixed, the reflection points of the rays lie in the neighborhood of the midpoint between the transmitter (T) and receiver (R) for the one-hop mode of propagation, irrespective of the frequency, as shown in Fig. 3.1(a). Similarly, for multiple-hop mode of propagation, reflection for each hop occurs in the neighborhood of a single location, for all frequencies capable of propagating in that mode. A typical shape of the oblique ionogram trace for the one-hop mode is shown in Fig. 3.1(b).

It can be easily conjectured from Fig. 3.1(a) that a one-hop mode oblique ionogram is capable of providing an equivalent vertical profile of electron density valid near the midpoint between the transmitter and the receiver, but is not by itself too useful for deriving the horizontal gradients of electron density. How-

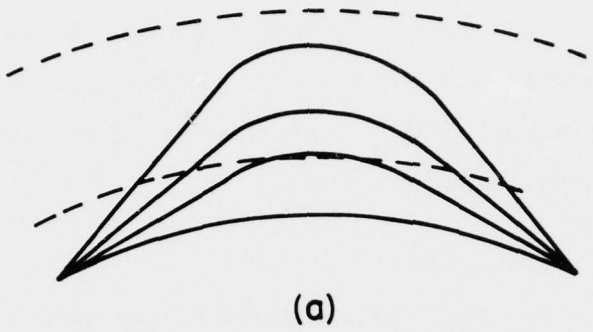

Figure 3.1a. Ray paths for point-to-point oblique ionograms at different frequencies.

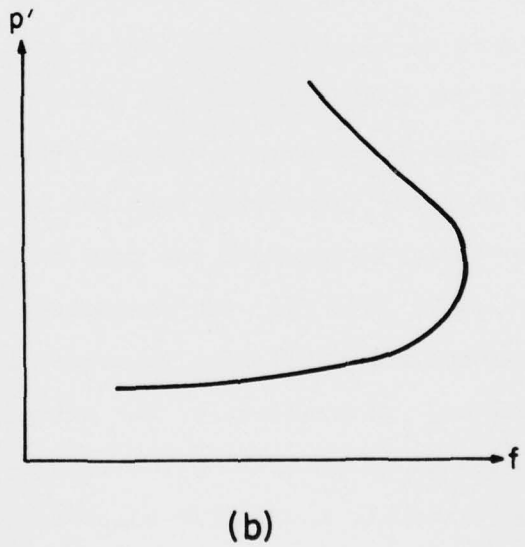

Figure 3.1b. Typical shape of one-hop trace for point-to-point oblique ionogram.

ever, for a particular assumed model with horizontal gradients near the midpoint, it is in principle possible to deduce the parameters so as to achieve a best fit with the experimental ionogram. The use of a two-hop mode trace in addition to the one-hop mode trace would increase the utility of the ionogram for deriving the horizontal gradients but still does not permit their determination continuously along the azimuthal direction from the transmitter to the receiver.

3.1.2 Backscatter Leading Edge

In the case of the backscatter mode of propagation, the transmitter and the receiver are located at about the same location and the time delay of the transmitted signal backscattered from the ground and then received at the receiver is measured as a function of frequency. For a given frequency, many returns are possible corresponding to all elevation angles of transmission and reception within the bandwidths of the transmitting and receiving antennas. There is however a minimum value for the time delay which occurs near the transition from the low angle ray mode of propagation to the high angle ray mode of propagation, as shown in Fig. 3.2(a). Thus for each frequency, a continuum of backscattered returns beginning with the minimum time delay return will be received. Alternatively, the situation can be thought of as a continuum of point-to-point oblique ionograms corresponding to continuously increasing values of ground range of the backscatter location away from the transmitter. Such a continuum of oblique ionograms is shown in Fig. 3.2(b). The tangent curve to these ionograms is the "backscatter leading

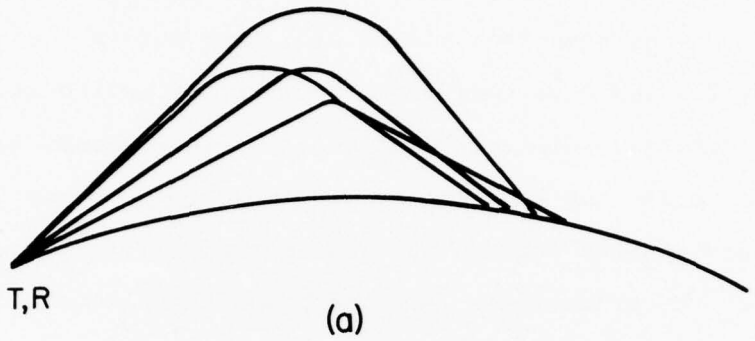

Figure 3.2a. Ray paths for backscattered rays at one frequency.

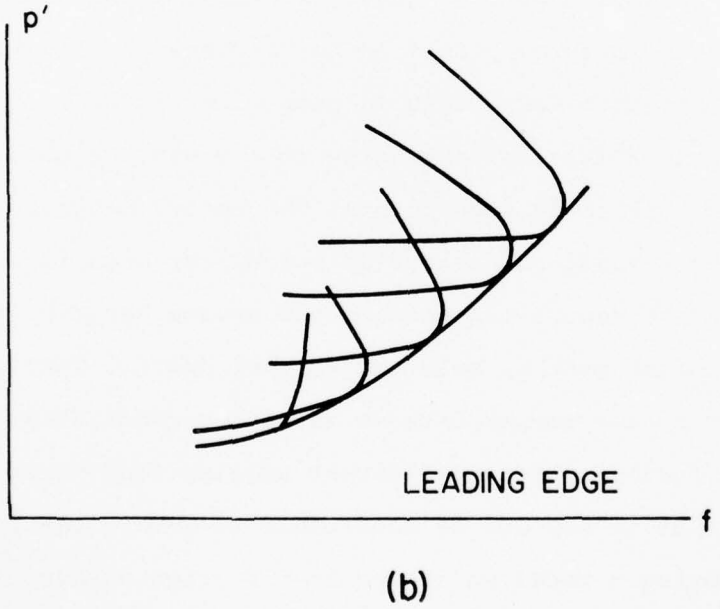

Figure 3.2b. Showing that the leading edge of a backscatter ionogram is the tangent curve to a continuum of point-to-point oblique ionograms.

edge", and is generally the only useful portion of the backscatter ionogram since all the other returns cannot be distinguished from one another, except in the case of high resolution ionograms.

It can now be seen from Fig. 3.2(b) that points along the backscatter leading edge correspond to rays reflecting at continuously increasing distant locations from the transmitter. Hence in principle the backscatter leading edge contains information concerning the horizontal gradients of electron density outward from the transmitter location along the azimuthal direction corresponding to the backscatter ionogram. However to obtain a vertical profile of electron density at any given location, a minimum number of points along an ionogram are required. For example, a quasi-parabolic profile requires a minimum of three points. If we consider three points on the backscatter leading edge, we must select them very close to each other in order to obtain the vertical profile corresponding to a given location. On the other hand, for three such points, the apogee heights of reflection for the corresponding rays do not differ significantly. Hence to increase the separation between the apogee heights of reflection, the points must be selected farther apart. Thus it becomes necessary to compromise between these two conflicting requirements. A further point of interest arising from these requirements is that it may not be profitable to invert the leading edge by employing a model which requires a large number of parameters to describe its vertical profile. This observation is augmented by the fact that simulated backscatter leading edge data using three-dimensional ray tracing indicates that the apogee heights of rays corresponding to points on the leading

edge lie within a narrow range of values. This also makes it impossible for the backscatter leading edge to provide the vertical profile above the maximum apogee height. However, as will be pointed out later, this is not a limitation insofar as the application of the leading edge data is concerned, although it may be a limitation for the purpose of obtaining the complete ionospheric structure up to the layer peak.

Thus in this introductory section, we point out that while the point-to-point oblique ionograms are capable of providing the vertical profile near the midpoint between the transmitter and the receiver, they are not too useful for obtaining horizontal gradients over wide ranges of distance along the line from the transmitter to the receiver. On the other hand, the backscatter leading edge, while unable to provide the vertical profiles above certain heights, is useful for deducing the horizontal gradients over wide ranges of distance along the azimuthal direction from the transmitter corresponding to the backscatter ionogram.

3.2 The Quasi-Parabolic Layer

Since the primary purpose of this investigation is to devise techniques capable of yielding the horizontal gradients of electron density, it can be seen from the discussion in the previous section that it becomes necessary to attach more importance to the backscatter leading edge data than to the point-to-point oblique ionogram data. Hence for selecting a model, the limitation of the leading edge data need to be considered. In view of this, an ionospheric model requiring a small number of parameters to

define its vertical profile is more suitable. Such a model, which is also amenable to analytical solution for the ray path parameters, as compared to numerical ray tracing, is the quasi-parabolic layer model. Hence this section is devoted to the quasi-parabolic layer.

3.2.1 The Earth Concentric Quasi-Parabolic Layer

The quasi-parabolic layer is defined by the variation of electron density N_e with the radial distance r from the center of the earth as given by

$$N_e(r) = \begin{cases} N_m\left[1 - \left(\frac{r-r_m}{y_m}\right)^2\left(\frac{r_b}{r}\right)^2\right] & \text{for } r_b < r < r_m\left(\frac{r_b}{r_b - y_m}\right) \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

where

$N_m = f_c^2/80.6$ = maximum value of electron density

f_c = critical frequency

r_m = value of r at which N_e is equal to N_m

r_b = value of r at the base of the layer

$y_m = r_m - r_b$ = semithickness of the layer

The slight modification over the parabolic model, which is defined by ignoring the factor $(r_b/r)^2$ in (3.1), enables the derivation of exact closed form expressions for the ray path parameters for the quasi-parabolic layer (Croft and Hoogasian, 1968), by application of Bouguer's rule for ray tracing in a spherically symmetric layer. For a signal of frequency f and elevation angle

of transmission β, these expressions for the ground range R, and group path P' are given as follows (Rao, 1974; Rao, 1975):

$$R = 2r_o \left\{ (\gamma-\beta) - \frac{F\, r_o \cos\beta}{2\sqrt{C}} \ln \frac{Ur_b^2}{W^2} \right\} \tag{3.2}$$

$$P' = 2\left(1 - \frac{F^2}{A}\right) r_b \sin\gamma - 2r_o \sin\beta - \frac{BF}{2A^{\frac{3}{2}}} \ln \frac{U}{V^2} \tag{3.3}$$

where

$F = f/f_c$

$\gamma = \cos^{-1}\left(\frac{r_o}{r_b}\cos\beta\right)$ = elevation angle at the base of the layer

r_o = radius of the earth

$U = B^2 - 4AC$

$V = 2Ar_b + B + 2r_bF\sqrt{A}\sin\gamma$

$W = 2\sqrt{C}\, Fr_b \sin\gamma + 2C + Br_b$

$A = F^2 - 1 + (r_b/y_m)^2$

$B = -2r_m r_b^2/y_m^2$

$C = \left(\frac{r_b r_m}{y_m}\right)^2 - F^2 r_o^2 \cos^2\beta$

To find the minimum group path, which is equal to the minimum time delay times the velocity of light in free space, we first note that for a given frequency and for a given set of layer parameters, the elevation angle of transmission corresponding to the minimum group path ray is given by the solution of the equation

$$\frac{\partial P'}{\partial \beta} = 2\left(1 - \frac{F^2}{A} + \frac{BF^2}{AV}\right) r_o \sin\beta \cos\gamma$$

$$- 2\left(1 - \frac{2B F^3 r_o \sin\beta}{\sqrt{A}\, U}\right) r_o \cos\beta$$

$$= 0 \qquad (3.4)$$

Recognizing that $\partial P'/\partial\beta$ varies continuously from a large negative value near zero elevation angle to a large positive value near maximum elevation angle corresponding to penetration of the ray through the layer, we can solve (3.4) for β in an iterative manner. The penetration condition occurs for the ray apogee radius equal to - B/2A and hence for the elevation angle of transmission given by

$$\beta_p = \cos^{-1}\left\{\frac{1}{Fr_o}\left[\left(\frac{r_b r_m}{y_m}\right)^2 - \frac{B^2}{4A}\right]^{\frac{1}{2}}\right\} \qquad (3.5)$$

Thus starting with values of β near zero and β_p, the value of β for which $\partial P'/\partial\beta$ is a small specified value can be found in an iterative manner. Substitution of this value of β in (3.2) then gives the value of the minimum group path, P'_{min}.

In the formulation of the procedures for inversion of the point-to-point oblique ionograms and backscatter leading edges for quasi-parabolic layer parameter, we will find later that the partial derivatives of R and P' with respect to the layer parameters are required. Expressions for these quantities, which can be easily derived from (3.2) and (3.3) are given in the appendix.

3.2.2 The Eccentric Quasi-Parabolic Layer

In papers by Rao (1968, 1973), the ionospheric layer has

been assumed to be still spherical but eccentric with respect to the earth. Figure 3.3 shows the geometry pertinent to such a layer with its center C' displaced by vector $\vec{D}$ from the earth's center C. Assuming quasi-parabolic distribution of electrons in the eccentric layer, the spherical geometry of the system with respect to its own center enables us to use the formulas for the concentric case (Section 3.2.1) with some additional computations introduced by Rao (1968). Thus the eccentric quasi-parabolic layer model, which permits the existence of gradients in the electron density, is completely defined by the following six parameters:

f_c : critical frequency of the layer

h_o : vertical distance between the base of the ionosphere and the surface of earth along vector $\vec{D}$

y_m : semi-thickness of the layer

D : the distance between the centers of the earth and the ionosphere

ω_t : the angle between the displacement vector $\vec{D}$ and the radius of earth through the transmitter

α : an angle pertinent to the azimuthal angle of transmission

For simplicity, we assume no gradient lateral to the transmission path ($\alpha = 0$) and the model is then defined by the first five parameters. The consideration of non zero α will not affect the presented techniques and will only add an additional dimension to the problem.

It is essential to have a feel for the type and amount of

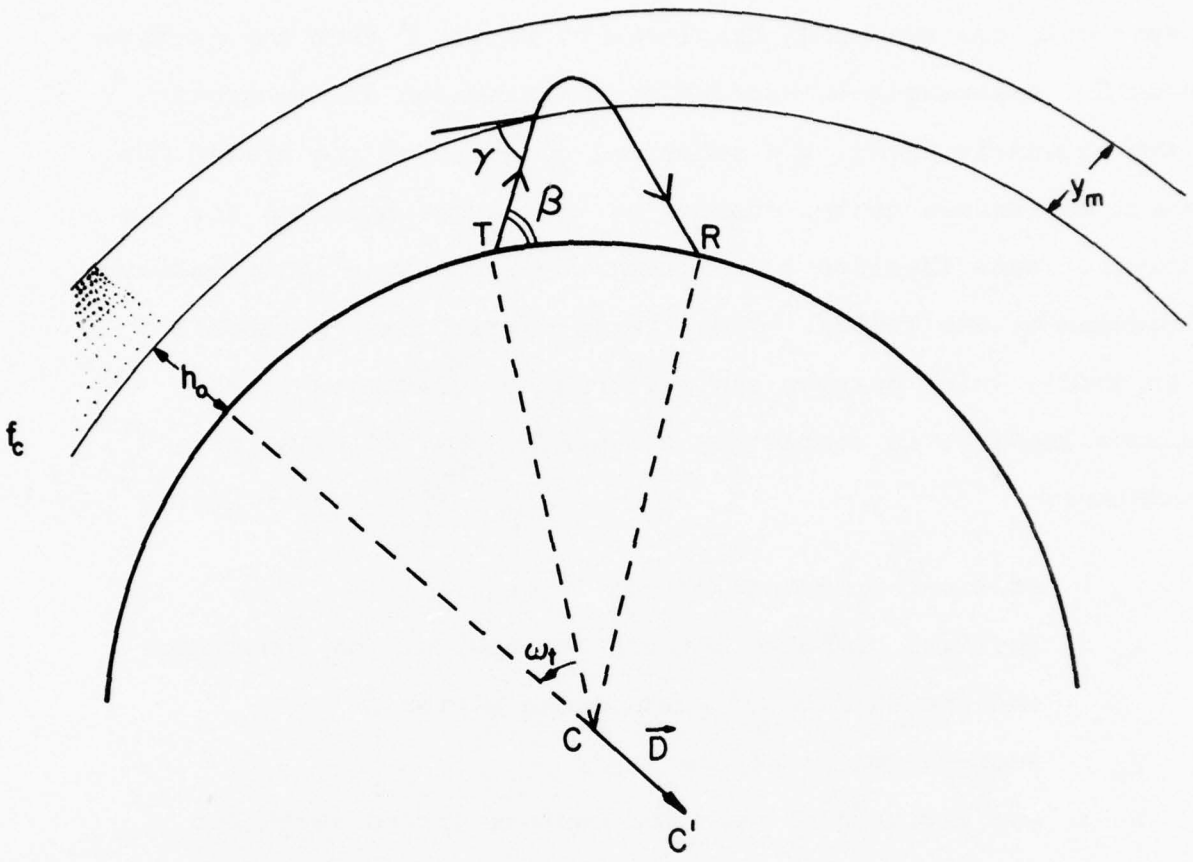

Figure 3.3. The eccentric quasi-parabolic layer with its defining parameters.

the gradients introduced by D and ω_t parameters of the eccentric layer. To accomplish this, we must make a few simplifying assumptions and approximations. The geometry of Figure 3.3 suggests that we have gradients in the base and peak heights of the layer. To obtain realistic relative values for these two gradients, we will assume that the base height of the layer above the transmitter be a constant value for all cases. In Figure 3.4, T and R are the locations of the transmitter and the receiver with M and θ_o being the location and the angle of their mid-point. The base heights of the layer above T and R are denoted by h_o' and h_o''. Expressions can now be derived for the gradients at angle θ_o (approximately where the reflection takes place).

The equation of the base-height circle can be written as

$$(x + D\cos\omega_t)^2 + (y + D\sin\omega_t)^2 = D^2 + (r_o + h_o')^2 + 2D(r_o + h_o')\cos\omega_t \qquad (3.6)$$

Writting (3.6) in polar coordinates, we get

$$r^2 + 2rD[\cos(\theta - \omega_t)] = (r_o + h_o')^2 + 2D(r_o + h_o')\cos\omega_t \qquad (3.7)$$

Differentiating 3.7 with respect to θ, we have

$$2r\frac{\partial r}{\partial\theta} + 2D\frac{\partial r}{\partial\theta}\cos(\theta - \omega_t) - 2Dr\sin(\theta - \omega_t) = 0$$

$$\left.\frac{\partial r}{\partial\theta}\right|_{\theta=\theta_o} = \frac{D\sin(\theta_o - \omega_t)}{1 + \frac{D}{r}\cos(\theta_o - \omega_t)} \qquad (3.8)$$

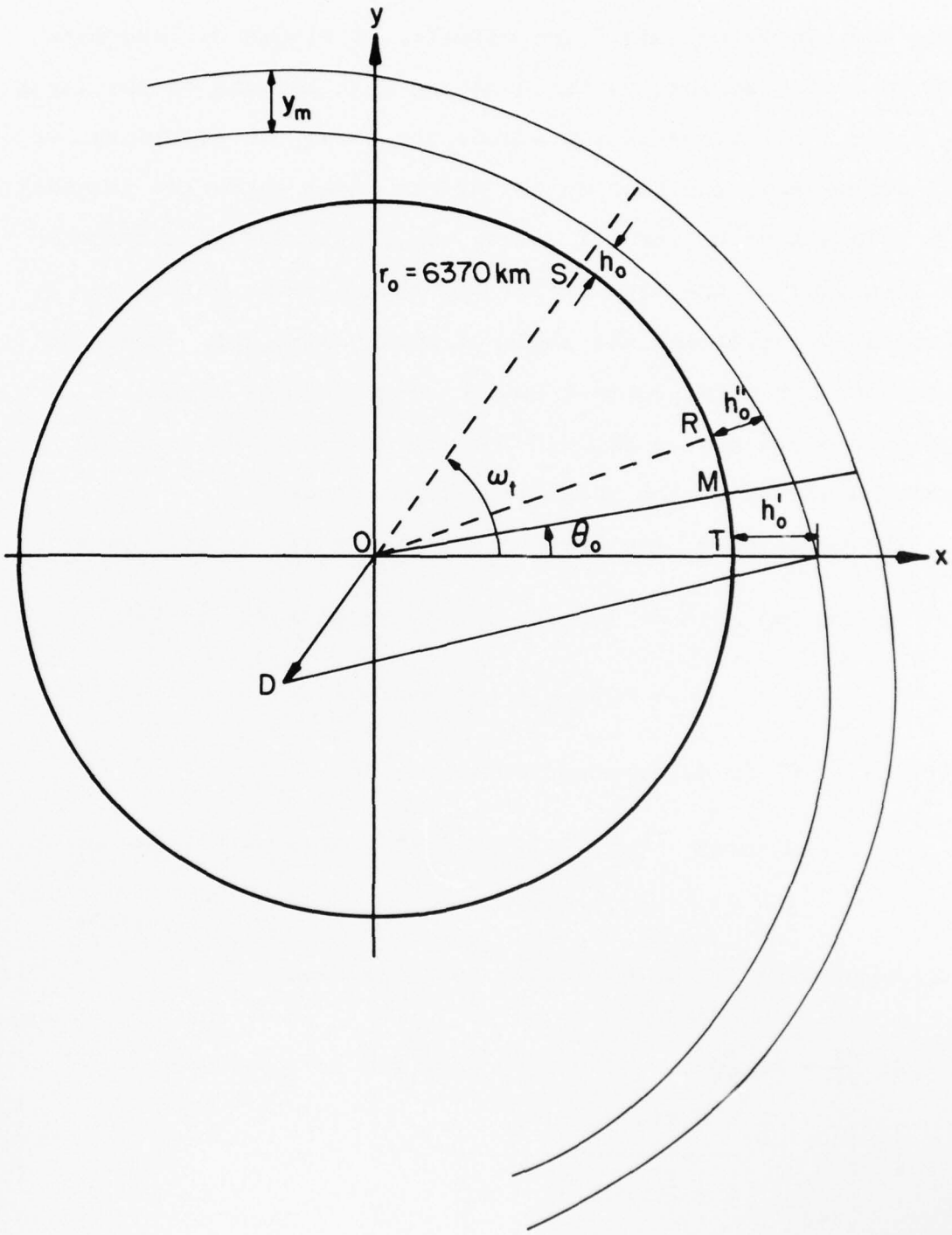

Figure 3.4. The geometry for computing gradients as a function of ω_t, D.

where

$$r' = r_o + h'_o \qquad \text{for the base height gradient}$$

$$r' = r_o + h'_o + y_m \qquad \text{for the peak height gradient.}$$

Since, in general $r_o >> h'_o$ and y_m, we can assume that the gradient is equal for both base and peak heights ($r' \cong r_o$).

As a typical example, let us assume $\hat{TOR} = 10°$ corresponding to a ground range of R = 1111.8 km ($\theta_o = 5°$). The gradient given by (3.8) is evaluated for several values of D and ω_t and Fig. 3.5 shows the peak height gradient (r' = 6370 + 150 + 200 = 6720 km as a function of D for several values of ω_t. Figure 3.5 also shows the forbidden regions, characterized by the ionosphere going underground at the receiver. This happens when the gradients are large, forcing the base height of the layer to be less than zero above the receiver ($h''_o < 0$). The quantity h''_o can be derived by solving (3.7) for r and setting $\theta = 2\theta_o$. Thus

$$r_o + h''_o = -D\cos(2\theta_o - \omega_t) + \sqrt{D^2\cos^2(2\theta_o - \omega_t) + (r_o + h'_o)^2 + 2D(r_o + h'_o)\cos\omega_t} \qquad (3.9)$$

The above equation can also be used to set an upper limit for h''_o as well as a lower limit. This will result in a usable region on Fig. 3.5 for the values of D and ω_t.

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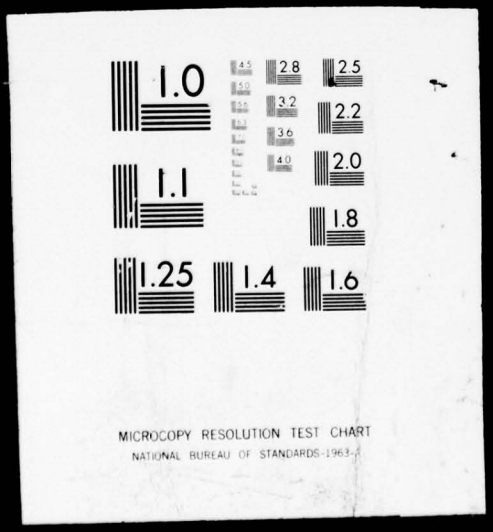


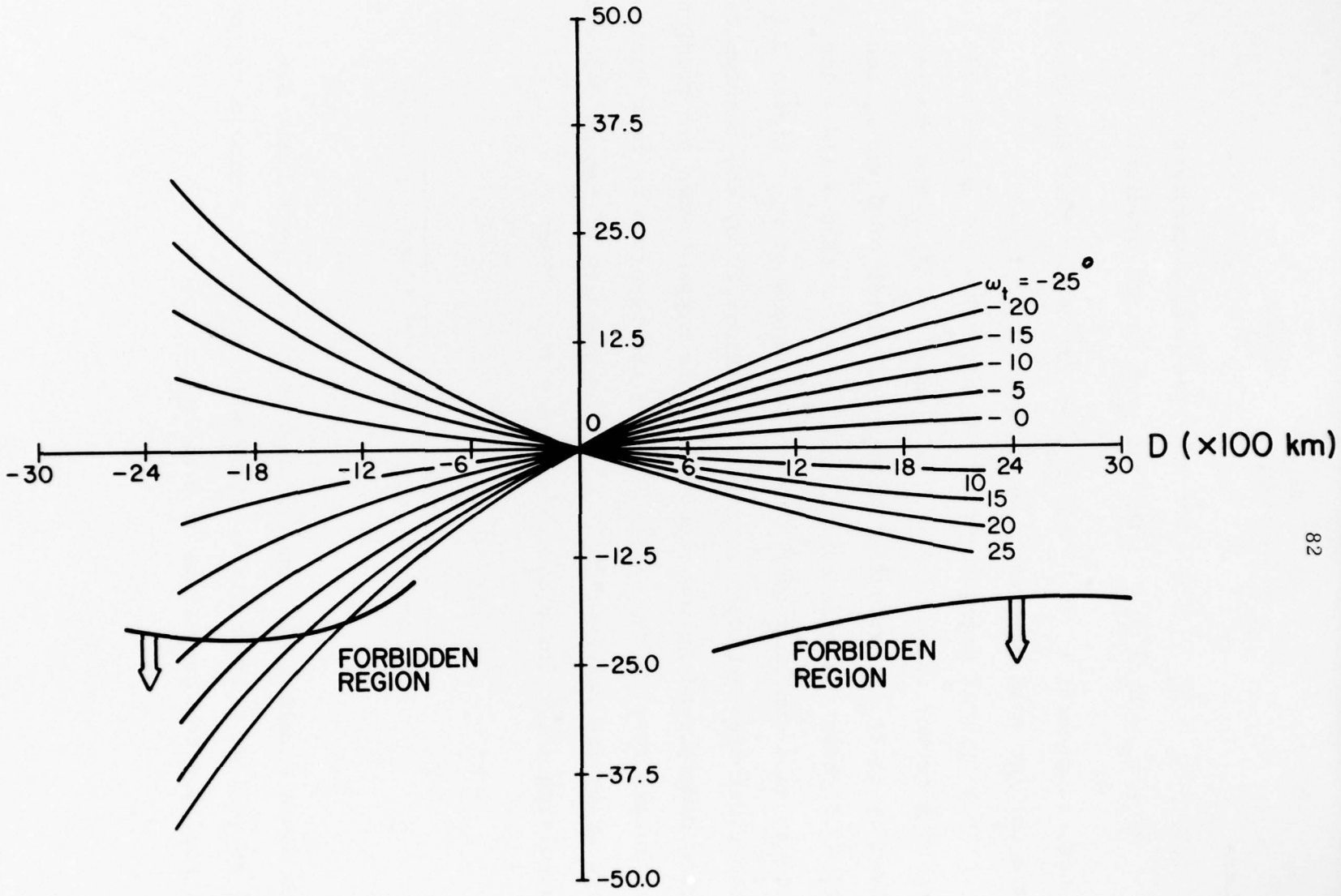


Figure 3.5. Peak height gradient as a function of D and ω_t.

3.3 Inversion of Point-to-Point Oblique Ionograms

In this section, we report our investigation of techniques for inversion of point-to-point oblique ionograms. The inversion of such ionograms for earth-concentric ionospheric layers already exist in the literature. Hence, we consider here their inversion to a layer with horizontal gradients, and in particular the eccentric quasi-parabolic layer discussed in Sec. 3.2.2.

3.3.1 Sensitivity Analysis

First we need to explore the individual behavior of the five parameters of the eccentric Q-P layer model with respect to numerically synthesized oblique ionograms, so that we can devise an efficient numerical procedure for inverting a given oblique ionogram to the model's parameters. In Figs. 3.6-3.10, four of the five parameters are held constant, with the fifth varying over a small range, and the resulting oblique ionograms are plotted. Investigation of these graphs shows that small changes in f_c, h_o, and y_m parameters have a relatively linear effect on their respective ionograms, that is, the curves are shifted by a similar amount for equal changes in each of the three parameters. However, for changes in the gradient parameters (ω_t, D), the shifted ionograms actually tend to converge to a certain curve and do not demonstrate the linear behavior associated with the variation of the first three parameters.

The above conclusion can also be verified from the geometric point of view. Since ω_t is a periodic variable, the constructed ionograms will not be linearly shifted as the ω_t parameter is increased. Also the constructed ionograms can be very insensi-

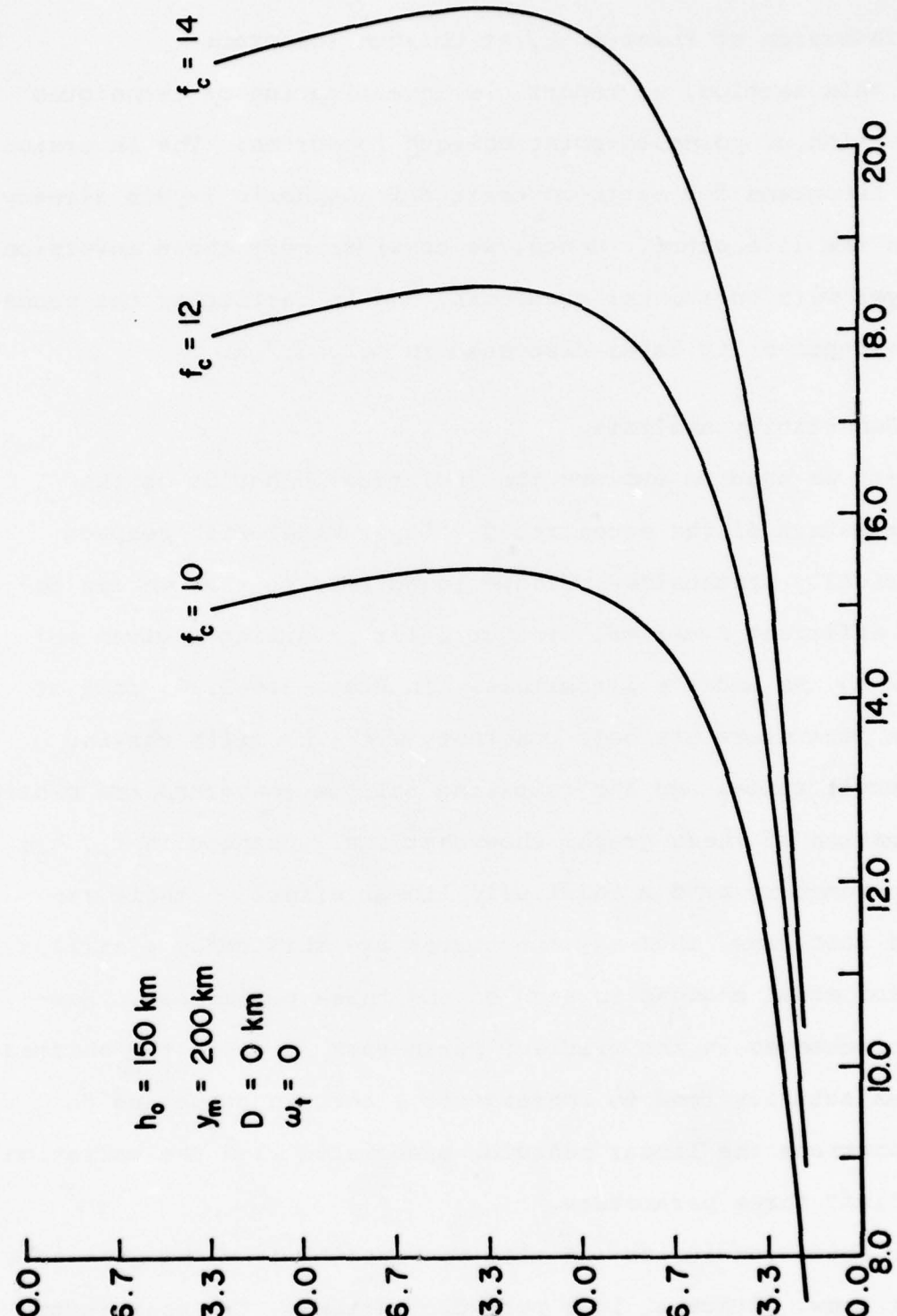


Figure 3.6. Effect of changing the f_c (critical frequency) parameter (in MHz).

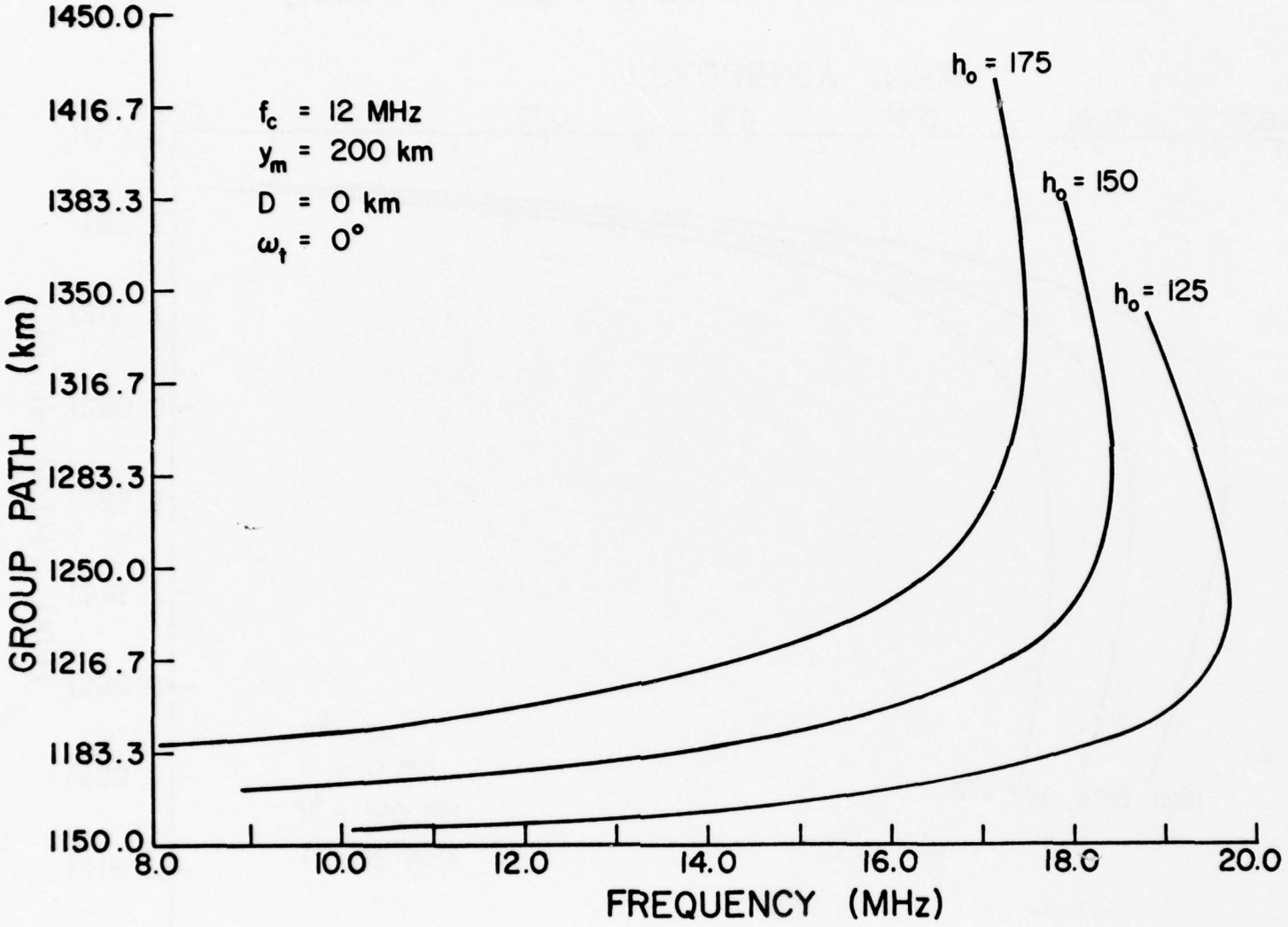


Figure 3.7. Effect of changing h_o; the layer base-height (in km).

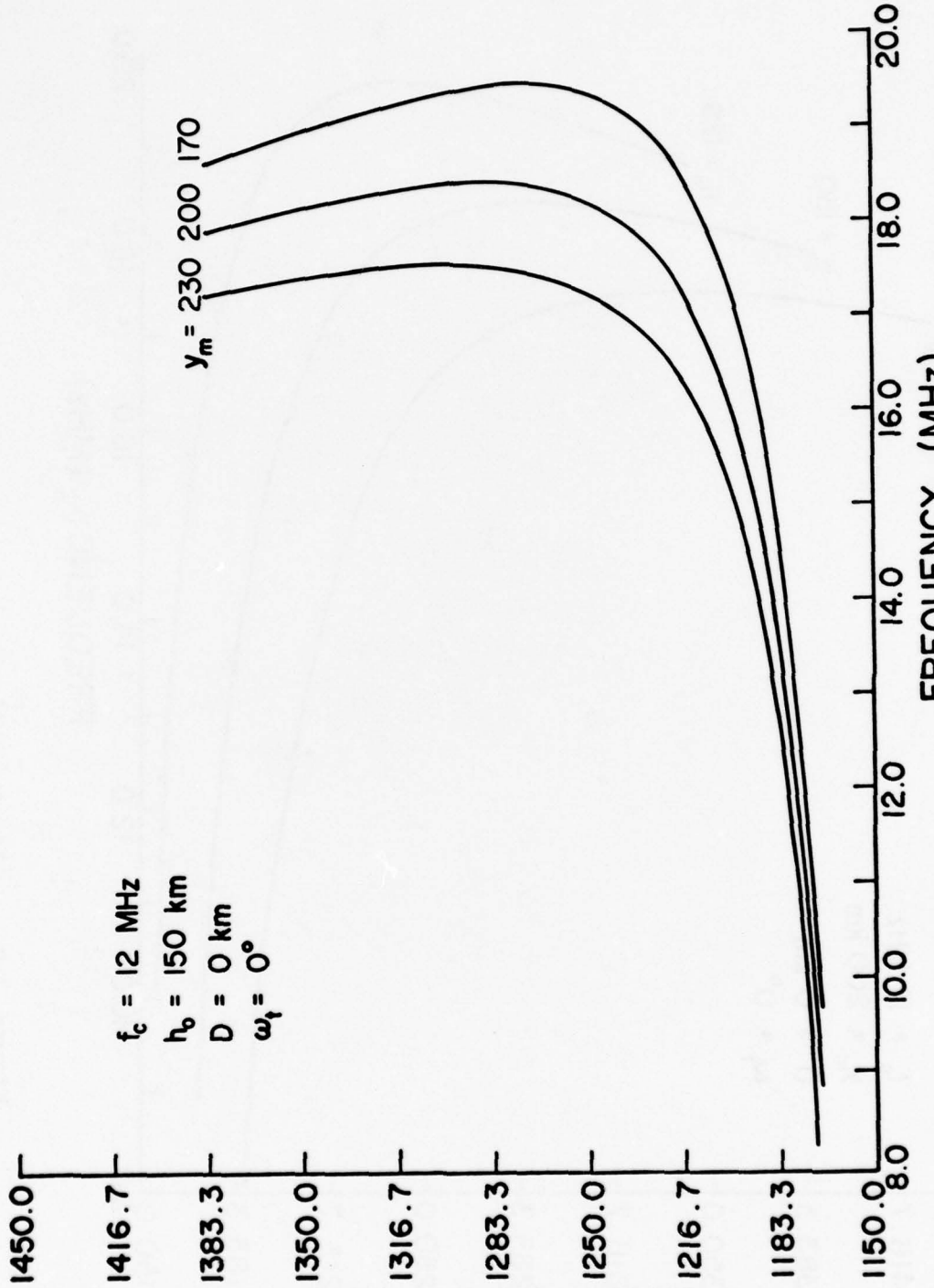


Figure 3.8. Effect of changing the layer semi-thickness y_m (in km).

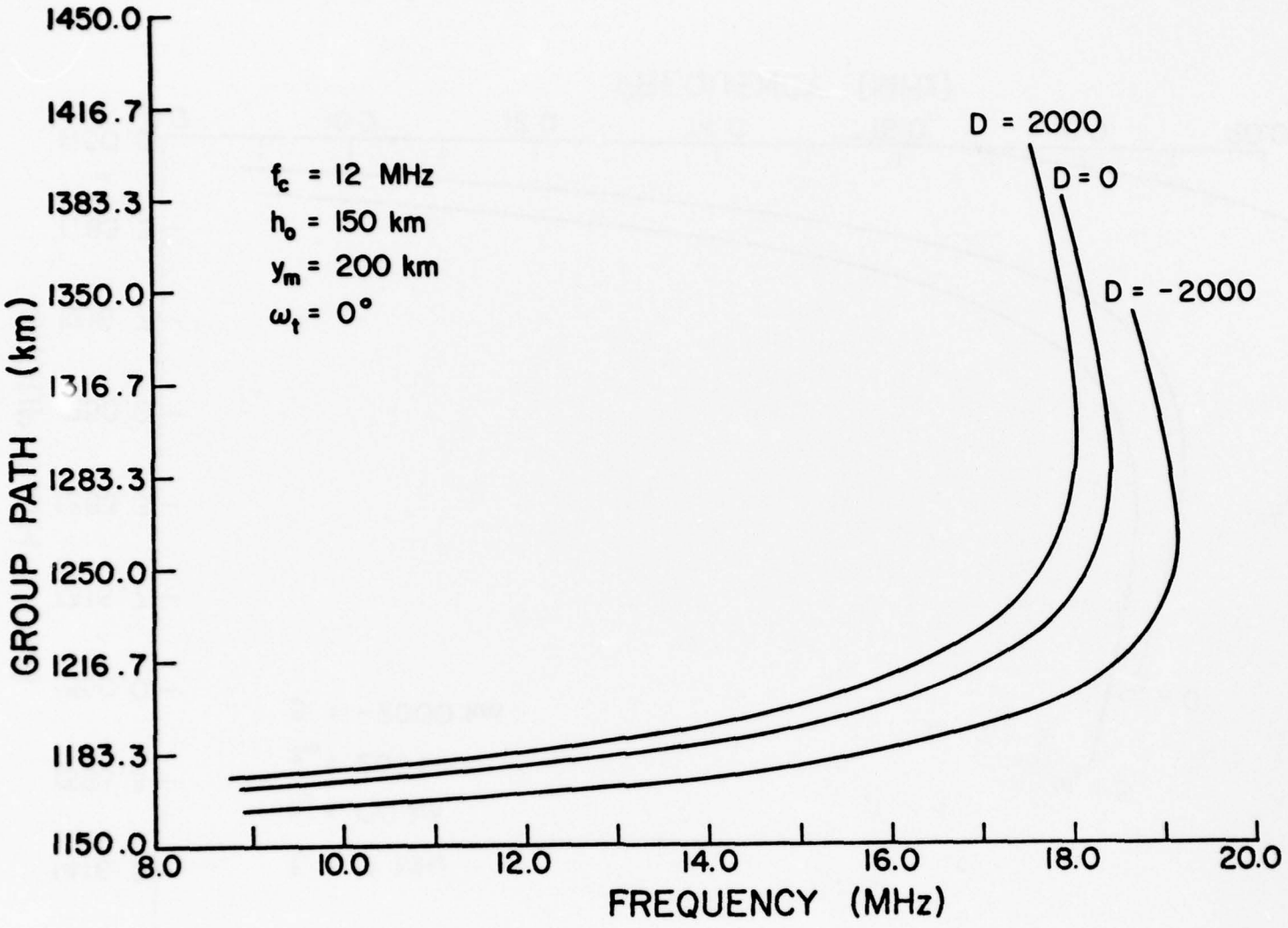


Figure 3.9. Effect of changing the D parameter (in km).

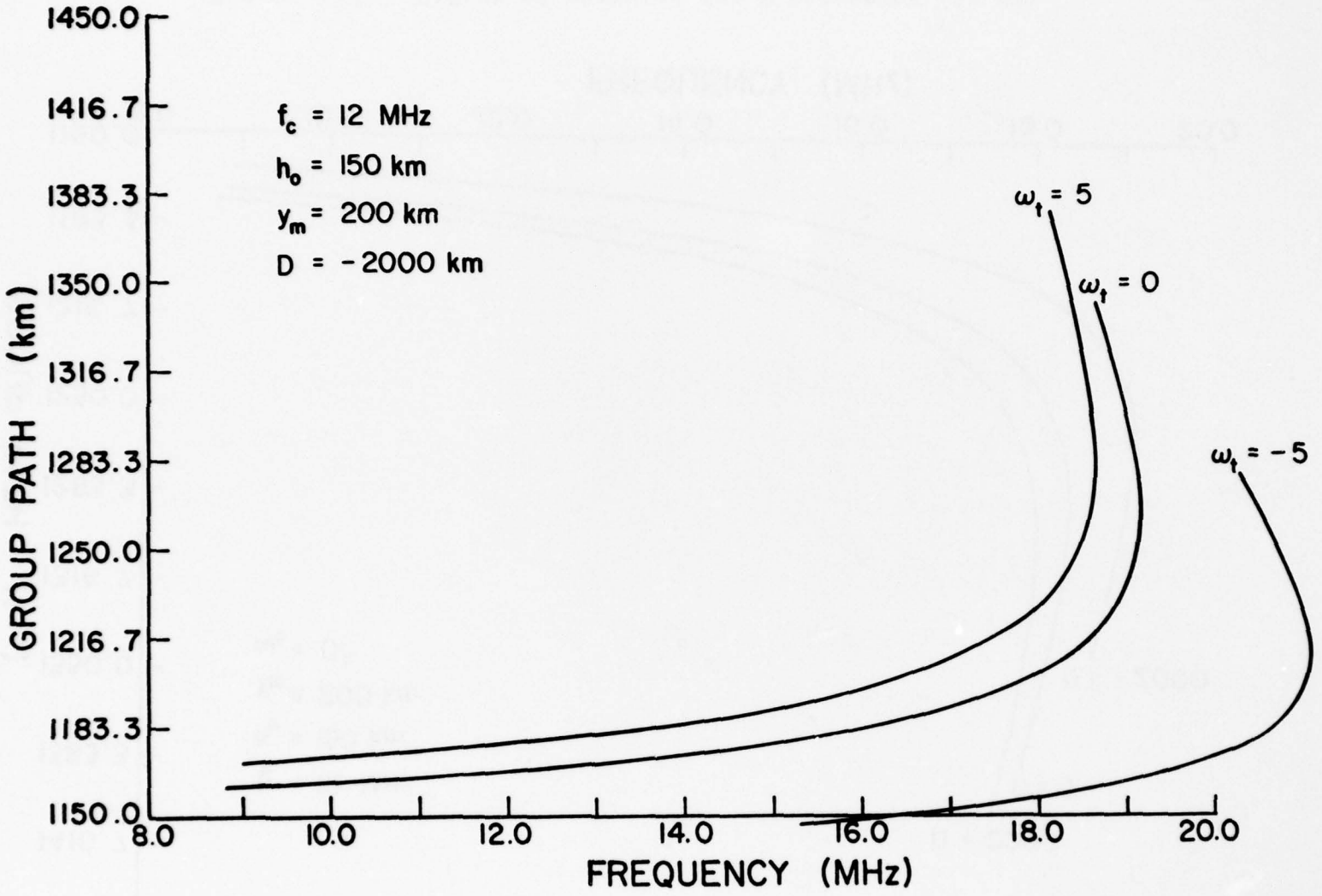


Figure 3.10. Effect of varying the ω_t parameter (in degrees).

tive to changes in D for large values of D (the layer approaches a plane slab) and be highly sensitive for negative D values. These facts tend to suggest that most standard numerical procedures have the potential for inverting for the f_c, h_o, and y_m parameters, but could easily encounter convergence problems when the gradient parameters are introduced.

Rao (1975) successfully inverted an oblique ionogram, using three data points, to the three defining parameters of a concentric quasi-parabolic layer using Newton's iterative technique (this procedure is described and used in Section 3.3.3). An attempt to extend this attractive technique to include two more parameters, D and ω_t, failed because of convergence problems caused by the nonlinear behavior with respect to the gradient parameters. These observations suggest that a slower and more controllable technique is needed to converge to the final solution.

3.3.2 Determination of Initial Set of Parameters

In every complex numerical convergence procedure, it is essential to have a reasonable set of approximate starting values in order to be able to converge to the desired solution. In the present problem we have five data points (frequency-group path pairs) corresponding to five rays with frequencies f_i, group paths P'_i and all having a constant range R. It is desired to find a set of parameters and five elevation angles β_i, so that the reflection of each ray (with frequency f_i and approximate elevation angle β_i) is ensured with small errors in their corresponding group paths and range.

A concentric layer is a practical choice for initial approximation, thus assuming $D = \omega_t = 0$. Using the geometry of Fig. 3.11(a) and applying Martyn's theorem, we can derive approximate expressions for γ_i, the angle of ray at the reflection point and then β_i, the elevation angle:

$$\gamma_i = \sin^{-1}\left[\frac{r_o}{p'_i}\sin\left(\frac{R}{2r_o}\right)\right] \tag{3.10a}$$

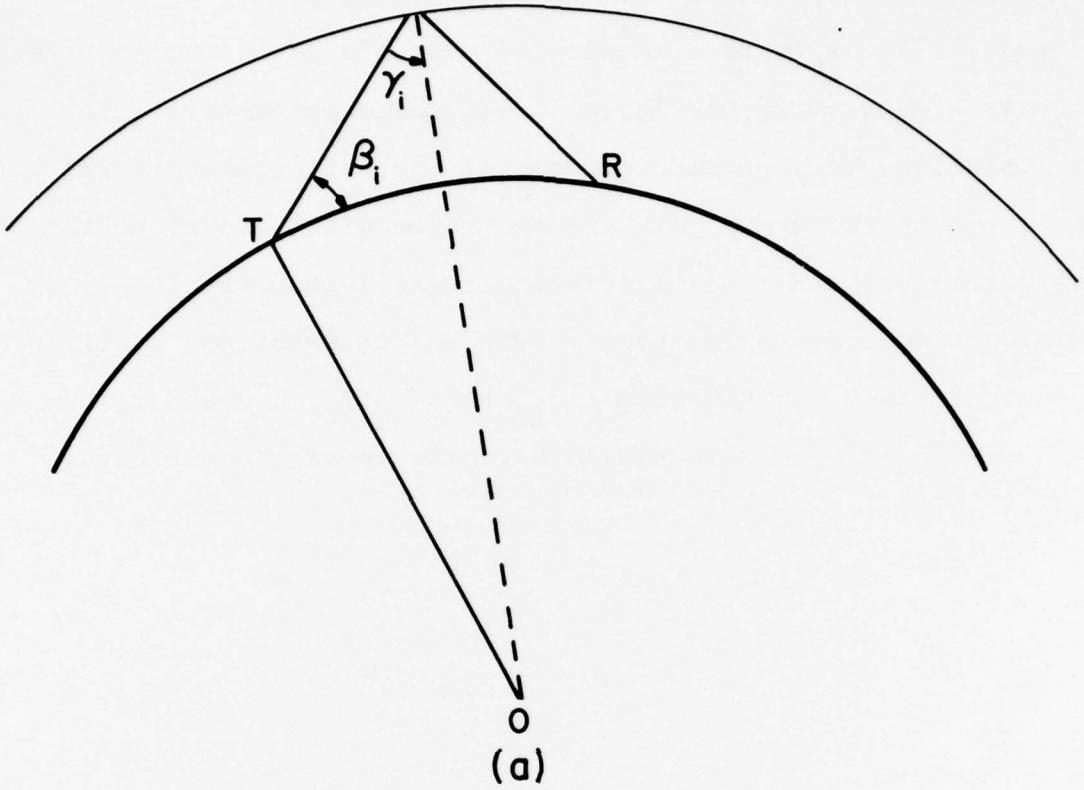
$$\beta_i = \pi/2 - \gamma_i - R/2r_o. \tag{3.10b}$$

The critical frequency f_c is chosen to be equal to the highest ray frequency (f_5) to ensure the reflection of all five rays by the modelled ionosphere. Since the zero frequency ray (with group path P'_o), reflects from the base of the layer, the geometry of Fig. 3.11(a) can be used for determining the initial base height h_o. A linear interpolation of two data points (preferably in the lower frequency region) is made to find an approximate value for P'_o [Fig. 3.11(b)] and then by using the simple geometry of Figure 3.11(a), h_o is determined:

$$P'_o = P'_1 - \left[\frac{P'_2 - P'_1}{f_2 - f_1}\right] f_1 \tag{3.11}$$

$$h_o = r_o \cos(R/2r_o) + \sqrt{\frac{P'_o}{4} - r_o^2 \sin^2(R/2r_o)} - r_o \,. \tag{3.12}$$

Initial value of y_m is not as critical as the other two parameters, we will simply use a physical constraint. Here, we have chosen $y_m = 300 - h_o$(in kilometers). Using these sets of approximate starting values, we are now ready to consider techniques in order



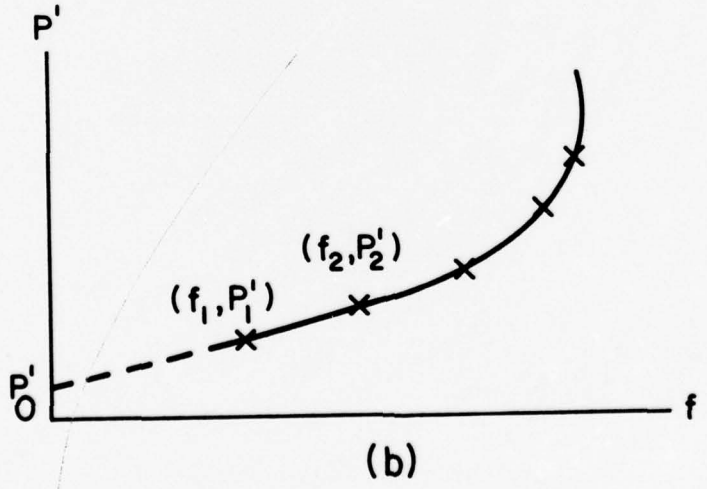


Figure 3.11. Determining an initial set of parameters.

for the solution to converge to an optimal set of parameters.

3.3.3 Inversion of Synthesized Oblique Ionograms

Rao (1975) employed a version of Newton's technique in order to invert a concentric Q-P layer in its three parameters. In each iteration, this procedure requires evaluating the partial derivatives of the group path and the range with respect to the three parameters and three elevation angles (subscript i = 1, 2, 3 refers to the data point used). The matrix equation (3.13) is then used to find the required increments in f_c, h_o, and r_m and the three elevation angles and then update these quantities as shown in (3.14):

$$\begin{bmatrix} \Delta f_c \\ \Delta h_o \\ \Delta y_m \\ \Delta \beta_1 \\ \Delta \beta_2 \\ \Delta \beta_3 \end{bmatrix} =$$

$$\begin{bmatrix} \left(\frac{\partial P'}{\partial f_c}\right)_1 & \left(\frac{\partial P'}{\partial h_o}\right)_1 & \left(\frac{\partial P'}{\partial y_m}\right)_1 & \left(\frac{\partial P'}{\partial \beta}\right)_1 & 0 & 0 \\ \left(\frac{\partial P'}{\partial f_c}\right)_2 & \left(\frac{\partial P'}{\partial h_o}\right)_2 & \left(\frac{\partial P'}{\partial y_m}\right)_2 & 0 & \left(\frac{\partial P'}{\partial \beta}\right)_2 & 0 \\ \left(\frac{\partial P'}{\partial f_c}\right)_3 & \left(\frac{\partial P'}{\partial h_o}\right)_3 & \left(\frac{\partial P'}{\partial y_m}\right)_3 & 0 & 0 & \left(\frac{\partial P'}{\partial \beta}\right)_3 \\ \left(\frac{\partial R}{\partial f_c}\right)_1 & \left(\frac{\partial R}{\partial h_o}\right)_1 & \left(\frac{\partial R}{\partial y_m}\right)_1 & \left(\frac{\partial R}{\partial \beta}\right)_1 & 0 & 0 \\ \left(\frac{\partial R}{\partial f_c}\right)_2 & \left(\frac{\partial R}{\partial h_o}\right)_2 & \left(\frac{\partial R}{\partial y_m}\right)_2 & 0 & \left(\frac{\partial R}{\partial \beta}\right)_2 & 0 \\ \left(\frac{\partial R}{\partial f_c}\right)_3 & \left(\frac{\partial R}{\partial h_o}\right)_3 & \left(\frac{\partial R}{\partial y_m}\right)_3 & 0 & 0 & \left(\frac{\partial R}{\partial \beta}\right)_3 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P'_1 \\ \Delta P'_2 \\ \Delta P'_3 \\ \Delta R_1 \\ \Delta R_2 \\ \Delta R_3 \end{bmatrix} \quad (3.13)$$

$$\left.\begin{aligned} [f_c]_{new} &= [f_c]_{old} + \Delta f_c \\ [h_o]_{new} &= [h_o]_{old} + \Delta h_o \\ &\cdots\cdots \\ &\cdots\cdots \end{aligned}\right\} \qquad (3.14)$$

where $\Delta P_i'$ and ΔR_i are defined to be the differences between the actual and the computed values of group path and range using the latest values of the parameters.

The iteration in (3.14) is repeated until the errors in the group path and range satisfy a given convergence condition. As mentioned in Section 3.3.1, our attempt to extend this convenient technique to include two more parameters, D and ω_t, failed to result in a stable procedure. However, by employing a bisection search on D and ω_t, and repeating the iteration for every change in these two parameters, the procedure becomes stable. Fig. 3.12 contains the flow chart for such a technique, and for simplicity, it only optimizes D with ω_t constant. This technique can also be demonstrated graphically in Fig. 3.13. Several oblique ionograms are synthesized for different values of the D parameter, satisfying three common data points at all times (step 3.14 is repeated for every value of D). It is clear that by selecting the fourth and the fifth data point in the high frequency region of the ionograms, which is the most sensitive region with respect to the gradient parameters, a bisection search can be used to optimize the values of D and ω_t.

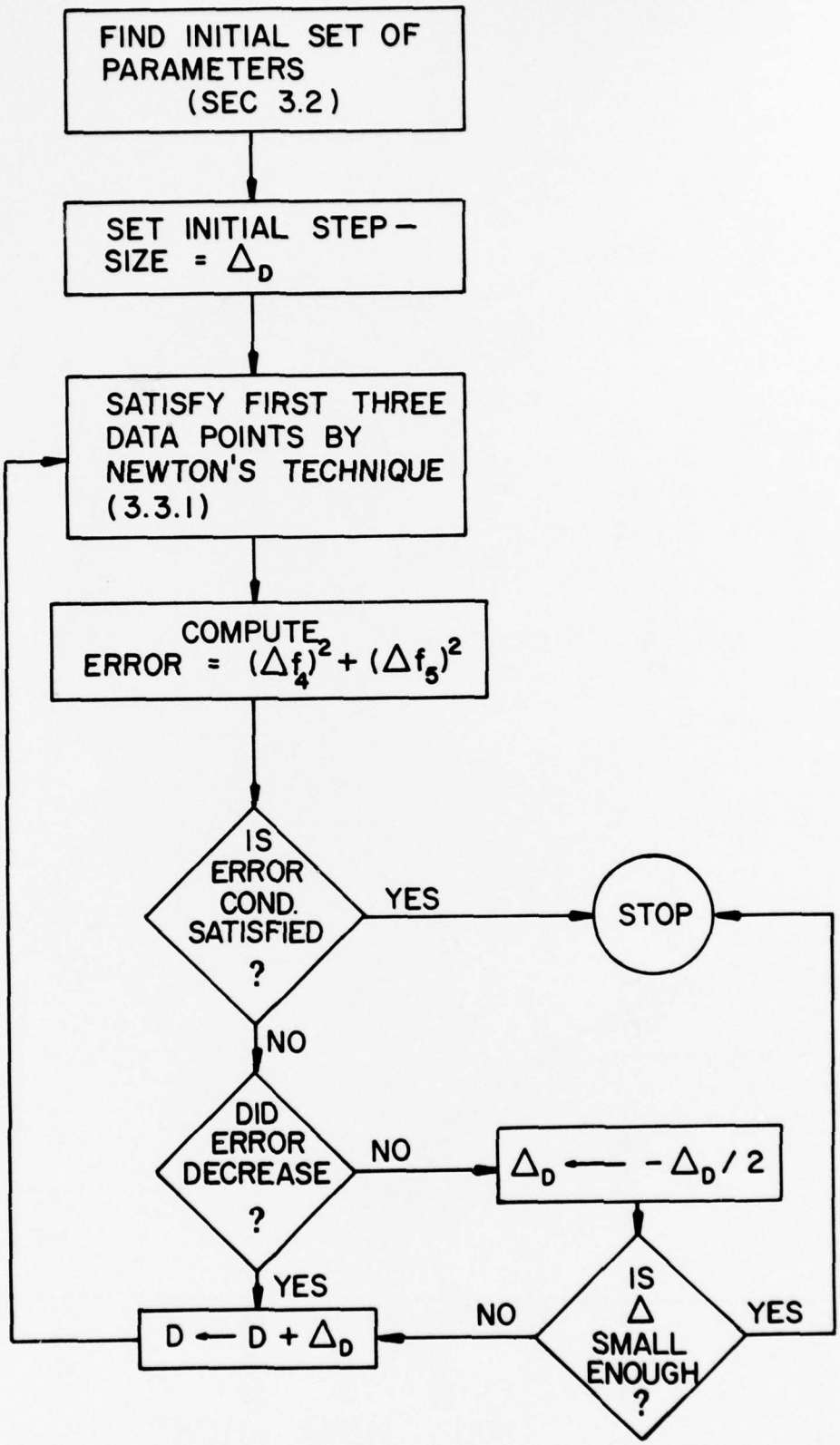


Figure 3.12. Inverting numerically synthesized data.

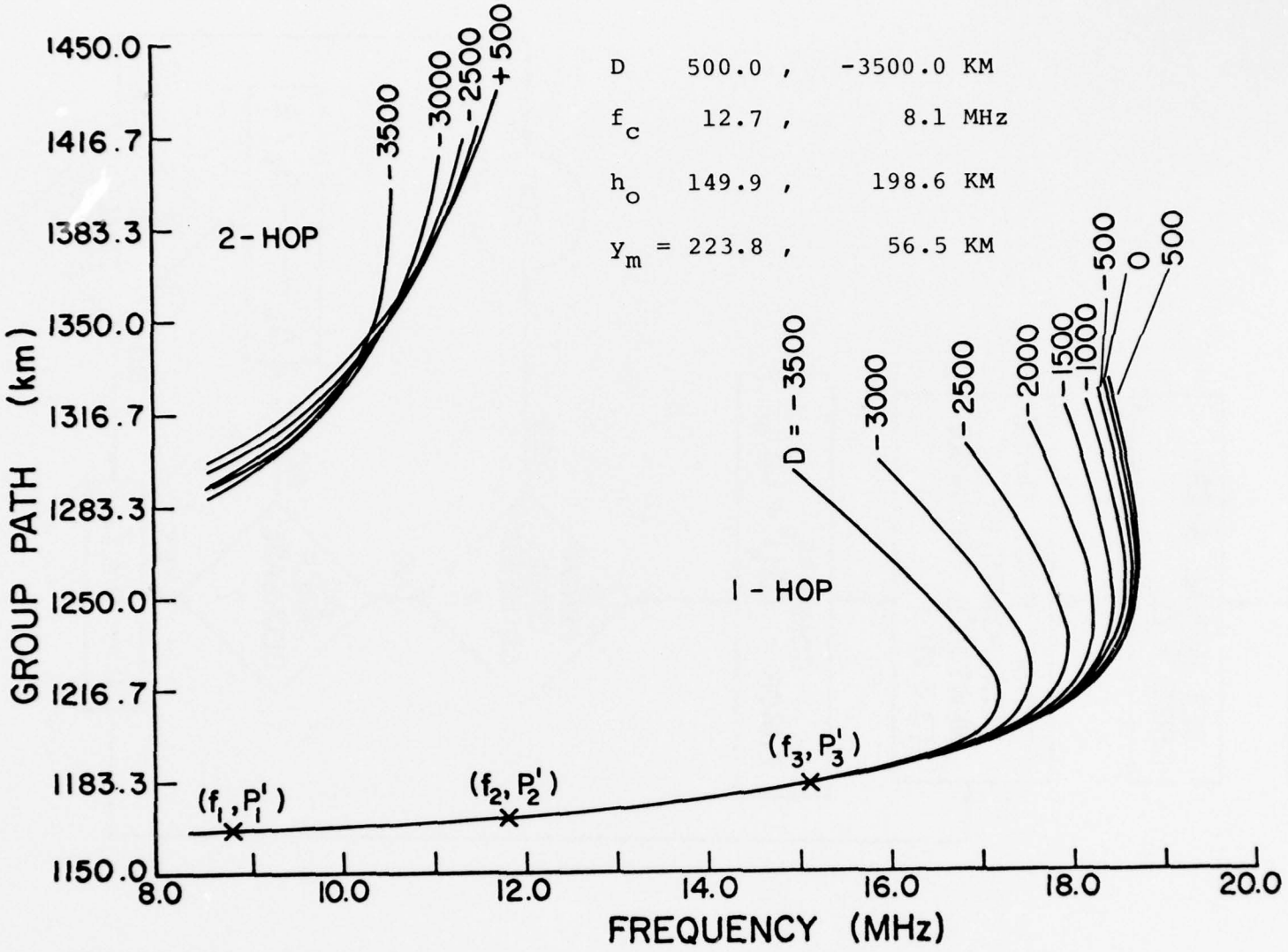


Figure 3.13. Synthesized oblique ionograms for Range = 1111.7 km and ω_t = 0°. The parameter D is successively changed by 500 km. For each D, the remaining three parameters are adjusted so as to force the curve through the three points marked by X. The parameters for the two extreme curves are given above.

The error function used for the bisection search was initially proposed to be:

$$\text{Error} = (\Delta P'_4)^2 + (\Delta P'_5)^2 \quad . \tag{3.15}$$

However, as apparent from Fig. 3.13, for a given set of parameters, a ray with frequency f_i might penetrate the ionosphere making $\Delta P'_i$ approach infinity. This problem can be solved by considering the ray with group path P'_i and finding the error in its frequency Δf_i (this value is always finite). Based on this explanation, the final error function used in the inversion technique is chosen to be:

$$\text{Error} = (\Delta f_4)^2 + (\Delta f_5)^2 \quad . \tag{3.16}$$

It is important to note that Δf_i ($i = 4, 5$) is defined to be the frequency deviation of the modelled ray, from the actual data value, which has its group path equal to P'_i and the range equal to R (the elevation angle β_i is adjusted so that the error in range vanishes). This technique is completely programmed and simulated on an IBM-360 digital computer, and Table 3.1 contains an example in which a numerically synthesized oblique ionogram is inverted with excellent accuracy to the model's parameters.

TABLE 3.1. Inversion of Synthesized Oblique Ionograms Using the Technique in Section 3.3.3

Model parameters: f_c = 12 MHz

h_o = 150 km

y_m = 100 km

D = -2000 km

ω_t = 0

Range = 1111.8 km

Synthesized data points used:

i	1	2	3	4	5
f_i	12.23	19.03	22.10	23.57	24.15 MHz
P'_i	1161.3	1173.1	1186.3	1200.9	1217.2 km

Inversion results:

	f_c MHz	h_o km	y_m km	D km	ω_t deg	$\Delta f_4^2+\Delta f_5^2$ $(MHz)^2$
1st iteration	13.07	136.4	125.3	0	0	.031
2nd iteration	12.96	138.5	123.5	-500	0	.026
3rd iteration	12.78	141.3	119.6	-1000	0	.018
4th iteration	12.48	144.9	112.3	-1500	0	.0081
5th iteration	11.99	150.0	99.89	-2000	0	.00004

Since Newton's technique attempts to converge the error to zero, problems can be encountered in case of real data because of the finite errors involved in the final solution. In order to adapt this technique for inversion of real data, a steepest descent method (discussed in detail later in Section 3.3.4) is employed to find an optimal set of starting parameters for three of our data points. Then the group paths of the three rays were computed using the parameters and used to replace the original group path values (Fig. 3.14). These adjusted data points are indeed well behaved and we can continue our procedure as before for minimizing $(\Delta f_4)^2 + (\Delta f_5)^2$. Table 3.2 contains a numerical example for this technique; it must be noted that in this example the fourth and the fifth data points are chosen in the nearly horizontal region of the ionogram, therefore making Δf_4, Δf_5 correspond to a much smaller error in group paths $(\Delta P'_4, \Delta P'_5)$.

Because of the two-step optimization involved in inverting real oblique ionograms, the final result is not guaranteed to be optimum. In fact, different combinations of the real data points used in the inversion procedure tend to converge to different sets of parameters (in that case, the combination with the least amount of error is the final solution). Even though the technique converges quite rapidly in the synthesized data case, the computation time becomes an order of magnitude larger for the real data case, which could be considered undesirable.

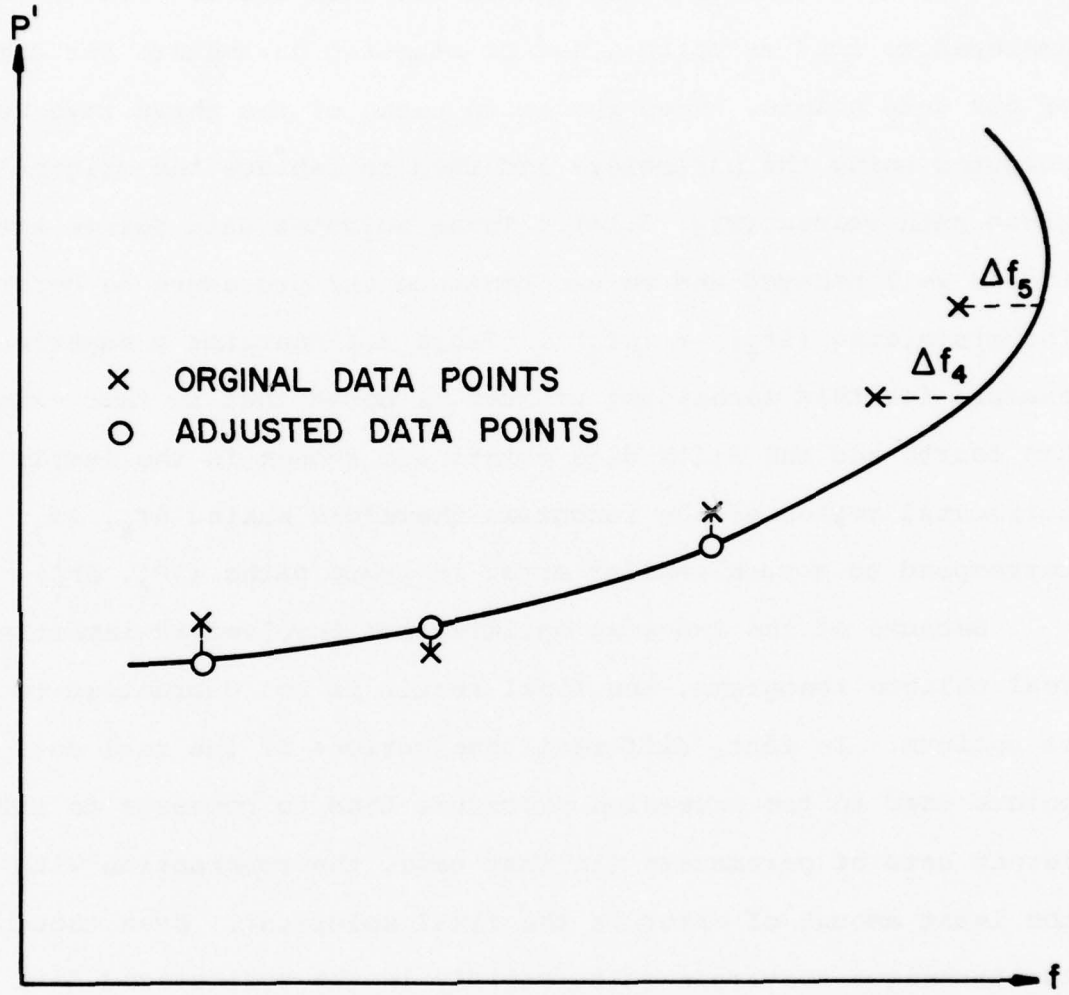


Figure 3.14. A technique for the inversion of real data. The first three data points are adjusted so that the technique in Section 3.3.3 can be employed for minimizing $(\Delta f_4)^2 + (\Delta f_5)^2$.

TABLE 3.2. Inversion of Real Oblique Ionograms,

Using the Technique in Section 3.3.3

Range = 1421.5 km

Frequency, MHz	14	18	22	16	20
actual (P') km	1532.0	1540.4	1556.1	1528.5	1539.0
adjusted (P') km (after steepest descent)	1527.0	1530.3	1559.4	same	same

Parameters	f_c MHz	h_o km	y_m km	D km	ω_t deg	$(\Delta f_4)^2+(\Delta f_5)^2$ $(MHz)^2$
Iteration #1	12.87	236.7	74.2	0	0	26.8
Iteration #2	12.73	244.6	72.1	-1000	0	27.4
Iteration #3	12.80	240.3	73.3	-500	0	26.6
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
Iteration #7	12.82	239.2	73.5	-375	0	26.2

Based on the controllable convergence behavior of the steepest descent method, a different approach to the inversion of real data is proposed and discussed in detail in the following section.

3.3.4 Inversion of Experimental Oblique Ionograms

Actual oblique ionograms are not smooth and nicely behaved as the numerically generated ones. In fact, one can frequently find discontinuities, and other peculiarities on the plots of actual data. Ideal zero error cannot be accomplished when actual data points are used, requiring the optimization of the error function to a minimum value.

It is convenient to approximate the gradient, in the actual data case, by using only one parameter. Referring back to Fig. 3.5, it is apparent that a wide range of gradients can be obtained by holding ω_t at a reasonable value and varying D. In the examples to follow, $\omega_t = 0$ is chosen not only to span a wide range of gradient values by varying D, but also to force h_o to represent the actual base height value of the layer above the transmitter. The steepest descent method, with it's controlled step size is an ideal choice for simultaneous inversion of the remaining four parameters. Figure 3.10 outlines the procedure in a simple flow chart.

Since, in fact, we are finding a best fit curve through the data points, we need not be limited to using five points, and obviously the more data points used the more descriptive the model will be. The error function is chosen to be:

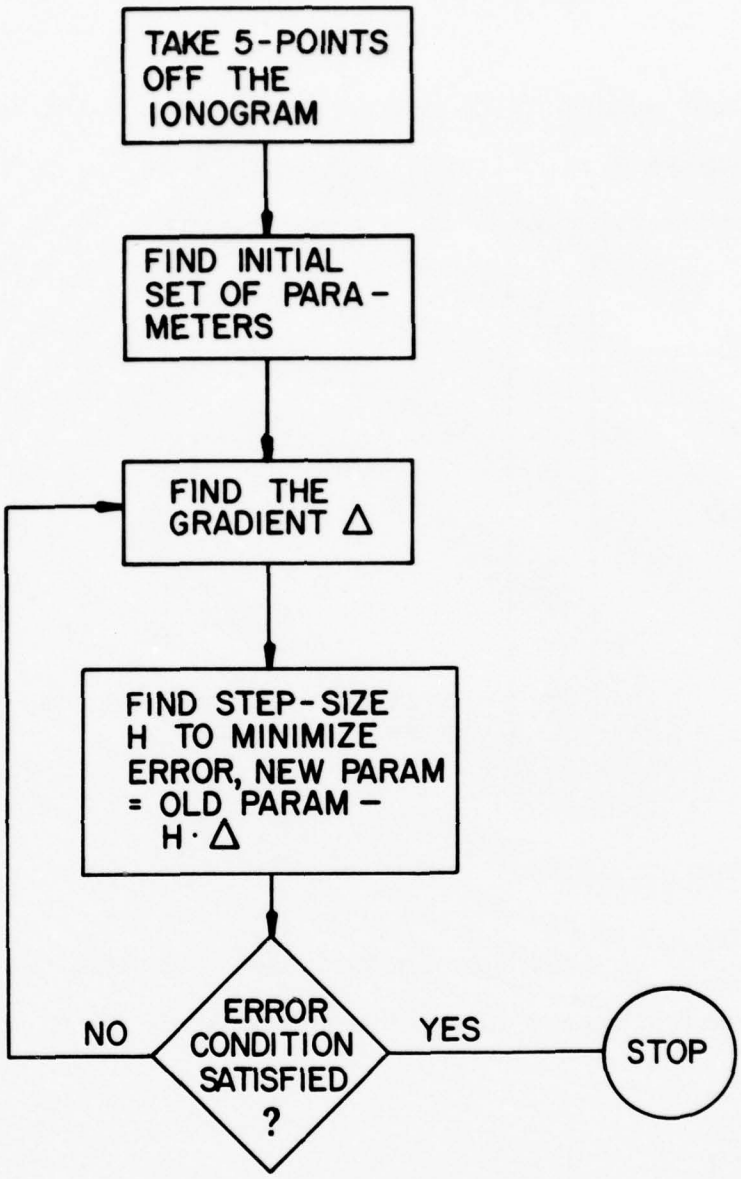


Figure 3.15. Method of steepest descent for inverting actual data.

$$E = \sum_{i=1}^{N} (\Delta P_i')^2$$

where N is the number of data points and $\Delta P_i'$ is the error in the i^{th} group path as defined in Section 3.3.

The gradient vector Δ is defined to be:

$$[\Delta] = \begin{bmatrix} \frac{\partial f_c}{\partial E} \\ \\ \frac{\partial h_o}{\partial E} \\ \\ \frac{\partial y_m}{\partial E} \\ \\ \frac{\partial D}{\partial E} \end{bmatrix}$$

Evaluated at the current parameter values

The theory of steepest descent uses the fact that continuous functions decrease along the negative of their gradient. Thus

$$\text{New} \begin{bmatrix} f_c \\ h_o \\ y_m \\ D \end{bmatrix} = \text{Old} \begin{bmatrix} f_c \\ h_o \\ y_m \\ D \end{bmatrix} - H \times [\Delta]$$

where H is a positive scalar step size. For this iteration to be most efficient, we like to choose an H such that the resultant error function from the new parameter is at a minimum. This can be accomplished by simply setting H to zero, and then incrementing it by an interval such that none of the resultant parameters change by more than a specified amount (this ensures a slower but smoother convergence to the final solution not accomplished by the Newton's technique in Section 3.3.3). Error is computed for each value of H and minimized by performing a simple interval halving routine on H. The old parameters are replaced by using the optimal step size H and if the convergence criterion is not met, the gradient is reevaluated and the procedure is repeated. This technique is also programmed on a digital computer and Tables 3.3 and 3.4 contain examples using five data points from actual oblique ionograms.

TABLE 3.3(a). Inversion of an Actual Ionogram

Data points scaled from oblique ionogram (Range = 1421.5 km):

i	1	2	3	4	5
f_i(MHz)	14.	16.	18.	20.	22.
P'_i(km)	1536	1536.5	1540.5	1545.3	1555.5

Inversion results:

Iteration#	f_c(MHz)	h_o(MHz)	y_m(km)	D(km)	Error = $\sum_{i=1}^{5} (\Delta P'_i)^2$
0	22.00	252.2	50.0	0.0	290.1
1	18.28	254.5	51.6	342.2	133.8
2	12.67	250.0	55.5	-797.7	14.5
3	12.47	249.9	55.5	-836.7	12.5
4	12.71	249.6	55.5	-831.4	12.3
.					
.					
.					
10	12.63	249.0	55.6	-803.6	11.1

The final individual group path errors (in km):

$\Delta P'_1 = 1.17$, $\Delta P'_2 = -1.46$, $\Delta P'_3 = -1.00$, $\Delta P'_4 = -1.11$,

$\Delta P'_5 = 2.30$

TABLE 3.3(b) Inversion of an Actual Ionogram

Data points scaled from oblique ionogram: (Range = 1421.5 km):

i	1	2	3	4	5
f_i(MHz)	14.	16.	18.	20.	22.
P'_i(km)	1533	1539.0	1546.2	1555.8	1566.0

Inversion results:

Iteration #	f_c(MHz)	h_o(km)	y_m(km)	D(km)	Error = $\sum_{i=1}^{5} (\Delta P'_i)^2$
0	22.0	220.0	80.0	0.0	5838.5
1	12.58	227.9	88.5	1237.3	20.6
2	12.60	227.9	88.6	1239.1	20.5
3	12.59	227.9	88.5	1239.3	20.4

The final individual group path errors (in km):

$\Delta P'_1 = .49$, $\Delta P'_2 = 1.6$, $\Delta P'_3 = 2.3$, $\Delta P'_4 = 2.5$,

$\Delta P'_5 = -2.3$

3.4 Inversion of Backscatter Leading Edge

The most important task of this project is to develop a technique of inversion of the sweep frequency backscatter leading edge data for various azimuthal directions, to an ionosphere varying in three dimensions, which can then be applied for the purpose of other ray path computations. In this section, we report our work on this problem. Several of the results can be found in the various progress reports but not in the order presented here. Here we discuss the solution of the problem in a step-by-step manner and its eventual application to the important problem of finding the ground range for a measured group path for reflection from a target.

3.4.1 Basic Inversion Technique

In a paper by Rao (1974), it has been demonstrated that three data points taken from the leading edge of a backscatter ionogram can be used to obtain the quasi-parabolic layer parameters f_c, r_b, and r_m, corresponding to those three points. Briefly, this method consists of starting with an initial estimate of the layer parameters and then calculating the minimum group paths, P'_{c_i}, corresponding to the three frequencies at which data points are taken from the leading edge. The differences between these computed minimum group path values and the actual minimum group path values, P'_i, from the leading edge are then used in an iterative procedure to obtain a final solution for the quasi-parabolic layer parameters. This final solution is such that the differences between P'_i and P'_{c_i} are less than a certain specified value (theoretically zero).

When the above described procedure was applied to synthesized leading edge data polluted intentionally by adding or subtracting a few kilometers from each of the P'_i values, thereby simulating the actual data case, it was found that in order for the differences between P'_i and P'_{c_i} to go down to less than a small specified value, one or more of the layer parameters often got incremented to physically unrealizable values. The reason for this was found to be that the backscatter leading edges corresponding to quasi-parabolic layer profiles have certain shapes and it is not in general possible to fit a synthesized leading edge exactly through three points on an experimental ionogram. In view of this, the method was modified for use in this work.

The modification involves the minimization of the following sum-squared error function:

$$E\ (r_b,\ r_m,\ f_c) = \sum_{i=1}^{3} [P'_{c_i}\ (r_b,\ r_m,\ f_c) - P'_i]^2 \tag{3.17}$$

Fig. 3.16 depicts the strategy graphically. Assume that (f_1, P'_1), (f_2, P'_2), and (f_3, P'_3) are the points we have chosen from the backscatter leading edge. As before, we start the procedure with an initial set of layer parameters, which are denoted by $(r_b, r_m, f_c)_0$. Corresponding to these starting parameters is a unique backscatter ionogram leading edge trace $[P'(f)]_0$. We note that the error function E is simply the sum of the squared distances between the ordinates of the $[P'(f)]_0$ curve corresponding to the frequencies f_1, f_2, and f_3, and the chosen backscatter ionogram minimum group paths P'_1, P'_2, and P'_3. As r_b, r_m, and f_c

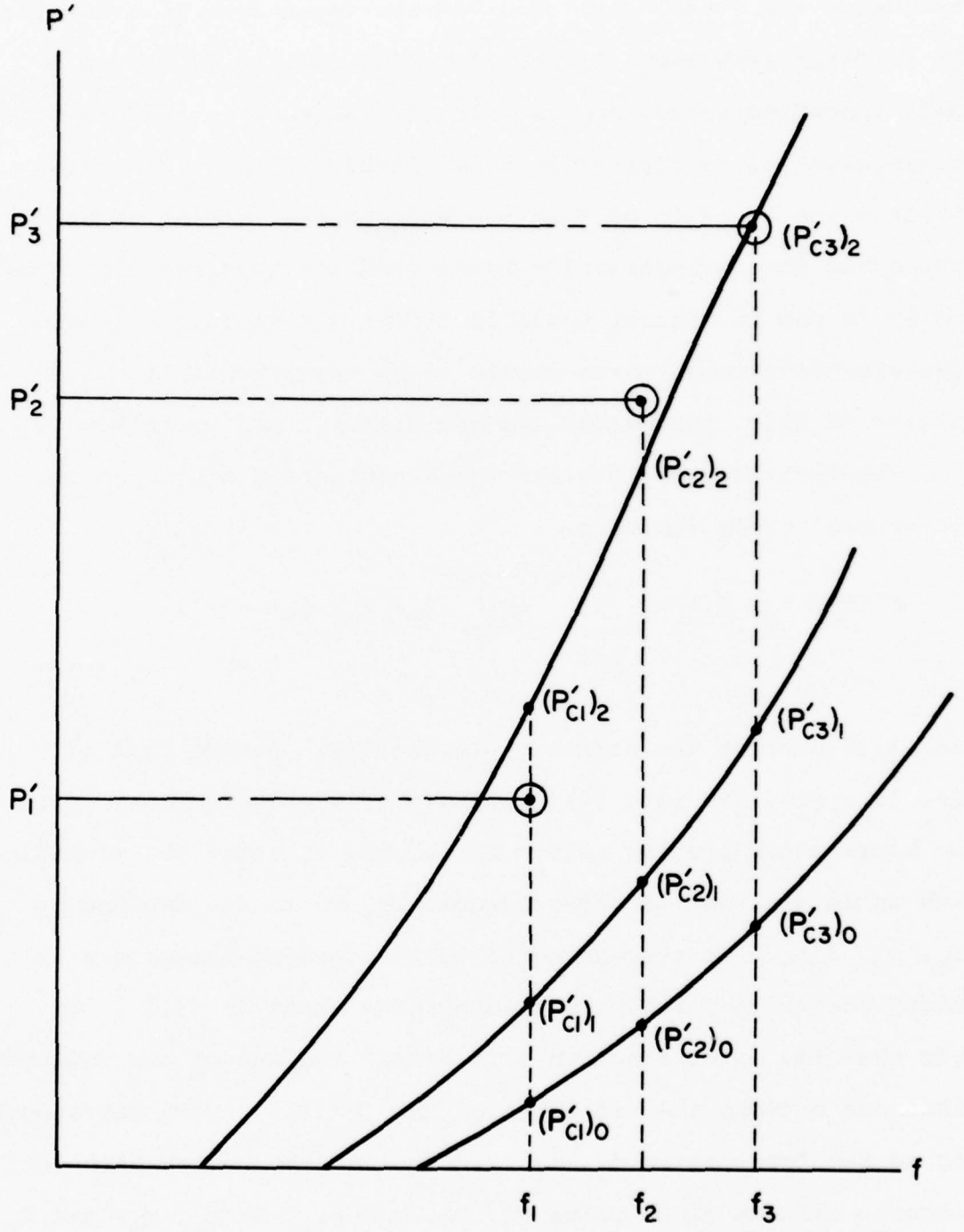


Figure 3.16. Graphical illustration of the least sum-squared error method of backscatter ionogram leading-edge inversion for a Q-P ionosphere.

are varied to minimize E, we see that we are effectively fitting the P'(f) curve to the three data points in the least-squared error sense. Furthermore, the magnitude of the minimum of E provides an indication of the noisiness of the data with reference to a synthetic leading edge. If the data points are from a synthetic curve, computed by assuming the Q-P model, the error E will then be zero.

The error E is in general a nonlinear function of r_b, r_m, and f_c and an iterative method is required to seek its minimum. Numerical minimization techniques are available which seek a local minimum for a given starting point in parameter space. The nonlinear minimization alogorithm employed in this study is described by Fletcher and Powell (1963) and is supplied in IBM's scientific subroutine package as subroutine DFMFP. This routine was modified for use with our computer. The Fletcher-Powell method performs, for each iteration step, a linear minimization along a direction determined by the current gradient and an updated estimate of the Hessian. The size of the steps taken through the parameter space as the minimum is sought is proportional to the difference between the current value of the function to be minimized (e), and the user-supplied minimum value (EST), typically taken to be 0.001 km^2, and inversely proportional to the current magnitude of the function gradient ($|\vec{\nabla}E|$). In this manner, rapid convergence is assured. The minimization procedure is terminated when the function value has not changed by more than a user-specified value (EPS), or if $|\vec{\nabla}E|$ has become less

than EPS. This routine requires an external function subprogram, which supplies DFMFP with E and $\vec{\nabla}E$ as the parameters r_b, r_m, and f_c are varied. The components of $\vec{\nabla}E$, which involve derivatives of P' with respect to the layer parameters are calculated using the expressions provided by Rao (1974) and repeated here in the appendix.

Thus far we have discussed the procedure for the inversion of three data points on a backscatter leading edge to Q-P layer parameters. Since each point on the leading edge corresponds to a different ground range, it is obvious that the three points must be chosen fairly close together if horizontal gradients are present. The three data points must correspond to three ionospheric propagation paths which are in such horizontal proximity as to validate the assumption of a locally horizontally uniform ionosphere. These considerations give rise to a method of deriving the horizontal ionization gradients from the leading edge. Three closely spaced points on the leading edge just above the critical frequency at the backscatter sounder site can be used to determine the layer parameters of the ionosphere at a range which is in close proximity to the site. Next, three closely spaced points further up the leading edge can be used to obtain the layer parameters at a greater range than that corresponding to the previous set of data points. In obtaining these layer parameters, the solution for the first set of three data points can be used as the starting solution. A repetition of this process continuously along the leading edge for successive sets of three data points yields the layer parameters as a function

of distance away from the sounder site. This is the technique we employ in the following sections.

3.4.2. Inversion of Synthesized QP Layer Data

The basic technique of inversion of points on the backscatter leading edge for quasi-parabolic layer parameters is tested by simulating the leading edge for assumed QP layer parameters and then polluting the simulated data by adding a few kilometers to one or more of the points. The assumed values of the layer parameters are r_b=6570 km, r_m=6720 km, and f_c=5.0 MHz. Minimum group paths for frequencies of 10, 11, and 12 MHz are synthesized to be P'_1=1866.1 km, P'_2=2133.8 km, and P'_3=2441.3 km, respectively. Table 3.4 shows the results of the test by indicating the errors introduced in P'_1, P'_2, and P'_3 and the corresponding solution obtained by inverting the polluted data. For each case, starting values of layer parameters used are r_b=6500 km, r_m=6650 km, and f_c=4.0 MHz. From Table 3.4, it can be seen that even small measurement errors seem to appreciably affect the results of inversion of the leading edge data.

To investigate further the sensitivity of the inversion process to measurement errors, the procedure has been generalized to permit the use of a variable number (N) of data points in the inversion technique, that is, for minimizing the error function E. Leading edge data are synthesized for an earth-concentric QP layer having the parameters r_b=6570 km, r_m=6720 km, and f_c=7.0 MHz and at increments of 0.25 MHz throughout the frequency range of 10.0 to 13.0 MHz. These data are then made noisy by the addition of samples of a random variable uniformly distributed over the interval

TABLE 3.4

SYNTHESIZED NOISY BACKSCATTER IONOGRAM LEADING EDGE DATA INVERSION

ASSUMING A SPHERICAL EARTH-CONCENTRIC QUASI-PARABOLIC IONOSPHERE MODEL

Synthesized points on backscatter ionogram leading edge:

Point Number	Frequency (MHz)	Minimum Group Path km
(1)	10.0	1866.1
(2)	11.0	2133.8
(3)	12.0	2441.3

Layer parameters used in synthesis:

r_b=6570.0 km r_m=6720.0 km f_c=5.0 MHz

Starting values of layer parameters:

r_b=6500.0 km r_m=6650.0 km f_c=4.0 MHz

Estimated Minimum Sum-Squared Error (EST) = .001 km^2

Specified Absolute Error (EPS) = .001 km

Error introduced in P_1'(km)	Error introduced in P_2'(km)	Error introduced in P_3'(km)	Resulting inverted layer parameter r_b(km)	r_m(km)	f_c(MHz)	Value of E$(km)^2$ Minimum
0	0	0	6570.6	6719.2	4.993	.00023
+1	-1	+1	6621.7	6700.0	4.743	.838
-1	0	-1	6550.2	6759.9	5.324	.240
-1	0	0	6543.9	6777.4	5.444	.00593
-1	-1	0	6564.0	6724.1	5.0311	.09162
0	0	+1	6566.6	6721.7	5.0105	.0844
+2	0	0	6625.4	6703.3	4.782	.109
0	+2	0	6555.0	6749.6	5.248	1.83
0	+2	+1	6535.7	6825.1	5.772	.411
0	4	0	6533.7	6853.7	5.966	6.01

-1.0 km to +1.0 km, as illustrated in Table 3.5. These random noise values were chosen to be the rightmost three digits from consecutive entries on a page of the local telephone directory. Using an initial estimate of layer parameters r_b=6600 km, r_m=6700 km, and f_c=9.0 MHz. These data are inverted by using different sets of points, as shown in Table 3.6. It can be seen that the percent error in determining the actual parameters of the QP layer decreases as the number of noisy data points used in the inversion process is increased.

Table 3.5. Noisy Backscatter Leading Edge Synthesis -- Assuming Earth-Centered Q-P Ionosphere r_b=6570 km, r_m=6720 km, and f_c=7.0 MHz

Point No.	f_i (MHz)	Precise P'_{M_i} (km)	Additive Noise (km)	Noisy P'_{M_i} (km)
1	10.00	1233.388	.114	1233.502
2	10.25	1268.959	-.096	1268.863
3	10.50	1304.946	-.257	1304.689
4	10.75	1341.369	-.332	1341.037
5	11.00	1378.250	-.576	1377.674
6	11.25	1415.611	.945	1416.556
7	11.50	1453.475	-.988	1452.487
8	11.75	1491.867	.062	1491.929
9	12.00	1530.814	-.600	1530.214
10	12.25	1570.344	-.760	1569.584
11	12.50	1610.486	-.271	1610.215
12	12.75	1651.272	+.931	1652.203

Table 3.6. Results of Inversion of the Data in Table 3.4, Using Various Numbers of Points in E.

Number of leading edge pts used	Optimized value of E (km^2)	% error in r_b	% error in r_m	% error in f_c
N=3 using points 1, 6, 12	.17203	.1040	.155	1.77
N=6 using points 1,3,5, 7,9,12	1.8730	.052	.0327	.659
N=9 using points 1,2,4,5, 7,8,10, 11,12	2.3943	.0472	.0193	.129

3.4.3 Inversion of Synthesized Data Involving Horizontal Gradients

To investigate the technique of inversion of successive sets of three data points on the leading edge for the horizontal gradients, the eccentric QP layer model discussed in Sec.3.2.2 is employed. First, for assumed layer parameters of r_b=6570 km, r_m= 6720 km, f_c=5.0 MHz, D=1000 km, and ω_T=0, data points on the leading edge trace are computed for every 0.5 MHz from 7.0 to 11.0 MHz. Next, the points are grouped in four sets of three, starting with (7.0, 7.5, 8.0), then (8.0, 8.5, 9.0), etc. Each group of three data points is then inverted for the parameters of a concentric QP layer. The first group is inverted by using an arbitrary starting set of layer parameters (r_b=6500 km, r_m=6650 km, and f_c= 4.0 MHz). Succeeding groups of data are inverted by starting with the layer parameters obtained for the previous group of data points. The synthesized data as well as the computed layer parameter values are shown in Table 3.7. It can be seen from this table that inverted layer parameters indeed exhibit horizontal gradient. When the magnitude of the gradient in r_b or r_m is compared with that predicted from the model, with the use of Fig. 3.5, they are found to be in good agreement.

For a practical example of the backscatter ionogram leading edge for horizontal gradients, we consider the data supplied by the contract sponsor. These data of minimum group paths are simulated by 3-D ray tracing for models of the ionosphere, derived from ITS-78 predictions by applying various linear tilts and gradients to a median profile. This median profile, shown in

TABLE 3.7.

INVERSION OF SYNTHESIZED BACKSCATTER LEADING EDGE DATA

FOR AN ECCENTRIC QUASIPARABOLIC LAYER

Frequency MHz	Synthesized Data P'(km)	Range (km)	Computed Layer Parameters r_b(km)	r_m(km)	f_c(MHz)	E (km^2)
7.0	1219.238	1004.889	6531.6	6660.3	4.1614	.00064
7.5	1323.839	1120.756				
8.0	1433.133	1238.934	6529.7	6659.3	4.1445	.00311
8.5	1547.939	1360.683				
9.0	1669.278	1487.355	6526.9	6657.3	4.1159	.01730
9.5	1798.458	1620.455				
10.0	1937.207	1761.826	6623.1	6652.5	4.0570	.120
10.5	2087.893	1913.875				
11.0	2253.933	2079.975				

Fig. 3.17, is assumed to represent the vertical profile at a range of 3500 km from a backscatter sounder site. A linear increase or decrease of the layer parameters along the sounder boresight is then introduced as follows:

(i) Gradients: $f_oF2 = (f_oF2)_{median} \pm na$

$h_mF_2 = (h_mF2)_{median}$

(ii) Tilts: $f_oF2 = (f_oF2)_{median}$

$h_mF2 = (h_mF2)_{median} \pm pb$

where n, p=0,1,2,3 and a=1 MHz/3500 km, and b=10 km/3500 km. The simulated minimum group values are shown in Table 3.8. The numbers in each box corresponding to one leading edge. Thus we have 31 leading edges.

Each set of leading edge data in table 3.8 are inverted for QP layer profiles along the boresight by considering overlapping sets of the three data points. Thus a set of five data points for the minimum group path yield three sets of QP layer parameters corresponding to different ranges from the sounder location. The inversion results are shown in Tables 3.9 through 3.12, which correspond to the ground range, peak height of the layer, base height of the layer, and the critical frequency of the layer, respectively. These results indicate that while it is difficult to compare quantitatively the gradients with those of the original model in view of the differences in the two vertical profiles, the directions of the gradients are generally determined correctly by this procedure.

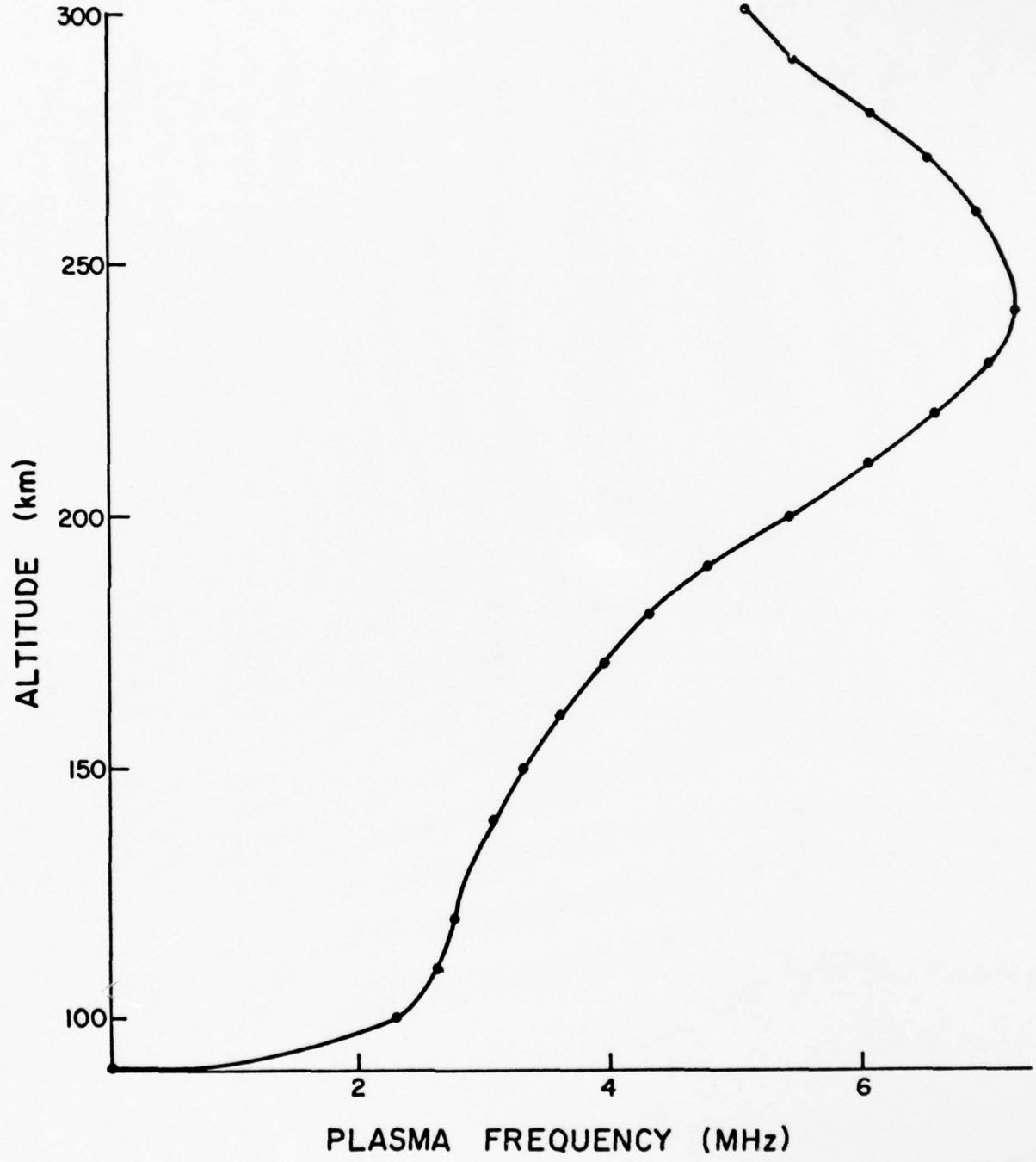


Figure 3.17. Nominal electron density profile.

Table 3.8

Group Path (km) to Leading Edge of Backscatter Ionogram for Various Linear Tilts and Gradients.

n	f	p = -3	-2	-1	0	1	2	3
	11 MHz	1036	1038	1041	1044			
	14	1388	1395	1402	1409			
-3	17	1815	1828	1838	1852			
	20	2398	2423	2459	2499			
	23	--	--	--	--			
	11	1025	1028	1031	1033			
	14	1364	1370	1377	1384			
-2	17	1766	1777	1788	1802			
	20	2288	2312	2334	2372			
	23	3128	3216	3305	3395			
	11	1015	1017	1021	1023			
	14	1343	1349	1354	1361			
-1	17	1724	1736	1743	1755			
	20	2194	2215	2235	2264			
	23	2855	2926	2992	3044			
	11	1005	1008	1011	1014	1017	1019	1021
	14	1322	1328	1334	1338	1345	1351	1357
0	17	1685	1674	1704	1715	1725	1743	1761
	20	2114	2132	2152	2184	2209	2232	2254
	23	2698	2735	2785	2848	2875	2905	2930
	11				1004	1006	1009	1012
	14				1320	1324	1330	1336
1	17				1677	1688	1705	1718
	20				2110	2133	2155	2176
	23				2679	2717	2751	2787
	11				995	998	1000	1003
	14				1302	1307	1312	1317
2	17				1643	1654	1668	1682
	20				2049	2069	2088	2107
	23				2550	2584	2617	2647
	11				987	989	992	994
	14				1284	1289	1294	1299
3	17				1611	1622	1636	1648
	20				1993	2012	2029	2047
	23				2449	2475	2503	2531

Table 3.9

Ground Range (km)

n \ p		-3	-2	-1	0	1	2	3
	f=14 MHz	1291	1302	1310	1320			
-3	17	1722	1738	1751	1768			
	14	1245	1256	1268	1282			
-2	17	1658	1671	1684	1706			
	20	2182	2208	2229	2278			
	14	1200	1218	1221	1237			
-1	17	1597	1616	1622	1641			
	20	2071	2099	2113	2157			
	14	1145	1104	1173	1184	1198	1220	1358
0	17	1530	1539	1563	1580	1595	1625	1755
	20	1969	2006	2018	2061	2085	2113	2251
	14				1132	1150	1178	1193
1	17				1517	1537	1569	1585
	20				1966	1993	2024	2046
	14				1061	1083	1122	1145
2	17				1450	1471	1508	1528
	20				1878	1902	1937	1957
	14				958	1006	1052	1084
3	17				1381	1404	1443	1466
	20				1795	1816	1849	1871

Table 3.10

Height of Maximum Plasma Frequency

($h_m F_2$)

(kilometers)

n \ p		-3	-2	-1	0	1	2	3
	f=14 MHz	235	236	239	239			
-3	17	230	232	220	214			
	14	244	243	247	249			
-2	17	245	239	238	231			
	20	245	239	236	238			
	14	264	262	260	262			
-1	17	274	282	258	250			
	20	276	262	258	256			
	14	289	347	283	270	269	249	234
0	17	303	276	290	266	260	263	268
	20	290	278	277	268	277	307	319
	14				305	288	269	259
1	17				292	281	286	280
	20				292	290	311	311
	14				340	324	302	283
2	17				321	311	305	306
	20				320	315	328	332
	14				391	364	336	315
3	17				351	339	330	324
	20				346	344	356	358

Table 3.11

Base Height of Ionosphere (km)

n \ p		-3	-2	-1	0	1	2	3
	f=14 MHz	136.7	129.4	126.4	122.1			
-3	17	142.6	135.1	142.2	139.1			
	14	164.5	156.8	147.7	136.6			
-2	17	161.3	163.2	159.9	151.1			
	20	160.9	163.3	169.9	135.4			
	14	186.7	169.5	174.7	158.9			
-1	17	174.7	159.1	179.5	173.2			
	20	174.7	177.8	202.0	156.1			
	14	215.9	224.1	196.3	201.1	189.3	181.7	175.7
0	17	207.6	211.8	187.0	204.2	205.2	165.5	157.3
	20	225.1	169.4	213.5	182.5	167.0	148.7	150.5
	14				215.4	212.3	198.9	195.5
1	17				228.1	222.4	178.9	177.0
	20				205.4	194.2	168.6	166.8
	14				248.9	245.0	222.4	216.1
2	17				262.3	256.4	205.3	191.3
	20				231.4	226.2	191.0	187.1
	14				291.4	274.2	258.1	247.7
3	17				286.7	283.6	233.4	221.0
	20				258.0	251.0	218.3	210.7

Table 3.12

Critical Frequency (f_0F_2)

(in MHz.)

n \ p		-3	-2	-1	0	1	2	3
	f=14 MHz	6.237	6.207	6.224	6.156			
-3	17	6.140	6.158	5.872	5.725			
	14	6.568	6.523	6.567	6.530			
-2	17	6.610	6.448	6.386	6.198			
	20	6.613	6.447	6.334	6.316			
	14	7.148	7.061	7.002	7.005			
-1	17	7.378	7.466	6.943	6.740			
	20	7.413	7.072	6.873	6.856			
	14	7.804	9.232	7.644	7.272	7.251	6.732	6.322
0	17	8.173	7.488	7.816	7.146	6.937	7.078	7.118
	20	7.814	7.614	7.444	7.265	7.400	7.830	7.990
	14				8.235	7.784	7.274	7.001
1	17				7.853	7.546	7.706	7.538
	20				7.937	7.868	8.184	8.125
	14				9.193	8.739	8.176	7.679
2	17				8.558	8.262	8.288	8.264
	20				8.713	8.547	8.763	8.782
	14				10.537	9.831	9.072	8.481
3	17				9.414	9.037	9.020	8.814
	20				9.450	9.378	9.595	9.559

3.4.4 Inversion of Synthesized Data for Three-Dimensional Ionosphere

For nonzero value of the parameter ω_t, that is, when the transmitter is not situated along the line joining the Earth's center to the center of the ionosphere, the eccentric quasi-parabolic layer provides a three-dimensional variation of the electron density with respect to the transmitter location. Hence when backscatter leading edge data are synthesized for various azimuthal directions from the transmitter, they will all be different. To test the inversion of synthesized data for a three-dimensional ionosphere, leading edges are synthesized by assuming eccentric QP layer parameters of r_b=6570 km, r_m=6720 km, f_c=7.00 MHz, D= 500 km, and ω_T=5°, and for azimuthal beam headings from 0° to 70° in steps of 10°. The leading edge data for each azimuth is then inverted in the usual manner. The resulting concentric QP layer parameters as a function of range from the transmitter and azimuth are shown in Table 3.13. The entries in each box of this table correspond to values of r_b, r_m, and f_c, respectively. It can be seen that the inversion results do indicate the three dimensional variability of the assumed ionospheric model.

As a further test of the inversion technique, the three-dimensional ionospheric model supplied by the contract sponsor is used and by three-dimensional ray tracing including magnetic field backscatter leading edges are generated for two azimuths (330° and 358°) with the transmitter located at the geographic coordinates of 40°N lat. and 175° long. The generated minimum group path values and other propagation parameters versus fre-

Table 3.13. Results of Inversion of Simulated Backscatter Ionogram Data

Q-P parameters: r_b=6570 km r_m=6720 km f_c=7.00 MHz

Eccentricity: $|\vec{D}|$=500 km ω_T=5°

Approx. Gnd Range km	Azimuthal Beam Heading 0°							
630	6586.5	6586.1	6583.7	6580.1	6575.0	6568.8	6562.8	6556.1
	6727.7	6727.5	6726.2	6724.4	6721.9	6719.5	6717.1	6714.8
	7.1851	7.1794	7.1471	7.0998	7.0339	6.9592	6.8845	6.8024
1150	6564.3	6563.9	6562.7	6560.8	6558.4	6555.0	6551.7	6548.3
	6716.8	6716.6	6716.1	6715.3	6714.3	6714.4	6713.2	6712.0
	6.9146	6.9087	6.8921	6.8659	6.8314	6.8065	6.7601	6.7716
2020	6553.7	6553.5	6552.8	6551.5	6550.0	6548.0	6545.8	6543.4
	6711.2	6711.1	6710.8	6710.2	6709.6	6708.4	6707.5	6706.5
	6.7816	6.7773	6.7651	6.7447	6.7178	6.6825	6.6457	6.6059
2170	6545.8	6545.7	6545.1	6544.4	6543.2	6542.4	6540.7	6537.8
	6701.1	6701.1	6700.7	6699.6	6698.8	6690.7	6695.7	6696.3
	6.6046	6.6006	6.5888	6.5640	6.5383	6.4940	6.4580	6.4383

quency are shown in Tables 3.14 and 3.15. To invert these leading edge data, a QP layer is fitted to the actual vertical electron density profile existing above the transmitter location. By using these QP layer parameters as the starting solution, overlapping sets of three data points along the leading edge are inverted for QP layer parameters. In inverting a given set other than the first one, the solution found for the previous set is used as the starting solution. For each set of data inverted, the corresponding ground range is computed and the QP layer parameters are assigned to a location half that distance from the transmitter and along the pertinent azimuth. The layer parameters and the corresponding ranges generated in this manner are listed in Tables 3.16 and 3.17. It can be seen that in this manner, actual experimental backscatter leading edge data for various azimuthal directions from the transmitter location can be inverted for a three dimensional ionosphere specified by QP layer parameters at a set of grid points located along the azimuthal directions.

Table 3.14. Computed minimum group path data and other propagation parameters for transmitter location of 40°N, 175°W, and 330° azimuth.

Frequency, MHz	Minimum Group path, km	Ground range, km	Apogee height, km
10	798.35	468.81	220.26
11	884.79	590.29	222.25
12	972.60	708.63	222.02
13	1062.92	826.38	220.63
14	1155.56	929.67	223.15
15	1251.66	1041.29	222.89
16	1349.12	1150.64	222.91
17	1450.55	1261.44	223.20
18	1555.26	1370.37	225.13
19	1665.43	1483.68	227.10
20	1780.63	1604.12	227.60

Table 3.15. Same as Table 3.14 except for 358° azimuth

Frequency, MHz	Minimum Group path, km	Ground range, km	Apogee height, km
10	805.04	474.50	222.20
11	894.16	598.72	224.30
12	984.55	719.84	224.14
13	1077.68	840.37	222.70
14	1173.13	954.99	222.23
15	1273.45	1062.07	225.29
16	1376.13	1172.57	227.20
17	1483.70	1293.45	225.99
18	1596.70	1417.13	224.96
19	1714.86	1531.40	230.46

Table 3.16. Computed QP layer parameter values and the corresponding ground ranges for 330° azimuth

Range, km	f_c, MHz	r_b, km	r_m, km
327.56	9.2406	6556.4	6676.2
377.18	9.3493	6561.9	6679.5
431.21	9.3344	6561.2	6679.1
486.87	9.2511	6557.4	6676.4
531.07	9.7489	6565.8	6694.9
577.99	10.0895	6573.9	6707.4
632.21	10.1063	6574.6	6708.0
689.80	10.0427	6572.1	6705.8
749.46	9.9592	6568.2	6702.9

Table 3.17. Same as Table 3.16 except for 358° azimuth

Range, km	f_c, MHz	r_b, km	r_m, km
373.28	8.3981	6509.2	6661.6
414.73	8.9054	6519.1	6680.1
461.30	9.0253	6526.2	6681.9
515.75	8.9677	6522.8	6680.9
568.43	8.9757	6523.2	6681.1
619.54	9.0693	6529.0	6682.8
676.62	9.0482	6527.7	6682.4
732.96	9.2278	6528.4	6690.8

3.4.5 Application of the Backscatter Inversion Technique

It has been pointed out in Sec. 3.1, that while the backscatter leading edge is not capable of providing the ionospheric vertical profiles above certain heights, this is not a limitation in so far as the application of the inversion technique is concerned. In this section, we shall prove this point and provide an explanation for the applicability of the inversion technique despite the limitation for obtaining the complete ionospheric structure. To do this, we consider an important application, which consists of finding the ground range for a given group path (not a minimum group path) at a known frequency, and compare the applicability of the deduced three-dimensional ionosphere by inversion of the leading edge data, relative to that of the originally assumed three-dimensional model.

Referring back to Sec. 3.4.4 in which we presented the result of inversion of synthesized leading edges for two azimuths for the Air Force supplied model, we now trace rays at one frequency (16 MHz) in the original as well as the computed ionospheres for an azimuth of 340° (lying between 330° and 358° corresponding to the inverted backscatter data) and for several elevation angles of transmission. For tracing rays in the computed ionosphere, it should be noted that the rectangular grid of the originally assumed model is uniformly spaced, whereas the grid generated by the inversion of the leading edge data is nonuniformly spaced, in addition to being in azimuth-range space, rather than in the latitude-longitude space. Hence it has become necessary to generate vertical profiles at the uniformly spaced

rectangular grid of points by applying an interpolation procedure to the QP profiles at the grid point in the azimuth-range space. The result of these ray tracings are shown in Tables 3.18 and 3.19, which correspond to the originally assumed model and the computed ionosphere, respectively. The quantity (P'-R) is shown plotted versus P' for the two cases in Fig. 3.18. A comparison of the two curves indicates that the computed ionosphere gives the correct range to within ± 2 km for a wide range of group path values. The discrepancies greater than ± 2 km toward the low end of the curves are attributable to the fact that the region is close to the minimum group path, whereas the discrepancies greater than ± 2 km toward the high end of the curves are attributable to the fact that the region is beyond the range corresponding to the inverted backscatter data.

We shall now provide a simple explanation for the success of the technique in finding the range *for a given* group path to a good accuracy, despite the fact that the minimum group path rays all reflect from below a certain apogee height, as can be seen from Tables 3.14 and 3.15 and hence cannot yield the vertical profiles above those apogee heights. To do this, we refer to Fig. 3.19 which illustrates the fact that the backscatter ionogram is a continuum of oblique ionograms corresponding to sucessively increasing values of ground range away from the transmitter. Let us suppose we have found the range corresponding to a particular point A (f_1, P_1') on the leading edge. Then a point B, corresponding to the same group path as that of A but to a frequency $f_2 < f_1$, corresponds to a range greater than that of A. Alternatively, it corresponds to a range corresponding to

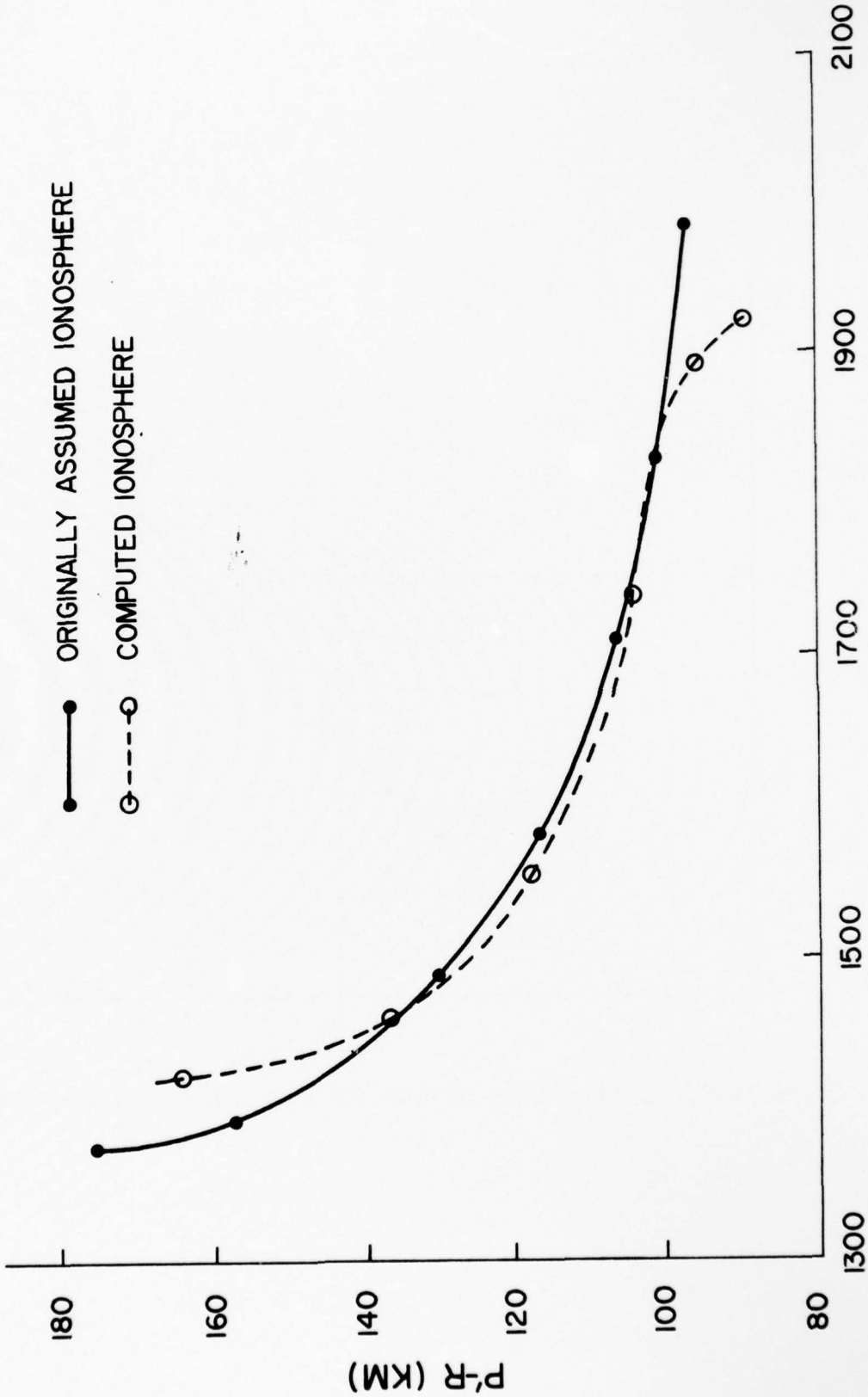


Figure 3.18. Reconstruction of ground range

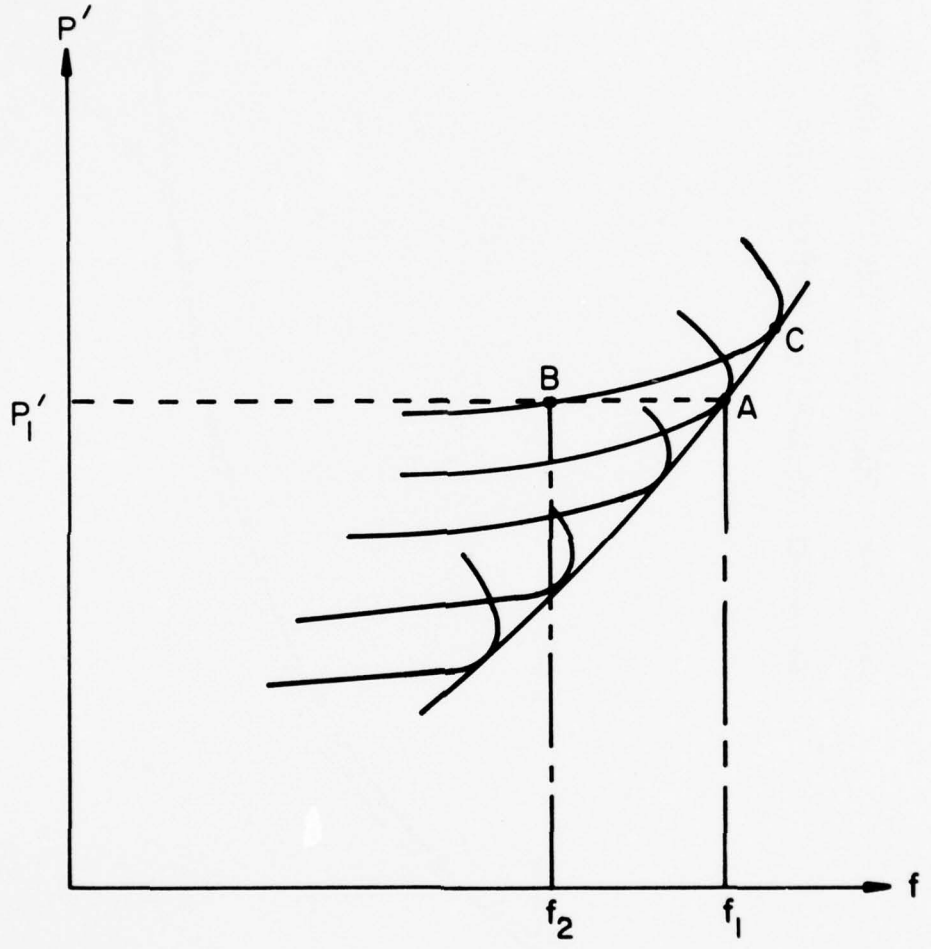


Figure 3.19. Limitations of information contained in data.

Table 3.18. Group path and ground range versus elevation angle for 16 MHz and for 340° azimuth, using the originally assumed model.

Elevation angle, deg.	Group path P', km	Ground range R, km	(P'-R) km
10	1985.06	1888.14	96.92
12	1795.51	1693.76	101.76
14	1661.89	1552.97	108.92
16	1563.03	1444.75	118.28
18	1490.80	1361.39	129.41
20	1435.69	1293.32	142.37
22	1393.93	1236.40	157.53
24	1371.43	1196.75	174.68

Table 3.19. Same as Table 3.18 except using the computed ionosphere

Elevation angle, deg.	Group path P', km	Ground range R, km	(P'-R) km
14	1921.58	1833.31	88.27
16	1892.55	1796.50	96.05
18	1725.20	1619.59	105.61
20	1555.49	1437.47	118.02
22	1458.60	1321.65	136.95
24	1422.14	1257.31	176.83

that of a point C on the leading edge, lying above the point A. Hence the ray of frequency f_2 and group path P_1' reflects below the apogee height corresponding to the point C. Since this argument is true for any point on the low-ray portions of the oblique ionograms, it follows that one can always find the range corresponding to such a point, although the backscatter leading edge is not capable of yielding the ionospheric profile above the apogee heights corresponding to the minimum group path rays.

Spurred by the success of the application of the backscatter leading edge inversion technique in the determination of the ground range for a given group path, we decided to combine the inversion problem and the subsequent range determination into one computer program. Thus, provided with the input of backscatter leading edge data for several azimuths and the group path values (non-minimum) at several frequencies for a different azimuth, the program computes the ground ranges. To carry out the range computation, we decided to employ a simple homing technique in conjunction with the exact ray path computation for the concentric QP layer rather than the elaborate three-dimensional ray tracing including the Earth's magnetic field. The details of the entire program are presented in the appendix.

To describe the procedure briefly, first each backscatter leading edge is inverted for a series of QP layer profiles along the corresponding azimuth, in the usual manner. For each specified group path value (P') for which the ground range is to be determined, the program then computes a rough estimate of the range, based on specular reflection at an assumed value of base

height of the ionosphere. By an interpolation procedure involving two azimuths and two ranges along each azimuth as shown in Fig. 3.20, the program then computes the QP layer parameters at half the estimated ground range along the azimuth (α) corresponding to the group path data. Using analytical expressions for ray path parameters and the interpolated QP layer parameters, a ground range value is computed for the specified group path, employing an iterative technique. If this range value is close to the original estimate to within a specified tolerance, it is the desired result; otherwise, the procedure is repeated until a final range value is obtained which agrees with the prvious value to within the specified tolerance. The technique is illustrated in the block diagram of Fig. 3.21.

As a test of the above discussed simplified method of application of backscatter ionograms, the leading edge inversion results presented in Tables 3.16 and 3.17 are used to compute the ground ranges for various values of group path for f=16 MHz and azimuthal angle of 340°. The results are shown in Table 3.20, which indicates larger errors in (P'-R) than in Table 3.19, but still less than 15 percent of the actual values of (P'-R) given in Table 3.18.

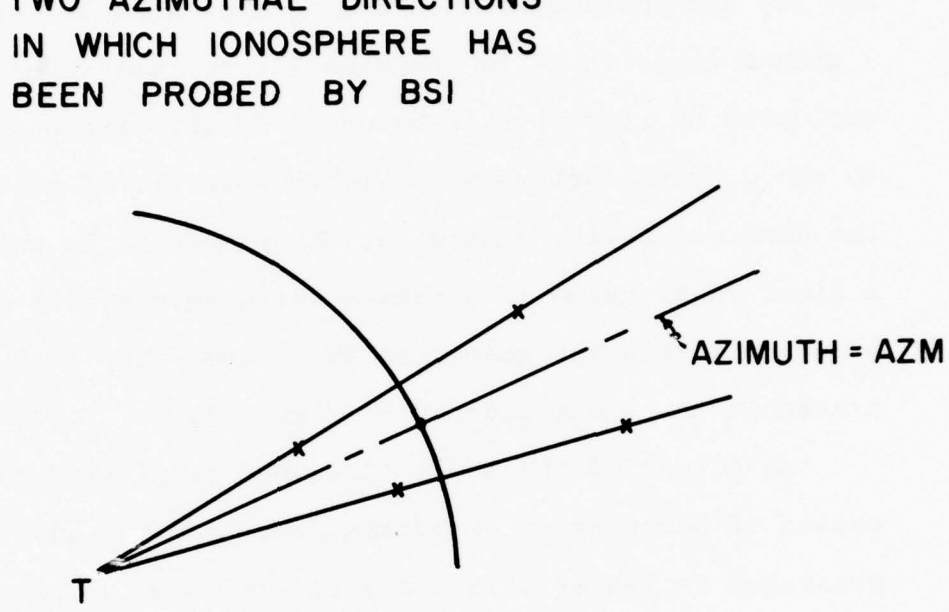


Figure 3.20. Interpolation procedure.

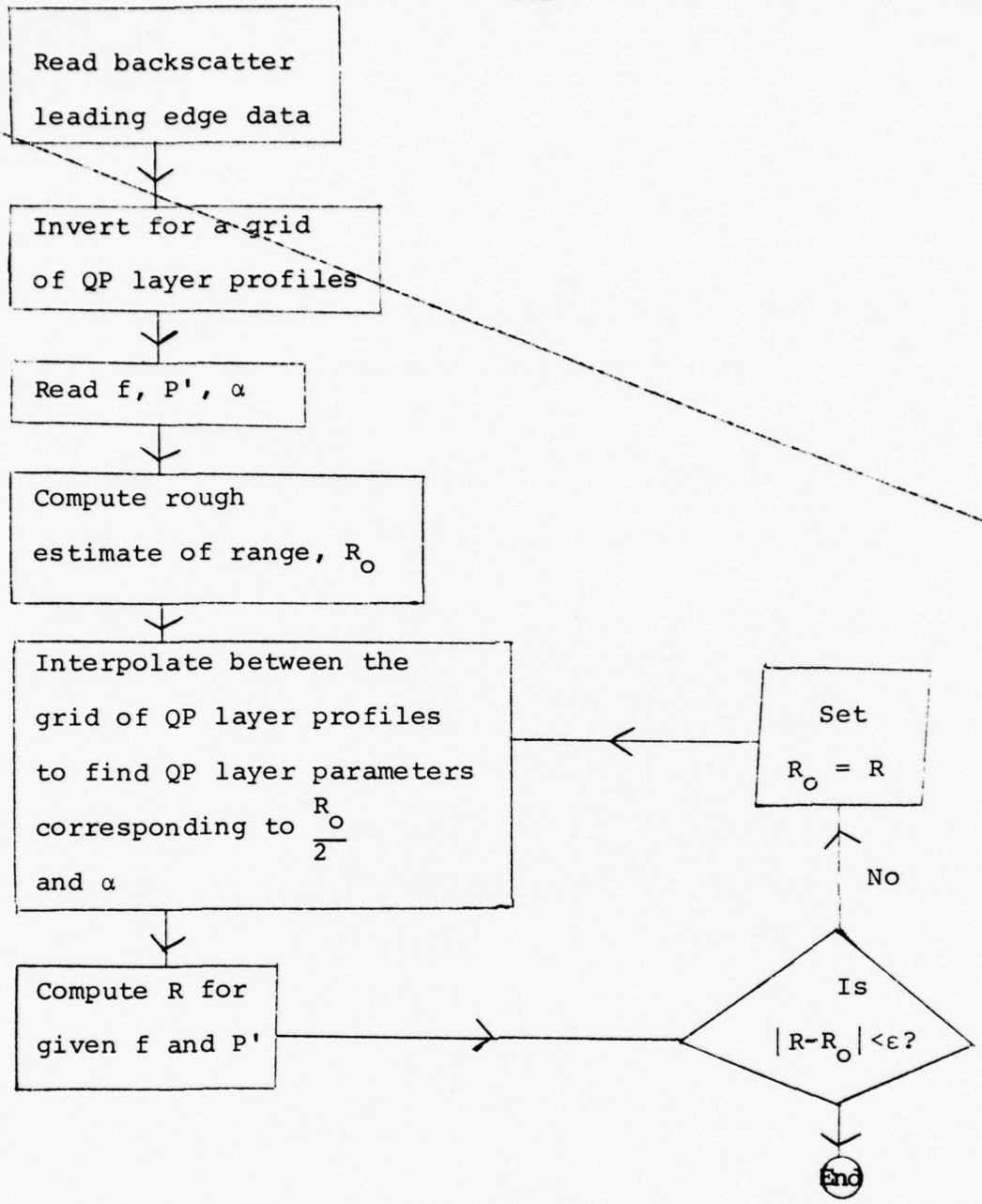


Figure 3.21. Block diagram of steps involved in backscatter leading edge inversion and application.

Table 3.20. Same as Table 3.18 except using the Simplified Ionospheric Model

Group path P', km	Ground range R km	(P'-R) km
1985.06	1904.16	80.50
1795.51	1707.41	88.10
1661.89	1565.75	96.14
1563.03	1457.76	105.27
1490.80	1375.62	115.18
1435.69	1308.68	127.01
1393.93	1252.20	141.73
1371.43	1216.52	154.91

3.4.6 Uniqueness of the Application of the Inversion Technique

In this section we will consider the effect of the initial estimate of QPP (QP layer parameters) upon the inversion of group delay into ground range. The motivation for this simulation was to determine the possible effects of the nonuniqueness of the inversion of backscatter ionogram data into ionospheric refractive index. Also we were interested in the reliability of the present technique in providing a reasonably consistent inversion between group delay and ground range, despite the uncertainty involved in the initial estimate of QPP.

The results of this analysis, to date, are presented in the following set of tables. The procedure used in generating these tables was to vary one of the three initial QPP while keeping the other two parameters fixed, and observe the effect of this variation upon the inversion of a fixed value of group path into a computed value of ground range. For our present purposes, we will consider the QPP to consist of the semithickness of the layer (y_m), the height of the base of the ionosphere (h_b), and the critical, or maximum, plasma frequency (f_oF_2). Other parameters such as $r_b(h_b+6370$ km) and $r_m(y_m+h_b+6370$ km) may be obtained easily once y_m and h_b are specified. Table 3.21 specifically indicates the effect of varying r_b which is equivalent to varying h_b. The last column illustrates the small variation in the computed ground range thereby induced in the case of an untilted (no horizontal gradient) ionosphere. Table 3.22 demonstrates the effect of varying the initial estimate of the maximum plasma frequency (or critical frequency), and again we

UNTILTED IONOSPHERE

Table 3.21: The Effect of Varying r_b Initial Estimate Upon Inversion of Group Path into Ground Range.

Theoretical Grp. Path (km)	Initial Est. of r_b	Computed Grnd. Range (km)
1700	6510	1591.89
	6520	1591.26
	6530	1591.93
	6540	1590.95
	6550	1590.53
1900	6510	1809.76
	6520	1809.26
	6530	1809.79
	6540	1808.96
	6550	1808.65
2100	6510	2020.76
	6520	2020.43
	6530	2020.79
	6540	2020.15
	6550	2019.98

see that the effect upon the computed ground range is small. Table 3.23 demonstrates the effect of variations in the initial estimate of r_m while keeping r_b fixed. This corresponds to expansion and contraction of the vertical dimensions of the ionosphere through the variation of y_m. The variation in the computed ground range is somewhat larger than the variation obtained in the previous two tables. Nevertheless, it should be noted that even though r_m is varying by less than 1% from its nominal value of 6610 km, the really significant parameter is the semithickness (y_m) of the ionospheric layer, and this latter parameter is varying over a proportionately larger range--i.e. r_m= 6600 km corresponds to a 70 km semithickness, while r_m=6620 km corresponds to a 90 km semithickness.

This type of analysis has also been performed for a tilted ionosphere (an ionosphere with a horizontal gradient). Tables 3.24, 3.25, and 3.26 correspond to Tables 3.21, 3.22, and 3.23 respectively, and in comparing these two sets of tables, it would appear that the inversion of group path into ground range is more sensitive to the variability of the initial QPP in a tilted ionosphere. This effect would seem to be most pronounced in the case of variations of r_m.

We therefore decided to consider the possible causes of the relatively large variation in ground range resulting from the simulated uncertainty in the initial value of r_m. By narrowing the uncertainty in the initial estimate of r_m by a factor of 2, it can be seen that the variability of the ground range

UNTILTED IONOSPHERE

Table 3.22: The Effect of Varying the Initial Estimate of Critical Frequency Upon Inversion of Group Path Into Ground Range.

Theoretical Grp. Path (km)	Initial Est. of r_b (km)	Computed Grnd. Range (km)
1700	6.75	1592.09
	7.00	1591.35
	7.25	1591.93
	7.50	1591.17
	7.75	1591.04
1900	6.75	1809.92
	7.00	1809.33
	7.25	1809.79
	7.50	1809.19
	7.75	1809.11
2100	6.75	2020.28
	7.00	2020.46
	7.25	2020.79
	7.50	2020.39
	7.75	2020.35

UNTILTED IONOSPHERE

Table 3.23: The Effect of Varying Layer Semithickness Upon Inversion of Group Path to Ground Range.

Theoretical Grp. Path (km)	Initial Est. of r_m (km)	y_m (km)	Computed Grnd. Range (km)
1700	6600	70	1590.60
	6605	75	1590.56
	6610	80	1591.93
	6615	85	1592.26
	6620	90	1591.96
1900	6600	70	1808.73
	6605	75	1808.67
	6610	80	1809.79
	6615	85	1810.06
	6620	90	1809.81
2100	6600	70	2020.08
	6605	75	2020.14
	6610	80	2020.79
	6615	85	2020.98
	6620	90	2020.80

TILTED IONOSPHERE

Table 3.24: The Effect of the Initial Estimate of r_b (Radius to Ionospheric Base) Upon the Inversion of Group Path into Ground Range.

Theoretical Grp. Path (km)	Initial Est. of r_b (km)	Computed Grnd. Range (km)
1700	6510	1584.67
	6520	1583.57
	6530	1586.57
	6540	1586.86
	6550	1583.67
1900	6510	1798.66
	6520	1792.39
	6530	1794.79
	6540	1795.77
	6550	1792.42
2100	6510	1996.30
	6520	1994.87
	6530	1996.97
	6540	1998.44
	6550	1994.96

TILTED IONOSPHERE

Table 3.25: The Effect of Initial Estimate of Critical Frequency (f_c) Upon Inversion.

Theoretical Grp. Path (km)	Initial Est. of f_c	Computed Grnd. Range (km)
1700	6.75	1586.41
	7.00	1586.89
	7.25	1586.57
	7.50	1587.24
	7.75	1586.37
1900	6.75	1795.11
	7.00	1796.12
	7.25	1794.79
	7.50	1796.61
	7.75	1795.26
2100	6.75	1997.62
	7.00	1999.02
	7.25	1996.97
	7.50	1999.62
	7.75	1997.90

TILTED IONOSPHERE

Table 3.26: The Effect of Initial Estimate of r_m Upon Inversion of Group Path into Ground Range. Program was allowed to go 40 Iterations

Theoretical Grp. Path (km)	Initial Est. of r_m (km)	y_m (km)	Computed Grnd. Range (km)
1700	6590	60	1578.35
	6600	70	1582.48
	6610	80	1586.57
	6620	90	1586.67
	6630	100	1590.76
1900	6590	60	1786.76
	6600	70	1790.84
	6610	80	1794.79
	6620	90	1798.14
	6630	100	1801.60
2100	6590	60	1988.81
	6600	70	1992.96
	6610	80	1996.97
	6620	90	2001.28
	6630	100	2005.76

SUBROUTINE TRACE

TILTED IONOSPHERE

Table 3.27: The Effect of Varying the Layer Semithickness Upon the Inversion of Group Path to Ground Range.

Theoretical Grp. Path (km)	Initial Est. of r_m (km)	y_m	Computed Grnd. Range (km)
1700	6600	70	1582.48
	6605	75	1582.13
	6610	80	1586.57
	6615	85	1587.62
	6620	90	1588.67
1900	6600	70	1790.84
	6605	75	1790.62
	6010	80	1794.79
	6615	85	1797.08
	6620	90	1798.14
2100	6600	70	1992.96
	6605	75	1992.82
	6610	80	1996.97
	6615	85	2000.16
	6620	90	2001.28


```
H=R(1)-EARTHR                                                    0237
WASNT=.NOT.HOMR                                                  0238
```

is also reduced, as shown in Table 3.27. In order to consider the effect of allowing the program to perform a greater number of iterations, we have repeated the calculation of Table 3.26 by doubling the number of iterations which the program uses in computing a final set of QPP. The result is shown in Table 3.28. Although this increase in the number of iterations generally improves the result, this is not always the case, as can be seen in Table 3.29. The significant quantity, σ, denotes the standard deviation of the ground range values from their respective means. For the smaller values of simulated group path, increased iterations produce a smaller standard deviation. For the largest value of group path (2100 km) the standard deviation is actually larger in the case of 80 iterations, as opposed to the case of 40 iterations.


```
C                                                                    0296
C          RAY PENETRATED                                            0297
30 NUTEST=1                                                          0298
```

TILTED IONOSPHERE

Table 3.28: The Effect of Initial Estimate of r_m Upon Inversion of Group Path into Ground Range. Program was allowed to go 80 Iterations in Fitting QPP.

Theoretical Grp. Path (km)	Initial Est. of r_m (km)	y_m (km)	Computed Grnd. Range (km)
1700	6590	60	1580.18
	6600	70	1582.72
	6610	80	1586.97
	6620	90	1587.40
	6630	100	1588.40
1900	6590	60	1778.40
	6600	70	1781.92
	6610	80	1788.25
	6620	90	1789.21
	6630	100	1790.48
2100	6590	60	1973.34
	6600	70	1977.90
	6610	80	1986.09
	6620	90	1987.49
	6630	100	1989.10


```
      SUBROUTINE BACK UP(HS)                                              0332
      DIMENSION NWORD1(2),NWORD2(2),NWORD(2)
      COMMON /BK/ NN,STEP,MODE,H1MAX,H1MIN,H2MAX,H2MIN,FACT,HSTART        0333
```

TILTED IONOSPHERE

Table 3.29: Variance Resulting from Different Values of r_m as Affected by Number of Iterations (IT).

Theoretical Grp. Path (km)	IT	$\bar{D}$	σ
1700	40	1585.37	4.45
	80	1585.13	3.15
1900	40	1794.43	5.23
	80	1785.65	4.39
2100	40	1997.16	5.97
	80	1982.78	6.10


```
      SUBROUTINE RKAM                                                   0888
C       NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS                 0889
      COMMON/FLAGS/IFLAG
```

3.5 General Consideration of the Inversion Problem

In the past two sections of this chapter we have presented several techniques that can be applied to deduce the ionospheric structure by using point-to-point oblique ionograms (section 3.3) or backscatter leading edges (section 3.4). In both cases our computations have been carried out for quasi-parabolic profiles. Recently an inversion technique developed in connection with geophysical problems has shown great promise. This technique, known as Backus-Gilbert technique, can be adapted to inverting oblique ionograms of both kinds. The problem has been formulated in a paper "A Method for Inverting Oblique Sounding Data in the Ionosphere" which is attached as Appendix 4. Initial application to the vertical incidence data demonstrates its rapid convergence. The technique has yet to be applied to inverting oblique ionograms. It is desirable to carry out many computations in order to understand the capability, the characteristics and the limitation of this inversion technique. So far the problem has been formulated for inverting either the backscatter leading edge or the oblique ionogram. In both cases pherical stratification is assumed. Details can be found in Appendix 4.


```
      DO 150 I=1,NN                                                    0937
150   FX(KM-I)=FXIT(I)                                                 0938
```

3.6 Conclusions

In conclusion, we have in this section discussed our efforts to devise techniques for the determination of ionospheric structure from oblique radio propagation data. Particular attention was devoted to the problem of obtaining the horizontal gradients. The major findings of the effort are: a) point-to-point oblique ionograms, by themselves, are not very useful for obtaining the horizontal gradients, while b) backscatter leading edge data are capable of providing information concerning the horizontal gradients.

We have provided several examples of inversion of the oblique radio propagation data and even investigated the application of the backscatter technique. We have demonstrated that despite the fact that the inversion technique does not necessarily yield a unique solution for the ionospheric structure, this uncertainty is not an important factor in the application.

4. Recommendations for Future Work

Based on the efforts of our work reported in sections 2 and 3 and the conclusions reached therein, we make the following recommendations for continuing work on this project:

1. Since the backscatter data and the corresponding inversion results involve a grid of points in the azimuth-range space, it is desirable that the electron density model employed in the three-dimensional ray tracing be specified in the azimuth-range space rather than in the latitude-longitude space.

2. The basic inversion techniques of backscatter leading edge data need to be tested by employing simulated data for progressively complicated ionospheres. In this connection, it is necessary that the simulated data points be closely spaced in range, in view of the fact that successive data points correspond to successively increasing values of range away from the transmitter. Hence for the derivation of local horizontal gradients, it is necessary to consider sets of data points, closely spaced in range, and hence in frequency.

3. In addition to the inversion of backscatter leading edge for ionospheric structure, further tests are necessary with regard to the application of the inversion results for computing the ranges for non-minimum group paths. In this connection, it is necessary to have simulated ray tracing data for non-minimum group path rays for the same model ionospheres for which the backscatter leading edges are simulated.

4. The dependence of the inversion result upon the choice of the initial set of layer parameters should be further investigated

with a view to determining how in actual practice, the initial set of layer parameters must be chosen in order to minimize the uncertainty of the inversion technique.

5. The present scheme for homing the ray after one reflection from the ionosphere is very successful as demonstrated by several examples in this report. However, in extending the present scheme to multi-hop modes some difficulties are encountered. Some of these difficulties have been identified: the coarseness of the grid points at which the electron density is given, and inflexibility in the computer program to handle mixed and complex modes. To remove these difficulties requires further refinement of the ray tracing program.

6. Preliminary analysis shows promise in applying the Backus-Gilbert technique to inversion of the oblique propagation data. This technique has been applied to the vertical ionogram and very rapid convergence is obtained. Even though the mathematical formulation for the inversion of oblique data has been carried out, its application to data has yet to be made. One possibility is to choose a model profile (such as piece-wise parabolic profile) for which analytic expressions are available so as to reduce the computation time. Another area of interest is to generalize the Backus-Gilbert technique by including the horizontal gradient. In this latter case the Bouguer's rule is no longer valid, one must reformulate the problem anew.

APPENDIX 1

The computer programs for ray homing are listed in the following pages. These programs are discussed in section 2.5.

```
C                    DECIDES WHICH RAYS ARE TO BE TRACED AND SETS THE IN    0002
CONDITIONS FOR EACH RAY BEFORE CALLING  TRACE, WHICH CALCULATES THE RAY     0003
      DIMENSION TYPE(3),TYPE2(3)                                            0004
      DIMENSION MODELT(4)
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,C,LOGTEN                       0005
      COMMON /PK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART           0006
      COMMON /PIN/ N2,MUX,PN2(8),PCLAP(4),SPACE,COLL,FIELD                  0007
      COMMON /PLG/ NTYP,RNEWW,PNEWW,RAYBEG,RAYSET,LINES,IHOP,NOTEST,APHT    0008
      COMMON /MODELS/ MODEL(4)                                              0009
      COMMON /XX/ X,PXPB,PXPTH,PXPPH,PXPT,HMAX                              0010
      COMMON B(12),T,STP,DRDT(12) /WW/ ID(10),WC,W(400)                     0011
      COMMON /VPRN1/ VN(250),VNPB(250),VNPTH(250),VNPPH(250)             11 KV 1
      COMMON /VPRN2/ VRANGE(250),VHT(250)                               11 KV 2
      COMMON /NDOPT/  NPRINT,KOPT,MPRT                                  11 KV 3
      COMMON /DENSC/ DUM2(683),NDUM(2),JFLAG,NSOL,DBMIN(21),JTEMP       UOFI 1
      EQUIVALENCE (P,W(3)),(PLON,W(13)),(LON,W(14)),(PLAT,W(15)),           0012
     1 (LAT,W(16)),(BETA,W(17)),(AZ1,W(18)),(EARTHR,W(19)),                 0013
     2 (XMTRH,W(20)),(INTYP,W(41)),(MAXERR,W(42)),(ERATIO,W(43)),           0014
     3 (STEP1,W(44)),(STPMAX,W(45)),(STPMIN,W(46)),(FACTR,W(47)),           0015
     4 (RAY,W(67)),(MAXSTP,W(93)),(SKIP,W(180)),(HOP,W(254)),               0016
     5 (ELBEG,W(255)),(ELEND,W(256)),(ELSTEP,W(257)),(FBEG,W(258)),         0017
     6 (FEND,W(259)),(FSTEP,W(260)),(AZBEG,W(263)),(AZEND,W(264)),          0018
     7 (AZSTEP,W(265)),(PIT,W(272)),(CNLY,W(279)),(WN,W(299))               0019
      EQUIVALENCE (RC,W(375)),(ALPHA,W(376)),(THC,W(377)),(PHC,W(378))  UOFI 2
      REAL LOGTEN,K,N2,MUX,MAXSTP,INTYP,MAXERR,MU,ION,LAT                   0020
      DATA MODELT,KOLL1,KOLL2/2*1H ,4HDIFC,1H ,4HWITH,4H  NO/
      DATA TYPE,TYPE2/4HX-CR,4HNC F,4HORDI,4HNARY,2*1H /
      DATA MCREW/0/                                                     UOFI 2A
      SECOND(A)=10.0
C     CALL QSEXUN                                                           0023
C     NDATE=IDATE(0)                                                        0024
C     SECOND=KLOCK(0)*.001                                                  0025
      TIME=SECOND(A)
      LINES=0
      NPAGE=0
      DO 5 I=1,400
5     W(I)=0.0
      FIELD=1.0
      COLL=0.0
      KOLL=KOLL2
      NDATE=0
      IF(COLL.NE.0.0) KOLL=KOLL1
C
C          CONSTANTS                                                        0029
      PI=3.1415926536                                                       0030
      PIT2=2.*PI                                                            0031
      PID2=PI/2.                                                            0032
      DEGS=180./PI                                                          0033
      RAD=PI/180.                                                           0034
      K=80.6E-6                                                             0035
      C=2.997925E5                                                          0036
      LOGTEN=ALOG(10.)
      DO 2 I=1,4
2     MODEL(I)=MODELT(I)
C                                                                           0038
C          INITIALIZE SOME VARIABLES IN THE W ARRAY                         0039
      PLON=0.                                                               0040
      PLAT=PID2                                                             0041
      EARTHR=6370.                                                          0042
      INTYP=3.                                                              0043
```

```
      MAXERR=1.E-8                                                          0044
      ERATIO=50.                                                            0045
      STEP1=1.                                                              0046
      STPMAX=100.                                                           0047
      STPMIN=1.E-4                                                          0048
      FACTR=0.5                                                             0049
      MAXSTP=1000.                                                          0050
      HOP=1.                                                                0051
      PLT=0.                                                                0052
C                                                                           0053
    1 CALL READ W                                                           0054
C
      MPRT = 0                                                        KV 054
C
      F=0.0
      BETA=0.0
      AZ1=0.0
      IF (WN.LT.7.) WN=7.                                                   0056
      IF (WN.GT.12.) WN=12.                                                 0057
      IF (SKIP.EQ.0.) SKIP=MAXSTP                                           0058
      NTYP=2.+FIELD*RAY+0.01                                                0059
      PRINT 7,ID,NDATE,MODEL,TYPE(NTYP),TYPE2(NTYP),KOLL
    7 FORMAT(1H1,10A4,20X,I8/5X,4(1X,A4),2X,2A4,' COLLISIONS'/)
      PRINT 8                                                               0063
    8 FORMAT (85H INITIAL VALUES FOR THE W ARRAY -- ALL ANGLES IN RADIAN     0064
     1S, ONLY NONZERO VALUES PRINTED/)                                      0065
      DO 10 I=1,400                                                         0066
      IF (W(I).NE.0.) PRINT 9, I,W(I)                                       0067
    9 FORMAT (I4,E19.11)                                                    0068
   10 CONTINUE                                                              0069
C          LET SUBROUTINES PRINTR AND RAYPLT KNOW THERE IS A NEW W ARRAY    0070
      PNEWW=1.0
      RNEWW=1.0
C                                                                           0072
C          INITIALIZE PARAMETERS FOR INTEGRATION SUBROUTINE RKAM            0073
      N=WN+0.01                                                             0074
      MODE=INTYP +0.01
      STEP=STEP1                                                            0076
      STP=STEP1                                                       1/28/71
      E1MAX=MAXERR                                                          0077
      E1MIN=MAXERR/ERATIO                                                   0078
      E2MAX=STPMAX                                                          0079
      E2MIN=STPMIN                                                          0080
      FACT=FACTR                                                            0081
C                                                                           0082
C        DETERMINE TRANSMITTER LOCATION IN COMPUTATIONAL COORDINATE SYST    0083
C        (GEOMAGNETIC COORDINATES IF DIPOLE FIELD IS USED)                  0084
      R0=EARTHR+XMTRH                                                       0085
      SP=SIN(PIAT)                                                          0086
      CP=SIN(PID2-PIAT)                                                     0087
      SINDPH=SIN(LON-PLON)                                                  0088
      COSDPH=SIN(PID2-(LON-PLON))                                           0089
      SL=SIN(LAT)                                                           0090
      CL=SIN(PID2-LAT)                                                      0091
      ALPHA=ATAN2(-SINDPH*CP,-COSDPH*CP*SI+SP*CL)                           0092
      TH0=ARCOS(COSIPH*CP*CI+SP*SL)
      PH0=ATAN2(SINIPH*CL,COSDPH*SP*CI-CP*SL)                               0094
C     THIS PORTION IS ADDED FOR THE HOMING FEATURE. IT READS THE DENSITYUOFI 3
C     PROFILE FROM ELECTX                                             UOFI 4
      IF(W(380).EQ.0.0)  GO TO 555                                    UOFI 5
      IF(MOREW.EQ.1)  GO TO 555                                       UOFI 5A
```

```
      MOREW=1                                                          UCFI 5B
      CALL ELECTX                                                      UCFI 5C
C                                                                        0095
C          LOOP ON FREQUENCY, AZIMUTH ANGLE, AND ELEVATION ANGLE         0096
555   NFREQ=1                                                          UOFI 5D
      IF (FSTEP.NE.0.) NFREQ=(FEND-FBEG)/FSTEP+1.5                       0098
      NAZ=1                                                              0099
      IF (AZSTEP.NE.0.) NAZ=(AZEND-AZBEG)/AZSTEP+1.5                     0100
      NBETA=1                                                            0101
      IF (ELSTEP.NE.0.) NBETA=(ELEND-ELBEG)/ELSTEP+1.5                   0102
      DO 50 NF=1,NFREQ                                                   0103
      F=FBEG+(NF-1)*FSTEP                                                0104
      IF(W(380).EQ.0.0) GO TO 55                                       UOFI 6
      CALL HOME                                                        UOFI 7
55    DO 45 J=1,NAZ                                                    UOFI 9
      IF(W(380).NE.0.0) GO TO 60                                       UOFI 10
      AZ1=AZBEG+(J-1)*AZSTEP                                             0106
60    AZA=AZ1*DEGS                                                     UOFI 11
      GAMMA=PI-AZ1+ALPHA                                                 0108
      SGAMMA=SIN(GAMMA)                                                  0109
      CGAMMA=SIN(PID2-GAMMA)                                             0110
      DO 40 I=1,NBETA                                                    0111
      IF(W(380).NE.0.0) GO TO 70                                       UOFI 12
      BETA=ELBEG+(I-1)*ELSTEP                                            0112
70    CBETA=SIN(PID2-BETA)                                             UOFI 13
C     THIS IS USED FOR THE HOMING FEATURE ONLY                         UOFI 14
      IF(W(380).EQ.0.0) GO TO 32                                       UOFI 15
      CALL ADJUST                                                      UOFI 16
      MPRT=0                                                           UOFI 17
      GO TO 50                                                         UOFI 18
32    R(1)=RO                                                          UOFI 19
      R(2)=THO                                                           0115
      R(3)=PHO                                                           0116
      R(4)=SIN(BETA)                                                     0117
      R(5)=CBETA*CGAMMA                                                  0118
      R(6)=CBETA*SGAMMA                                                  0119
      T=0.                                                               0120
      RSTART=1.                                                          0121
      CALL RINDEX                                                        0122
      EL=BETA*DEGS                                                       0123
      IF (I.NE.1.AND.NPAGE.LT.3.AND.LINES.LE.17) GO TO 18                0124
      NPAGE=0
      LINES=0
      PRINT 7, ID(1),ID,NDATE,MODEL,TYPE(NTYP),TYPE2(NTYP),KCLL          0126
      PRINT 17, F,AZA                                                    0127
   17 FORMAT (18X,11HFREQUENCY =,F12.6,37H MHZ, AZIMUTH ANGLE OF TRANSMI 0128
     1SSION =,F12.6,4H DEG)                                              0129
   18 NPAGE=NPAGE+1                                                      0130
      PRINT 19, EL                                                       0131
   19 FORMAT (/31X,33HELEVATION ANGLE OF TRANSMISSION =,F12.6,4H DEG/)   0132
      IF (N2.GT.0.) GO TO 25                                             0133
      CALL ELECTX                                                        0134
      FN=SIGN(SQRT(ABS(X))*F,X)                                          0135
      PRINT 21, FN                                                       0136
   21 FORMAT (58HOTRANSMITTER IN EVANESCENT REGION, TRANSMISSION IMPOSSI 0137
     1BLE/20HOPLASMA FREQUENCY = ,E17.10)                                0138
      GO TO 44                                                           0139
   25 MU=SQRT(N2)                                                        0140
      R(4)=MU*R(4)                                                       0141
      R(5)=MU*R(5)                                                       0142
      R(6)=MU*R(6)                                                       0143
```

```
      DO 28 NN=7,N                                                          0144
   28 R(NN)=0.                                                              0145
      CALL TRACE                                                            0146
C     SECOND=KLOCK(0)*.001                                                  0148
      OSEC=TIME
      TIME=SECOND(A)
      DIFF=TIME-OSEC
      PRINT 30, DIFF                                                        0150
   30 FORMAT(34X,26HTHIS RAY CALCULATION TOOK ,F8.3,4H SEC )                150 KV 1
C
C
C     NDPRINT  IS THE OPTION FOR PRINTING ELEC. DENSITY AND DERIVATIVES
C
      IF(NPRINT.EQ.0) GO TO 370
C
C     PRINT ELECTRON DENSITY PROFILE AND THE DERIVATIVES                    151 KV 1
C     USING  APPLETON - HARTREE FORMULA                                     151 KV 2
C
      PRINT  331                                                            151 KV 5
  331 FORMAT (1H1, 48H ELECTRON DENSITY PROFILE AND ITS DERIVATIVES    , 151 KV 7
     1 40H COURTESY   ARCON CORP.   K.VANGURI     //                       151 KV 8
     2 60H N = C * X,  C = 12400*F**2  F IN MHZ,  N = EL. PER CC      , 151 KV 9
     3 40H APPLETON-HARTREE FORMULA              /                        151 KV10
     4 60H DN/DR = C * PXPR            IN  EL/CC/KM                   / 151 KV11
     5 60H DN/DTH = C * PXPTH * RAD    IN  EL/CC/DEG                  / 151 KV12
     6 60H DN/DPH = C * PXPPH * RAD    IN  EL/CC/DEG                  / 151 KV13
     7 6X,5HRANGE,5X,6HHEIGHT,10X,1HN,9X,5HDN/DR,7X,6HDN/DTH,7X,        151 KV14
     8  6HDN/DPH,7X,4HSTEP/ 9X,2HKM,8X,2HKM,8X,5HEL/CC  /)                151 KV15
C
      DO  365 MJ = 1,MPRT                                                   151 KV25
      PRINT  345, VRANGE(MJ),VHT(MJ),                                       151 KV26
     2                  VN(MJ),VNPR(MJ),VNPTH(MJ),VNPPH(MJ),MJ          151 KV26
  345 FORMAT (1X,F10.4,2X,F10.4,4(2X,E11.4),2X,I5)                          151 KV27
  365 CONTINUE                                                              151 KV28
  370 CONTINUE
      MPRT = 0                                                              151 KV29
C
      IF (NOTEST.EQ.1.AND.ONLY.NE.0..AND.IHOP.EQ.1) GO TO 44                0152
   40 CONTINUE                                                              0153
   44 IF (PLT.NE.0.) CALL ENDPLT                                            0154
   45 CONTINUE                                                              0155
   50 CONTINUE                                                              0156
      GO TO 1                                                               0157
         END                                                                0158
```

```
      SUBROUTINE TRACE                                                    0186
      DIMENSION ROLD(12),DROLD(12)                                        0187
      COMMON /FK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART         0188
      COMMON /FIN/ N2,P2(8),TPOLAR,RPOLAR,SPACE,CCLL,FIELD                0189
      COMMON /TRAC/ GROUND,PERIGE,THERE,MINDIS,UNDER                      0190
      COMMON /FIG/ NTYP,RNEWW,ENEWW,RAYBEG,RAYSET,LINES,IHOP,NOTEST,APHT  0191
      COMMON /XX/ X,PXPR,PXPTH,PXPPH,PXPT,HMAX                            0192
      COMMON/FIAGS/IFIAG
      COMMON B(12),T,STP,DRDT(12) /WW/ ID(10),WC,W(400)                   0193
      EQUIVALENCE (EARTHR,W(19)),(HS,W(40)),(MAXSTP,W(93)),(SKIP,W(180))  0194
     1,(HCP,W(254)),(PLT,W(272))                                          0195
      EQUIVALENCE (CNIY,W(371)),(STEP1,W(44))                        UOFI 1
      REAL MAXSTP                                                         0196
      COMPLEX N2,TPOLAR,RPOLAR                                            0197
      LOGICAL SPACE,HOME,WASNT,PASSED,UNDRGD,GROUND,PERIGE,THERE,         0198
     1 MINDIS,UNDER                                                       0199
      CNIY=C.C                                                       UOFI 2
      STEP=STEP1                                                     UOFI 3
      STP=STEP1                                                      UOFI 4
      NHCP=HCP+0.01                                                       0200
      MAX=MAXSTP+C.01                                                     0201
      NSKIP=SKIP+C.C1                                                     0202
      RSTART=1.                                                           0203
      CALL HASEL                                                          0204
      IF(IFLAG.NE.0) GO TC 50
      H=R(1)-EARTHR                                                       0205
      HOME=DRDT(1)*(H-HS).GE.0.                                           0206
      RAYBEG=1.                                                           0207
      RAYSET=C.                                                           0208
      CALL PRINTR (6HXMTR    )                                            0209
      IF (PLT.NE.0.) CALL RAYPLT                                          0210
      RAYBEG=C.                                                           0211
C                                                                         0212
C          LCCP ON NUMBER CF HOPS                                         0213
      DO 45 JHOP=1,NHCP
      IHOP=JHCP
      RAYSET=C.                                                           0215
      NOTEST=C                                                            0216
      APHT=HS                                                             0217
      TPOLAR=RPOLAR                                                       0218
C                                                                         0219
C          LCCP ON MAXIMUM NUMBER OF STEPS PER HOP                        0220
      DO 28 J=1,MAX                                                       0221
      IF(ABS(R-HS).GT.ABS(APHT-HS))    APHT=H                             0222
      IF (.NOT.SPACE) GC TO 12                                            0223
      CALL REACH                                                          0224
      IF(IFLAG.NE.0) GO TC 50
      RSTART=1.                                                           0225
      IF (IHOP*J.EQ.1) TPOLAR=RPOLAR                                      0226
      IF (GROUND) GC TO 22                                                0227
      IF (PERIGE) CALL PRINTR (6HPERIGEE )                                0228
      IF (THERE) GO TC 40                                                 0229
      IF (MINDIS) GC TO 32                                                0230
      IF (PLT.NE.0.) CALL RAYPLT                                          0231
   12 DO 13 I=1,N                                                         0232
      ROLD(I)=R(I)                                                        0233
   13 DROLD(I)=DRDT(I)                                                    0234
      TOLD=T                                                              0235
      CALL RKAM                                                           0236
      IF(IFLAG.NE.0) GO TC 50
```

```
      H=R(1)-EARTHR                                                          0237
      WASNT=.NOT.HOME                                                        0238
      HOME=DRDT(1)*(H-HS).GE.0.                                              0239
      X=(DRDT(1)-DROLD(1))*(T-TOLD)                                          0240
      SMT=0.                                                                 0241
      IF (X.NE.0.) SMT=0.5*(R(1)-ROLD(1)+0.5*X)**2/ABS(X)                    0242
      UNDRGD=H.LT.0..OR.DRDT(1).GT.0..AND.DROLD(1).LT.0..AND.SMT.GT.H        0243
      PASSED=(H-HS)*(ROLD(1)-EARTHR-HS).LT.0.                                0244
      IF (PASSED.AND.(.NOT.UNDRGD.OR.HS.GT.0.)) GO TO 35                     0245
      IF (HS.EQ.ROLD(1)-EARTHR.AND.DROLD(1)*DRDT(1).LT.0..AND.HOME)          0246
     1 GO TO 39                                                              0247
      IF (HOME.AND.WASNT.AND.(.NOT.UNDRGD.OR.HS.GT.0.)) GO TO 17             0248
      IF (UNDRGD) GO TO 18                                                   0249
      GO TO 23                                                               0250
C                                                                            0251
C          RAY MAY HAVE MADE A CLOSEST APPROACH                              0252
   17 IF (SMT.GT.ABS(H-HS)) GO TO 36                                         0253
      NOTEST=4                                                               0254
      IF (R(5).NE.0..OR.R(6).NE.0.) CALL GRAZE(HS)                           0255
      IF (UNDER) GO TO 19                                                    0256
      IF (NOTEST.EQ.0.) GO TO 36                                             0257
      GO TO 32                                                               0258
C                                                                            0259
C          RAY WENT UNDERGROUND                                              0260
   18 IF (DRDT(1).LT.0.) GO TO 21                                            0261
   19 UNDER=.FALSE.                                                          0262
C     CALL PRINTR(8HUNDERGRD)                                                0263
      DO 20 L=1,N                                                            0264
      R(L)=ROLD(L)                                                           0265
   20 DRDT(L)=DROLD(L)                                                       0266
      T=TOLD                                                                 0267
      CALL RINDEX                                                            0268
      IF(IFLAG.NE.0) GO TO 50
   21 CALL BACK UP(0.)                                                       0269
   22 R(1)=EARTHR                                                            0270
      R(4)=ABS(R(4))                                                         0271
      DRDT(1)=ABS(DRDT(1))                                                   0272
      RSTART=1.                                                              0273
      CALL PRINTR(8HGRND REF)                                                0274
      IF (HS.EQ.0.) GO TO 41                                                 0275
      H=0.                                                                   0276
      GO TO 25                                                               0277
C                                                                            0278
   23 IF (DROLD(1).LT.0..AND.DRDT(1).GT.0.) CALL PRINTR(8HPREIGEE )          0279
      IF (DROLD(1).GT.0..AND.DRDT(1).LT.0.) CALL PRINTR(8HAPOGEE  )          0280
      IF (DROLD(2)*DRDT(2).LT.0.) CALL PRINTR(8HMAX LAT )                    0281
      IF (DROLD(3)*DRDT(3).LT.0.) CALL PRINTR(8HMAX LONG)                    0282
      DO 24 I=4,6                                                            0283
      IF (ROLD(I)*R(I).LT.0.) CALL PRINTR(8HWAVE REV)                        0284
   24 CONTINUE                                                               0285
   25 IF (PIT.NE.0.) CALL RAYPLT                                             0286
      IF(IFLAG.NE.0) GO TO 50
      IF (H.GT.HMAX.AND.H.GT.HS.AND.DRDT(1).GT.0.) GO TO 30                  0287
      IF (MOD(J,NSKIP).EQ.0) CALL PRINTR(8H         )                        0288
   28 CONTINUE                                                               0289
C                                                                            0290
C          EXCEEDED MAXIMUM NUMBER OF STEPS                                  0291
      NOTEST=2                                                               0292
      RAYSET=1.                                                              0293
      CALL PRINTR(8HMAX STEP)                                                0294
      RETURN                                                                 0295
```

```
C                                                                       0296
C           RAY PENETRATED                                              0297
   30 NUTEST=1                                                          0298
      RAYSET=1.                                                         0299
      ONLY=1.0                                                        UOFI 5
      CALL PRINTR(8HPENETRAT)                                           0300
      RETURN                                                            0301
C                                                                       0302
C           RAY MADE A CLOSEST APPROACH                                 0303
   32 NUTEST=4                                                          0304
      DRDT(1)=0.                                                        0305
      RAYSET=1.                                                         0306
      CALL PRINTR(8HMIN DIST)                                           0307
      IF (PLT.NE.0.) CALL RAYPLT                                        0308
      IF (IHOP.GE.NHOP) GO TO 45                                        0309
      IHOP=IHOP+1                                                       0310
      CALL PRINTR (8HMIN DIST)                                          0311
      GO TO 45                                                          0312
C                                                                       0313
C           RAY CROSSED RECEIVER HEIGHT                                 0314
   35 IF (HOME) GO TO 39                                                0315
   36 CONTINUE                                                          0316
C        CALL PRINTR(8HX RCVRHT)                                        0317
      DO 37 I=1,N                                                       0318
      R(I)=ROLD(I)                                                      0319
   37 DRDT(I)=DROLD(I)                                                  0320
      T=TOLD                                                            0321
      CALL RINDEX                                                       0322
      RSTART=1.                                                         0323
   39 CALL BACK UP(HS)                                                  0324
   40 R(1)=EARTHR+HS                                                    0325
   41 RAYSET=1.                                                         0326
      CALL PRINTR(8HRCVR    )                                           0327
      IF (PLT.NE.0.) CALL RAYPLT                                        0328
   45 HOME=.TRUE.                                                       0329
      RETURN                                                            0330
   50 NUTEST=1
      RYTEST=1.0
      CALL PRINTR(8H CUT ICN   )
      IFLAG=0
      RETURN
         END                                                            0331
```

```
      SUBROUTINE BACK UP(HS)                                                0332
      DIMENSION NWORD1(2),NWORD2(2),NWORD(2)
      COMMON /BK/ NN,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART          0333
      COMMON /TRAC/ GROUND,REFIGE,THERE,MINDIS,UNDER                        0334
      COMMON /FIG/ NTYP,RNEWW,ENEWW,RAYBEG,RAYSET,LINES,IHOP,NUTEST,APHT    0335
      COMMON R(12),T,STP,DRDT(12) /WW/ ID(10),W0,W(400)                     0336
      EQUIVALENCE (EARTHR,W(19)),(INTYE,W(41)),(STEP1,W(44))                0337
      REAL INTYE                                                            0338
      LOGICAL UNDER                                                         0339
      DATA NWORD1,NWORD2/4HBACK,4BUP  ,4HGBAZ,4HE   /
      STPS=STP
      NWORD(1)=NWORD1(1)
      NWORD(2)=NWORD1(2)
      DO 1 I=1,10                                                           0340
      IF (DRDT(1).EQ.0.) GO TO 5                                            0341
      STEP=-(R(1)-EARTHR-HS)/DRDT(1)                                        0342
       STP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF (ABS(R(1)-EARTHR-HS).LT..5E-4.AND.STEP.LT.1.) GO TO 5              0344
C        CALL PRINTR(8BECMING  )                                            0345
      MODE=1                                                                0346
      CALL RKAM                                                             0348
1     CONTINUE
C                                                                           0350
      ENTRY GBACE                                                           0351
      STPS=STP
      NWORD(1)=NWORD2(1)
      NWORD(2)=NWORD2(2)
      DO 2 I=1,10                                                           0352
      IF (DRDT(4).EQ.0.) GO TO 5                                            0353
      STEP=-R(4)/DRDT(4)                                                    0354
       STP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF(ABS(R(4)).LE.1.E-6.AND.STEP.LT.1.) GO TO 5                         0356
C        CALL PRINTR(8BECMING  )                                            0357
      ROLD=R(1)                                                             0358
      MODE=1                                                                0359
      CALL RKAM                                                             0361
      IF (R(1)-EARTHR.LT.0.) GO TO 4                                        0363
      IF ((R(1)-EARTHR-HS)*(ROLD-EARTHR-HS).LT.0.) GO TO 3                  0364
  2   CONTINUE                                                              0365
      GO TO 5                                                               0366
  3   NUTEST=0                                                              0367
      GO TO 5                                                               0368
    4 UNDER=.TRUE.                                                          0369
    5 MODE=INTYE+0.01                                                       0370
      STEP=STEP1                                                            0371
      STP=STPS
      RSTART=1.0
      CALL PRINTR(NWORD)
      RETURN                                                                0372
         END                                                                0373
```

```
      SUBROUTINE RKAM                                                    0888
C        NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS                 0889
      COMMON/FLAGS/IFLAG
      COMMON /RK/ NN,SPACE,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART      0890
      COMMON Y(12),T,STEP,DYDT(12) /WW/ ID(10),WC,W(400)
      DIMENSION DELY(4,12),BET(4),XV(5),FV(4,12),YU(5,12)                0892
      REAL*8 YU
      EQUIVALENCE (EARTHR,W(19)),(HMIN,W(390)),(HMAX,W(391))
      DATA XMM/1.0E-6/
      IF (RSTART.EQ.0.) GO TO 1000                                       0894
      LL=1                                                               0895
      MM=1                                                               0896
      IF (MODE.EQ.1) MM=4                                                0897
      ALPHA=1                                                            0898
      EPM=0.0                                                            0899
      BET(1)=0.5                                                         0900
      BET(2)=0.5                                                         0901
      BET(3)=1.0                                                         0902
      BET(4)=0.0                                                         0903
      R=19.0/270.0                                                       0905
      XV(MM)=T                                                           0906
      IF (E1MIN.LE.0.) E1MIN=E1MAX/55.                                   0907
      IF (FACT.LE.0.) FACT=0.5                                           0908
      CALL HASEL                                                         0909
      IF(MODE.NE.1) STEP=SPACE                                          1/28/71
      IF(IFLAG.NE.0) GO TO 100
      DO 320 I=1,NN                                                      0910
      FV(MM,I)=DYDT(I)                                                   0911
  320 YU(MM,I)=Y(I)                                                      0912
      RSTART=0.                                                          0913
      GO TO 1001                                                         0914
 1000 IF (MODE.NE.1) GO TO 2000                                          0915
C                                                                        0916
C           RUNGE-KUTTA                                                  0917
      DR1=DYDT(1)*STEP
      DR2=HMIN+EARTHR-Y(1)
      IF((DR1.LT.DR2).AND.(DR2.LT.0.0)) STEP=DR2/DYDT(1)+XMM
      DR3=HMAX+EARTHR-Y(1)
      IF((DR1.GT.DR3).AND.(DR3.GT.0.0)) STEP=DR3/DYDT(1)+XMM
 1001 DO 1034 K=1,4                                                      0918
      DO 1350 I=1,NN                                                     0919
      DELY(K,I)=STEP*FV(MM,I)                                            0920
      Z=YU(MM,I)                                                         0921
 1350 Y(I)=Z+BET(K)*DELY(K,I)                                            0922
      T=BET(K)*STEP+XV(MM)                                               0923
      CALL HASEL                                                         0924
      IF(IFLAG.NE.0) GO TO 100
      DO 1034 I=1,NN                                                     0925
 1034 FV(MM,I)=DYDT(I)                                                   0926
      DO 1039 I=1,NN                                                     0927
      DEL=(DELY(1,I)+2.0*DELY(2,I)+2.0*DELY(3,I)+DELY(4,I))/6.0          0928
 1039 YU(MM+1,I)=YU(MM,I)+DEL                                            0929
      MM=MM+1                                                            0930
      XV(MM)=XV(MM-1)+STEP                                               0931
      DO 1400 I=1,NN                                                     0932
 1400 Y(I)=YU(MM,I)                                                      0933
      T=XV(MM)                                                           0934
      CALL HASEL                                                         0935
      IF(IFLAG.NE.0) GO TO 100
      IF (MODE.EQ.1) GO TO 42                                            0936
```

```
      DO 150 I=1,NN                                                    0937
  150 FV(MM,I)=DYDT(I)                                                 0938
      IF (MM.LE.3) GO TO 1001                                          0939
C                                                                      0940
C          ADAMS-MOULTON                                               0941
 2000 DO 2048 I=1,NN                                                   0942
      DEL=STEP*(55.*FV(4,I)-59.*FV(3,I)+37.*FV(2,I)-9.*FV(1,I))/24.    0943
      Y(I)=YU(4,I)+DEL                                                 0944
 2048 DELY(1,I)=Y(I)                                                   0945
      T=XV(4)+STEP                                                     0946
      CALL BASEI                                                       0947
      IF(IFLAG.NE.0) GO TO 100
      XV(5)=T                                                          0948
      DO 2051 I=1,NN                                                   0949
      DEL=STEP*(9.*DYDT(I)+19.*FV(4,I)-5.*FV(3,I)+FV(2,I))/24.         0950
      YU(5,I)=YU(4,I)+DEL                                              0951
 2051 Y(I)=YU(5,I)                                                     0952
      CALL BASEI                                                       0953
      IF(IFLAG.NE.0) GO TO 100
      IF (MODE.LE.2) GO TO 42                                          0954
C                                                                      0955
C          ERROR ANALYSIS                                              0956
      SSE=0.0                                                          0957
      DO 3033 I=1,NN                                                   0958
      EPSIL=R*ABS(Y(I)-DELY(1,I))                                      0959
      IF (MODE.EQ.3.AND.Y(I).NE.0.) EPSIL=EPSIL/ABS(Y(I))              0960
      IF (SSE.LT.EPSIL) SSE=EPSIL                                      0961
 3033 CONTINUE                                                         0962
      IF (E1MAX.GT.SSE) GO TO 3035                                     0963
      IF (ABS(STEP).LE.E2MIN) GO TO 42                                 0964
      LL=1                                                             0965
      MM=1                                                             0966
      STEP=STEP*FACT                                                   0967
      GO TO 1001                                                       0968
 3035 IF (LL.LE.1.OR.SSE.GE.E1MIN.OR.E2MAX.LE.ABS(STEP)) GO TO 42      0969
      LL=2                                                             0970
      MM=3                                                             0971
      XV(2)=XV(3)                                                      0972
      XV(3)=XV(5)                                                      0973
      DO 5363 I=1,NN                                                   0974
      FV(2,I)=FV(3,I)                                                  0975
      FV(3,I)=DYDT(I)                                                  0976
      YU(2,I)=YU(3,I)                                                  0977
 5363 YU(3,I)=YU(5,I)                                                  0978
      STEP=2.0*STEP                                                    0979
      GO TO 1001                                                       0980
C                                                                      0981
C        EXIT ROUTINE                                                  0982
   42 LL=2                                                             0983
      MM=4                                                             0984
      DO 12 K=1,3                                                      0985
      XV(K)=XV(K+1)                                                    0986
      DO 12 I=1,NN                                                     0987
      FV(K,I)=FV(K+1,I)                                                0988
   12 YU(K,I)=YU(K+1,I)                                                0989
      XV(4)=XV(5)                                                      0990
      DO 52 I=1,NN                                                     0991
      FV(4,I)=DYDT(I)                                                  0992
   52 YU(4,I)=YU(5,I)                                                  0993
      IF (MODE.LE.2) RETURN                                            0994
      E=ABS(XV(4)-ALPHA)                                               0995
```

```
      IF (E.LE.EPM) GO TO 2000                                          0996
      EPM=E                                                             0997
  100 RETURN
          END                                                           0999
```

```
      SUBROUTINE PRINTR(NWHY)                                                  0493
C                          PRINTS A LINE EACH TIME IT IS CALLED               0494
      DIMENSION G(3,3),G1(3,3),A1(3),E1(3),C1(3)
     1,TITLE1(10),TITLE2(10),UNIT(10)                                          0496
      DIMENSION NWHY(2),IBLNK(2)
      COMMON /CONST/ PI,PIT2,PIT2,DEGS,RAD,DUM(3)                              0497
      COMMON /BK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART              0498
      COMMON /BIN/ N2,MUX,BN2(8),TECLAB(2),POLAR(2),SPACE,COIL,FIELD           0499
      COMMON /FLG/ NTYP,RNEWW,ENEWW,RAYBEG,RAYSET,LINES,IHOP,NOTEST,APHT       0500
      COMMON B(12),T /WW/ ID(10),WC,W(400)                                     0501
      COMMON /XX/ X,EXPE,EXPTH,EXPPH,EXPT,EMAX                          501 KV 1
      COMMON /VERN1/ VN(250),VNPE(250),VNPTH(250),VNPPH(250)            501 KV 2
      COMMON /VERN2/ VRANGE(250),VHT(250)                               501 KV 3
      COMMON /NDOPT/  NPRINT,KOPT,MPRT                                  501 KV 5
      COMMON /EFLD/  SIDEG                                              501*KV 4
      EQUIVALENCE (THETA,B(2)),(PHI,B(3))                                  0502
      EQUIVALENCE (P,W(3)),(PLON,W(13)),(LCN,W(14)),(PLAT,W(15)),          0503
     1 (LAT,W(16)),(PITA,W(17)),(AZ1,W(18)),(EARTHR,W(19)),(XMTRH,W(20))   0504
     2,(P7,W(280)),(P8,W(281)),(P9,W(282))                                 0505
      EQUIVALENCE (RANGE,W(384)),(AZDEV,W(379)),(H,W(383))
      EQUIVALENCE (E2IT,W(372)),(E3LT,W(373)),(AZA,W(374))              UOFI 1
      REAL N2,ICN,LAT,MUX                                                  0506
      DATA A1,E1,C1/1HE,1HN,1HO,1HX,1HE,1HR,1HT,1HI,1HD/
      DATA TITLE1(7),TITLE2(7),UNIT(7)/4H EH.,4HPATH,4H  KM/
      DATA TITLE1(8),TITLE2(8),UNIT(8)/4H ABS,4HCEEN,4H  DB/
      DATA TITLE1(9),TITLE2(9),UNIT(9)/4H DOE,4HPLER,4H C/S/
      DATA IBLNK/2*4H
      P7=C.0
      P8=C.0
      P9=C.0
      IF (RNEWW.EQ.C.) GO TO 8                                             0512
C                                                                          0513
C          NEW W ARRAY -- REINITIALIZE                                     0514
      ENEWW=C.                                                             0515
      SPL=SIN(PION-LCN)                                                    0516
      CPL=SIN(PI2-(PICN-ICN))                                              0517
      SP=SIN(PIAT)                                                         0518
      CP=SIN(PI2-PLAT)                                                     0519
      SL=SIN(LAT)                                                          0520
      CL=SIN(PI2-LAT)                                                      0521
      G(1,1)=CPI*SP*CI-CP*SL                                               0522
      G(1,2)=SPL*SP                                                        0523
      G(1,3)=-SI*SP*CPL-CL*CP                                              0524
      G(2,1)=-SPL*CL                                                       0525
      G(2,2)=CPI                                                           0526
      G(2,3)=SI*SPL                                                        0527
      G(3,1)=CI*CP*CPI+SP*SL                                               0528
      G(3,2)=CP*SPI                                                        0529
      G(3,3)=-SL*CP*CPL+SP*CL                                              0530
      DENM=G(1,1)*G(2,2)*G(3,3)+G(1,2)*G(3,1)*G(2,3)+G(2,1)*G(3,2)*G(1,3   0531
     1)-G(2,2)*G(3,1)*G(1,3)-G(1,2)*G(2,1)*G(3,3)-G(1,1)*G(3,2)*G(2,3)     0532
      G1(1,1)=(G(2,2)*G(3,3)-G(3,2)*G(2,3))/DENM                           0533
      G1(1,2)=(G(3,2)*G(1,3)-G(1,2)*G(3,3))/DENM                           0534
      G1(1,3)=(G(1,2)*G(2,3)-G(2,2)*G(1,3))/DENM                           0535
      G1(2,1)=(G(3,1)*G(2,3)-G(2,1)*G(3,3))/DENM                           0536
      G1(2,2)=(G(1,1)*G(3,3)-G(3,1)*G(1,3))/DENM                           0537
      G1(2,3)=(G(2,1)*G(1,3)-G(1,1)*G(2,3))/DENM                           0538
      G1(3,1)=(G(2,1)*G(3,2)-G(3,1)*G(2,2))/DENM                           0539
      G1(3,2)=(G(3,1)*G(1,2)-G(1,1)*G(3,2))/DENM                           0540
      G1(3,3)=(G(1,1)*G(2,2)-G(2,1)*G(1,2))/DENM                           0541
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```
      RC=EARTHR+XMTRH                                                    0542
      XR=R0*G(1,1)                                                       0543
      YR=R0*G(2,1)                                                       0544
      ZR=R0*G(3,1)                                                       0545
      COSTHR=G(3,1)                                                      0546
      SINTHR=SIN(ARCOS(COSTHR))                                          0547
      PHIR= ATAN2(YR,XR)                                                 0548
      ALPH=ATAN2(G(3,2),G(3,3))                                          0549
      IF (P8.NE.0.) PUNCH 5, E1(NTYP),ID                                 0550
    5 FORMAT (F1,9A8,A7)                                                 0551
      IF (P9.NE.0.) PUNCH 5, C1(NTYP),ID                                 0552
      IF (P7.NE.0.) PUNCH 6, A1(NTYP),ID                                 0553
    6 FORMAT (A1,1X,9A8,A6)                                              0554
C                                                                        0555
    8 IF (RAYPRG.EQ.0.) GO TO 12                                         0556
      NN=MINO(N,9)                                                       0557
      PRINT 10, (TITLE1(NR),TITLE2(NR),NR=7,NN)                          0558
   10 FORMAT (61X,7HAZIMUTH/60X,9HDEVIATION,7X,9HELEVATION/20X,15HHEIGHT
     1    RANGE,3X,3HPSI,3X,       11HCOLAT  LONG,2(5X,12HXMTR   LOCAL), 560 KV
     2 2X,12HCLARIZATION,2X,8H GR.PATH,4A4)                              561 KV
      PRINT 11, (UNIT(NR),NR=7,NN)                                       0562
   11 FORMAT (16X,2(7X,2HKM),4X,
     2                    3HDEG, 6(5X,3HDEG),3Y,4HREAL,4X,4HIMAG,        563 KV
     1 5X,2HKM,1X,2(A6,2X))                                              564 KV
   12 V=0.                                                               0565
      IF (N2.NE.0.) V=(R(4)**2+R(5)**2+R(6)**2)/N2-1.                    0566
      H=R(1)-EARTHR                                                      0567
      SINTH=SIN(THETA)                                                   0568
      COSTH=SIN(PID2-THETA)                                              0569
      XP=R(1)*SINTH*SIN(PID2-PHI)-XR                                     0570
      YP=R(1)*SINTH*SIN(PHI)-YR                                          0571
      ZP=R(1)*COSTH-ZR                                                   0572
      EPS=XP*G1(1,1)+YP*G1(1,2)+ZP*G1(1,3)                               0573
      ETA=XP*G1(2,1)+YP*G1(2,2)+ZP*G1(2,3)                               0574
      ZETA=XP*G1(3,1)+YP*G1(3,2)+ZP*G1(3,3)                              0575
      RCE2 = ETA**2 + ZETA**2                                            0576
      RCE=SQRT(RCE2)                                                     0577
      RANGE=RC*ATAN2(RCE,R0+EPS)                                        UOFI 2
      ANGDEG=ATAN2(R(4),SQRT(R(5)**2+R(6)**2))*DEGS                      0579
      SR=SQRT(RCE2+EPS**2)                                               0580
C
C     CHANGES MADE BY  ARCON CORP.      BY       K.VANGURI               580 KV 1
C
C     STORE THE ELECTRON DENSITY PROFILE AND THE DERIVATIVES             580 KV 2
C     CALCULATE N THE  ELECTRON DENSITY IN APPLETON - HARTREE FORMULA    580 KV 3
C       N = 12400*F**2 * X        ELECTRONS/CUBIC CM.                    580 KV 4
C         DN/DR    - EL/CC/KM.
C         DN/DTH   - EL/CC/DEG
C       F = FREQUENCY IN MHZ                                             580 KV 5
C         DN/DPH   - EL/CC/DEG
C     DERIVATIVES  OF EL. DEN.  W.R.T. R,TH,PH ARE COMPUTED BY
C        MULTIPLYING  PXPR, PXPTH, PXPPH  WITH  THE CONSTANT  C.
C
      C = 12400.0 * F ** 2                                               580 KV10
      MPRT = MPRT + 1                                                    580 KV11
      CALL ELECTX                                                        580 KV12
      VN(MPRT) = 12400.0*F**2 *X                                         580 KV13
      VNPR (MPRT) = C * PXPR                                             580 KV14
      VNPTH(MPRT) = C * PXPTH * RAD                                      580 KV15
      VNPPH(MPRT) = C * PXPPH * RAD                                      580 KV16
      VRANGE(MPRT) = RANGE                                               580 KV17
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      VHT(MPET) = H                                                    580 KV18
      IF(KOPT.EQ.0.AND.NWHY(1).EQ.IBLNK(1).AND.NWHY(2).EQ.IBLNK(2)) RETU
     1RN
C     PRINT  COLATITUDE  AND  LONGITUDE                                580 KV21
C     R2LT IS COLATITUDE    AND  R3LT  IS LONGITUDE                    580 KV22
      R2LT = R(2) * DEGS                                               580 KV23
      R3LT = R(3) * DEGS                                               580 KV24
C
C     COMPUTE  SIDEG - ANGLE BETWEEN  WAVE NORMAL AND THE MAGNETIC FIELD580KV27
C     ** CALL MAGV FOR OPTIONS 1 + 3.  **  CALL TRUFLD FOR OPTION 2. ** 583 V28
      CALL TRUFLD                                                      OPTION 2
C
      IF (SR.GE.1.E-2) GO TO 20                                           0581
      PRINT 15, V,NWHY,H,RANGE, SIDEG,R2LT,R3LT,                       582 KV 1
     1                          ANGDEG,POLAR,T,(R(I),I=7,NN)           582 KV 2
   15 FORMAT (1X,E8.1,1X,2A4,F8.2,1X,F8.2,3F7.1, 23X,  6F8.2,F6.1)
      GO TO 40                                                            0584
   20 ANGE=ATAN2(EPS,PCE)                                                 0585
      EL=ANGE*DEGS                                                        0586
      IF (PCE.GE.1.E-2) GO TO 30                                          0587
      PRINT 25, V,NWHY,H,RANGE, SIDEG,R2LT,R3LT,
     1                          EL,ANGDEG,POLAR,T,(R(I),I=7,NN)        588 KV 2
   25 FORMAT (1X,E8.1,1X,2A4,F8.2,1X,F8.2,3F7.1, 15X,  7F8.2,F6.1)
      GO TO 40                                                            0590
   30 ANGA=ATAN2(ETA,ZETA)                                                0591
      ANA=ANGA-ALPH                                                       0592
      SINANA=SIN(ANA)                                                     0593
      SINPHI=SINANA*SINTHR/SINTH                                          0594
      COSPHI=-SIN(PID2-ANA)*SIN(PID2-(PHI-PHIR))+SINANA*SIN(PHI-PHIR)*    0595
     1 COSTHR                                                             0596
      AZDEV=180.-AMOD(540.-(AZ1-ANGA)*DEGS,360.)                          0597
      IF (R(5).NE.0..OR.R(6).NE.0.) GO TO 34                              0598
      PRINT 33, V,NWHY,H,RANGE, SIDEG,R2LT,R3LT,
     1                          AZDEV,EL,ANGDEG,POLAR,T,(R(I),I=7,NN)  599 KV 2
   33 FORMAT (1X,E8.1,1X,2A4,F8.2,1X,F8.2,3F7.1,F8.2,7X, 7F8.2,F6.1)
      GO TO 40                                                            0601
   34 AZA=180.-AMOD(540.-(ATAN2(SINPHI,COSPHI)-ATAN2(R(6),R(5)))*DEGS,    0602
     1 360.)                                                              0603
      PRINT 35, V,NWHY,H,RANGE, SIDEG,R2LT,R3LT,
     1                          AZDEV,AZA,EL,ANGDEG,POLAR,T,(R(I),I=7,NN)604 KV 2
   35 FORMAT (1X,E8.1,1X,2A4,F8.2,1X,F8.2,4F7.1,  8F8.2, F6.1)
C
C
   40 LINES=LINES+1                                                       0606
      IF (N.LE.9) GO TO 45                                                0607
      PRINT 42, (R(I),I=10,N)                                             0608
   42 FORMAT (110X,3F8.2)                                              609 KV
      LINES=LINES+1                                                       0610
   45 IF (RAYSET.EQ.0.) RETURN                                            0611
      ELA=BETA*DEGS                                                       0612
      IF (H.LE.1.)    ANGDEG=ABS(ANGDEG)                                  0613
C                                                                         0614
C        3-D RAYSETS                                                      0615
      IF (P8.EQ.0.) GO TO 55                                              0616
      TLON=LON*DEGS                                                       0617
      IF (TLON.LT.0.) TLON=TLON+360.                                      0618
      TLAT=LAT*DEGS                                                       0619
      IF (TLAT.LT.0.) TLAT=TLAT+360.                                      0620
      AZ=ANGA*DEGS                                                        0621
      IF (AZDEV.LE.-100.) AZDEV=AZDEV+360.                                0622
      IF (AZA.LE.-100.) AZA=AZA+360.                                      0623
```

```
      PUNCH 50, B1(NTYP),ID(1),XMIBH,TLAT,TLON,H,RANGE,AZ,F,IHOP,          0624
     1 AZDEV,AZA,ELA,ANGDEG,TPCIAB,ECIAB,NUTEST                            0625
   50 FORMAT (A1,A3,2PF6.C,1P2E4.0,2PF6.0,1PF6.0,F5.0,3PF7.0,I1,2P4F5.0,   0626
     1 1P4E4.C,I1)                                                         0627
C                                                                          0628
C        SUBR. BAYSETS                                                     0629
   55 IF (P9.NE.0.) PUNCH 60, C1(NTYP),ID(1),F,IHOP,ELA,(R(I),I=7,10),     0630
     1 T,NUTEST                                                            0631
   60 FORMAT (A1,A3,3PF7.C,I1,2PF5.C,CP5F12.4,I3)                          0632
C                                                                          0633
C        2-D BAYSETS                                                       0634
      IF (NUTEST.NE.0) APET=H                                              0635
      IF (P7.NE.0.) PUNCH 70, A1(NTYP),ID(1),F,IHOP,ELA,ANGDEG,T,R(7),     0636
     1 RANGE,APHT,NUTEST                                                   0637
   70 FORMAT( A1,1X,A3,3PF7.0,I2,CPF1C.5,F9.5,3F11.4,F12.5,1X,I1)          0638
      RETURN                                                               0639
         END                                                               0640
```

```
      SUBROUTINE ADJUST
C     A SUBROUTINE TO ADJUST THE ELEVATION ANGLE BETA
C     AZIMUTH TOLERANCE IS SPECIFIED IN KM THEN CONVERTED TO RADIANS
      DIMENSION A1(3,3),B1(3),C1(3),DUM1(9)
      COMMON/CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)
      COMMON Y(12),T /WW/ ID(10),WO,W(400)
      COMMON/DENSC/ DENS(52),HEIT(52),ELRANJ(91,3,2),APROX(8,2,2),PRATIO
     1,NMAX,NSTART,JFIAG,NSOL,DBMIN(21),JJJM1
      COMMON/BK/ N,STEP,MODE,DUMY(5),ESTART
      COMMON/BIN/ N2,MUX,EN2(8),POLAR(4), SPACE,COIL,FIELD
      COMMON /NDOPT/ NPRINT,KOPT,MPRT
      REAL N2,MUX,MU
      EQUIVALENCE (GRANGE,W(385)),(CRANGE,W(384)),(BETA,W(17)),
     1(CONTRL,W(386)),(TOLERE,W(387)),(AZ1,W(18)),(RC,W(375)),
     2(ALPHA,W(376)),(THO,W(377)),(PHC,W(378)),(AZDEV,W(379)),
     3(AZA,W(374)),  (BETAT,W(389)),(H,W(383)),(R2IT,W(372)),
     4(R3LT,W(373)),(EARTHR,W(19))
      DATA A1(1,1),A1(2,1),A1(3,1)/3*1.0/
C     SET UP THE MAXIMUM HEIGHT FOR GROUND - SATELLITE CASE
      IF(NSOL.EQ.0)  RETURN
      NHOP=W(254)+0.1
      KONTRL=CONTRL
      RADIUS=EARTHR+W(40)
      IF(GRANGE.NE.0.0)  GO TO 806
      AZTOL=TOLERE/RADIUS
      GO TO 807
806   AZTOL=TOLERE/(RADIUS*SIN(GRANGE/RADIUS))
807   DAZTOL=AZTOL*DEGS
      PRINT 808,TOLERE,DAZTOL
808   FORMAT(1X,16HRANGE TOLERANCE=,F12.6,3H KM,3X,18HAZIMUTH TOLERANCE=
     1,F12.6,8H DEGREES)
      AZ2=AZ1
      DO 800 III=1,NSOL
      INCR=0
      CONT=0.0
      PRINT 805
805   FORMAT(1H1)
      IF(W(380).NE.1.0)  GO TO 249
      PRINT 252,LIL
252   FORMAT(2X,26HBEGINNING FOR SOLUTION NUMBER,I3)
C     SET UP AN INITIAL SLOPE BASED UPON PLANE GEOMETRY CONSIDERATION
C     FOR SATELLITE TO GROUND OR GROUND TO SATELLITE CASES
249   DINCR=-GRANGE/(SIN(APROX(LIL,1,1))*COS(APROX(LIL,1,1)))
      IF(W(380).NE.1.0) GO TO 253
      DINCR=0.0
      INDEX=APROX(LIL,2,1)
      DO 254 I=1,NHOP
      DINCR=DINCR+(ELRANJ(INDEX,2,I)-ELRANJ(INDEX-1,2,I))/
     1(ELRANJ(INDEX,1,I)-ELRANJ(INDEX-1,1,I))
254   CONTINUE
253   COUNT=0.0
      BETA=APROX(LIL,1,1)
      IF(W(380).NE.1.0.OR.NHOP.NE.1)  GO TO 300
      DO 260 I=1,JJJM1
      IF(BETA.GT.DBMIN(I+1))  GO TO 260
      JJK=I
      BLEFT=DBMIN(JJK)
      BRIGHT=DBMIN(JJK+1)
      GO TO 300
260   CONTINUE
```

```
C       ERROR IN COMPUTING HOMING ANGLE
        PRINT 261
261     FORMAT(2X,39H***ERROR IN COMPUTING  HOMING ANGLE ***)
        GO TO 800
300     CALL RAYINT(AZ1,BETA)
        EL=BETA*DEGS
        AZ3=AZ1*DEGS
        PRINT 55,EL,AZ3
        IF(W(380).NE.1.0.OR.NHOP.NE.1)  GO TO 301
        IF(BETA.LT.BLEFT.OR.BETA.GT.BRIGHT)  GO TO 700
301     CALL TRACE
        PRINT 75,R2LT,R3LT,AZDEV,AZA
75      FORMAT(1X,10HRAY COLAT=,F12.5,2X,9HRAY LONG=,F12.5,2X,
       114HR.AZDEV AT T.=,F12.5,2X,18HLOCAL AZDEV AT R.=,F12.5)
        IF(W(394).NE.0.0)  CRANGE=T
        IF(ABS(GRANGE-CRANGE).LE.TOLERR) GO TO 500
        IF(R.GT.(W(40)+0.1).OR.R.LT.(W(40)-0.1))  GO TO 780
        MPRT=0
        B1(1)=CRANGE
        A1(1,2)=BETA
        A1(1,3)=B1(1,2)**2
        IF(INCR.EQ.0) GO TO 310
        DINCR= (B1(1)-B1(2))/(A1(1,2)-A1(2,2))
C       INTERPOLATE LINEARLY IF POSSIBLE
        IF(B1(1).LT.GRANGE.AND.B1(2).LT.GRANGE) GO TO 310
        IF(B1(1).GT.GRANGE.AND.B1(2).GT.GRANGE) GO TO 310
        B=(B1(1)-B1(2))/(A1(1,2)-A1(2,2))
        A=B1(1)-B*A1(1,2)
        BETA=(GRANGE-A)/B
        GO TO 590
C       INCREMENT OR DECREMENT THE APPROXIMATE ELEVATION UTILIZING THE
C       SLOPE OF THE I-E CURVE
310     DIFFR=GRANGE-CRANGE
        BETA=BETA+DIFFR/DINCR
        IF(BETA.GT.0.0.OR.W(380).NE.1.0)  GO TO 312
        IF(CONT.EQ.1.0)  GO TO 2100
        BETA=0.0
        CONT=1.0
312     CALL RAYINT(AZ1,BETA)
        EL=BETA*DEGS
        AZ3=AZ1*DEGS
        PRINT 55,EL,AZ3
        IF(W(380).NE.1.0.OR.NHOP.NE.1)  GO TO 311
        IF(BETA.LT.BLEFT.OR.BETA.GT.BRIGHT)  GO TO 700
311     CALL TRACE
        PRINT 75,R2LT,R3LT,AZDEV,AZA
        IF(W(394).NE.0.0)  CRANGE=T
        IF(ABS(GRANGE-CRANGE).LE.TOLERR) GO TO 500
        IF(R.GT.(W(40)+0.1).OR.R.LT.(W(40)-0.1))  GO TO 780
        MPRT=0
        B1(2)=CRANGE
        A1(2,2)=BETA
        A1(2,3)=A1(2,2)**2
        DINCR= (B1(1)-B1(2))/(A1(1,2)-A1(2,2))
        INCR=INCR+1
C       IF THE GIVEN RANGE IS BETWEEN THE FIRST AND SECOND RANGE THUS
C       COMPUTED , INTERPOLATE LINEARLY ON BETA.
        IF(B1(1).LT.GRANGE.AND.B1(2).LT.GRANGE) GO TO 580
        IF(B1(1).GT.GRANGE.AND.B1(2).GT.GRANGE) GO TO 580
C       INTERPOLATE USING RANGE=A+B*BETA
        B=(B1(1)-B1(2))/(A1(1,2)-A1(2,2))
```

```
      A=B1(1)-B*A1(1,2)
      BETA=(GRANGE-A)/B
      GO TO 590
C     INCREMENT OR DECREMENT THE APPROXIMATE ELEVATION UTILIZING THE
C     SLOPE OF THE D-B CURVE
580   DIFFR=GRANGE-CRANGE
      BETA=BETA+DIFFR/DINCR
      IF(BETA.GT.0.0.OR.W(380).NE.1.0)  GO TO 581
      IF(CONT.EQ.1.0)  GO TO 2100
      BETA=0.0
      CONT=1.0
581   IF(INCR.GT.KONTRI)  GO TO 2000
      GO TO 300
590   CALL RAYINT(AZ1,BETA)
      EL=BETA*DEGS
      AZ3=AZ1*DEGS
      PRINT 55,EL,AZ3
      IF(W(380).NE.1.0.OR.NHOP.NE.1)  GO TO 591
      IF(BETA.LT.BLEFT.OR.BETA.GT.BRIGHT)  GO TO 700
591   CALL TRACE
      PRINT 75,R2LT,R3LT,AZDEV,AZA
      IF(W(394).NE.0.0)  CRANGE=T
      IF(ABS(GRANGE-CRANGE).LE.TOLERR) GO TO 500
      IF(R.GT.(W(40)+0.1).OR.R.LT.(W(40)-0.1))  GO TO 780
      MPRT=0
      B1(3)=CRANGE
      A1(3,2)=BETA
      A1(3,3)=A1(3,2)**2
      CALL EQSOLV(B1,A1,3,3,1,C1,DUM1,1.0E-8,IER)
C     USE QUADRATIC FORMULA TO COMPUTE BETA
C     C1(3)*BETA**2+C1(2)*BETA+C1(1)-GRANGE=0
      ARG=C1(2)**2-4.0*C1(3)*(C1(1)-GRANGE)
      IF(ARG) 200,210,210
C     THE ARGUMENT IS NEGATIVE , NO SOLUTION
200   IF(W(380).EQ.2.0)  GO TO 611
      PRINT 2010,III
      PRINT 610
      GO TO 1000
611   PRINT 2011
      PRINT 612
612   FORMAT(6X,53HDISCONTINUITY IN THE RAY TRACED RANGE-ELEVATION CURVE
     1   )
      GO TO 1000
210   BETAP=(-C1(2)+SQRT(ARG))/(2.0*C1(3))
      BETAM=(-C1(2)-SQRT(ARG))/(2.0*C1(3))
C     FIND THE CLOSEST RANGE
C     FIND THE CLOSEST ANGLE TO THE GIVEN RANGE
      DIFF1=ABS(GRANGE-B1(1))
      DIFF2=ABS(GRANGE-B1(2))
      DIFF3=ABS(GRANGE-B1(3))
      IF(DIFF1.LT.DIFF2.AND.DIFF1.LT.DIFF3)  ANG=A1(1,2)
      IF(DIFF2.LT.DIFF1.AND.DIFF2.LT.DIFF3)  ANG=A1(2,2)
      IF(DIFF3.LT.DIFF1.AND.DIFF3.LT.DIFF2)  ANG=A1(3,2)
      BETA=BETAM
      IF(ABS(ANG-BETAP).LT.ABS(ANG-BETAM)) BETA=BETAP
650   EL=BETA*DEGS
      AZ3=AZ1*DEGS
      PRINT 55,EL,AZ3
55    FORMAT(/ 5X,33HELEVATION ANGLE OF TRANSMISSION =,F12.6,4H DEG,5X,3
     11HAZIMUTH ANGLE OF TRANSMISSION =,F12.6,4H DEG/)
100   COUNT=COUNT+1.0
```

AD-A038 299 ILLINOIS UNIV AT URBANA-CHAMPAIGN DEPT OF ELECTRICAL --ETC F/G 17/2.1
TECHNIQUES OF DETERMINING IONOSPHERIC STRUCTURE FROM OBLIQUE RA--ETC(U)
DEC 76 N N RAO, K C YEH, M Y YOUAKIM F19628-75-C-0088

UNCLASSIFIED UILU-ENG-76-2559 RADC-TR-76-401 NL

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END

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```
      IF(CCUNT.GT.CCNTRI) GO TO 2000
      CALL RAYINT(AZ1,BETA)
      IF(W(38C).NE.1.C.CR.NHCE.NE.1)  GO TO 101
      IF(BETA.LT.BLEFT.CR.BETA.GT.BRIGHT)  GO TO 700
101   CALL TRACE
      PRINT 75,R2LT,R3LT,AZDFV,AZA
      IF(W(394).NE.C.C)  CRANGE=T
C     USE THE D-R CURVE TC ADJUST BETA
      DIFFR=GRANGE-CRANGE
      IF(ABS(DIFFR).LE.TOLERR)  GO TO 500
      IF(R.GT.(W(40)+C.1).OR .R.LT.(W(40)-0.1))  GO TO 780
      MPRT=0
C     SELECT THE CLOSEST RANGE TC THE GIVEN RANGE
      DIFF1=GRANGE-B1(1)
      DIFF2=GRANGE-B1(2)
      DIFF3=GRANGE-B1(3)
      IF(ABS(DIFF1)-ABS(DIFF2)) 10,20,20
10    IF(ABS(DIFF2)-ABS(DIFF3)) 50,60,60
C     REPLACE THE SECCNC RANGE AND ELEVATICN ANGLE
60    B1(2)=CRANGE
      A1(2,2)=BETA
      A1(2,3)=A1(2,2)**2
      GC TC 150
20    IF(ABS(DIFF1)-ABS(DIFF3)) 50,30,30
C     REPLACE THE FIRST RANGE AND ELEVATICN ANGLE
30    B1(1)=CRANGE
      A1(1,2)=BETA
      A1(1,3)=A1(1,2)**2
      GO TO 150
C     REPLACE THE THIRD RANGE AND ELEVATICN ANGLE
50    B1(3)=CRANGE
      A1(3,2)=BETA
      A1(3,3)=A1(3,2)**2
150   CALL EQSCLV(B1,A1,3,3,1,C1,DUM1,1.0E-8,IER)
C     USE QUADRATIC TC CCMPUTE BETA
      ARG=C1(2)**2-4.0*C1(3)*(C1(1)-GRANGE)
      IF(ARG) 160,170,170
C     THE ARGUMENT IS NEGATIVE , THERE IS NO SCLUTION
160   IF(W(38C).EQ.2.0)  GO TO 162
      PRINT 2C1C,III
      PRINT 61C
      GO TO 10C0
162   PRINT 2C11
      PRINT 612
      GO TO 1000
170   BETAP=(-C1(2)+SQRT(ARG))/(2.C*C1(3))
      BETAM=(-C1(2)-SQRT(ARG))/(2.0*C1(3))
C     FIND THE CLOSEST RANGE
C     FIND THE CLOSEST ANGLE TC THE GIVEN RANGE
      DIFF1=ABS(GRANGE-B1(1))
      DIFF2=ABS(GRANGE-B1(2))
      DIFF3=ABS(GRANGE-B1(3))
      IF(DIFF1.LT.DIFF2.AND.DIFF1.LT.DIFF3) ANG=A1(1,2)
      IF(DIFF2.LT.DIFF1.AND.DIFF2.LT.DIFF3) ANG=A1(2,2)
      IF(DIFF3.LT.DIFF1.AND.DIFF3.LT.DIFF2) ANG=A1(3,2)
      BETA=BETAM
      IF(ABS(ANG-BETAP).LT.ABS(ANG-BETAM)) BETA=BETAP
180   EL=BETA*DEGS
      AZ3=AZ1*DEGS
      PRINT 55,EL,AZ3
      GO TO 1CC
```

```
C     CORRECT FOR AZIMUTHAL DEVIATION
500   IF(H.LE.(W(40)+0.1).AND.H.GE.(W(40)-0.1))  GO TO 502
      GO TO 780
502   AZ4=AZ1-AZDEV*RAD
      IF(ABS(AZ4-AZ2).LE.AZTOL)  GO TO 3000
      INCR=0
      MPRT=0
      AZ5=AZ1+AZDEV*RAD
      AZ1=AZ5
      IF(ABS(AZ5-AZ2).GT.ABS(AZ4-AZ2))  AZ1=AZ4
      COUNT=0.0
      GO TO 300
2000  IF(W(380).EQ.1.0)  PRINT 2010,III
      IF(W(380).EQ.2.0)  PRINT 2011
2010  FORMAT(/// 2X,62H*** HOMING CANNOT BE ACHIEVED FOR APPROXIMATE SOL
     1UTION NUMBER , I3)
2011  FORMAT(/// 2X,29H*** HOMING CANNOT BE ACHIEVED  )
      PRINT 2020,CONTRI
2020  FORMAT(6X,35HEXCEEDED NUMBER OF SPECIFIED TRIES , F5.0)
      GO TO 1000
2100  PRINT 2010,III
      PRINT 2110
2110  FORMAT(6X,49HTHE RANGE IS HIGHER THAN THE ZERO ELEVATION RANGE  )
      GO TO 1000
C     CHECK IF THE DIAGNOSTIC IS FOR A HIGH RAY
700   JJKD2=JJK/2
      JJKR=JJK-JJKD2*2
      IF(JJKR.EQ.0)  GO TO 750
C     IT IS NOT A HIGH RAY
      PRINT 2010,III
      PRINT 610
610   FORMAT(6X,88HDISCONTINUITY IN THE RAY TRACED RANGE-BETA CURVE OR R
     1ANGE TOO CLOSE TO THE SKIP DISTANCE)
      GO TO 1000
C     IT IS A HIGH ANGLE RAY
750   PRINT 2010,III
      PRINT 755
755   FORMAT(6X,14HHIGH ANGLE RAY )
      GO TO 1000
780   IF(W(380).EQ.1.0)  PRINT 2010,III
      IF(W(380).EQ.2.0)  PRINT 2011
      PRINT 790
790   FORMAT(6X,52HRAY MISSED THE EARTH OR CANNOT REACH RECEIVER HEIGHT)
      GO TO 1000
C     WE FOUND AN EXACT SOLUTION
3000  IF(W(380).EQ.1.0)  PRINT 3010,III
      IF(W(380).EQ.2.0)  PRINT 3011
3010  FORMAT(/// 2X,39H*** HOMING ACHIEVED FOR SOLUTION NUMBER , I3)
3011  FORMAT(/// 2X,19H*** HOMING ACHIEVED   )
      PRINT 3020,EL
3020  FORMAT(6X,11HELEVATION =, F12.6,2X,7HDEGREES)
      PRINT 3030,AZ3
3030  FORMAT(6X,10HAZIMUTH  =, F12.6,2X,7HDEGREES )
      IF(W(394).EQ.0.0)  GO TO 3034
      PRINT 3031,CRANGE
3031  FORMAT(6X,21HHOMED IN GROUP PATH =,F12.2,2X,2HKM)
      GO TO 3039
3034  PRINT 3035,CRANGE
3035  FORMAT(6X,16HHOMED IN RANGE =, F12.2,2X,2HKM)
      IF(W(380).EQ.1.0)  GO TO 3039
      PRINT 3036,W(20)
```

```
3036  FORMAT(6X,20HTRANSMITTER HEIGHT =, F9.2,2X,2HKM )
      PRINT 3037,W(40)
3037  FORMAT(6X,17HRECEIVER HEIGHT =, F9.2,2X,2HKM )
3039  PRINT 3040,R2IT
3040  FORMAT(6X,33HHOMEI IN GEOMAGNETIC OOLATITUDE =, F12.5,2X,7HDEGREES
     1)
      PRINT 3050,R3LT
3050  FORMAT(6X,32HHOMEI IN GEOMAGNETIC LONGITUDE =, F12.5,2X,7HDEGREES)
1000  W(18)=AZ2
800   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE HOMES
      COMMON/CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)
      COMMON/DENSC/ DENS(52),HEIT(52),FLRANJ(91,3,2),APROX(8,2,2),FRATIO
     1,NMAX,NSTART,JFIAG,NSOL,DEMIN(21),JJJM1
      COMMON Y(6) /WW/ ID(10),WC,W(400)
      COMMON/DENX/ FH(19,52,5),THETA(19),FGT(52),FHI(5),FRES(4),HL
      COMMON/FG2/ LATMX,LONMX,IHTMX
      EQUIVALENCE(EARTHR,W(19)),(TLON,W(14)),(RLON,W(381)),(TLAT,W(16)),
     1(RLAT,W(382)),(BETA,W(17)),(TFREQ,W(3)),(AZ1,W(18)),(XMTRH,W(20))
      EQUIVALENCE (TRANGE,W(385)),(BETAT,W(389)),(PHC,W(378)),(THO,W(377
     1))
      EQUIVALENCE (PLON,W(13)),(PLAT,W(15)),(HCF,W(254))
C     SET NMAX TO ONE
      NMAX=1
      SP=SIN(PLAT)
      CP=SIN(PID2-PLAT)
C     TRANSFORM THE RECEIVER COORDINATES TO GEOMAGNETIC COORDINATES
      SINDPH=SIN(RLON-PLON)
      COSDPH=SIN(PID2-(RLON-PLON))
      SL=SIN(PLAT)
      CL=SIN(PID2-RLAT)
      GRLON=ATAN2(SINDPH*CL,COSDPH*SP*CL-CP*SL)
      GRLAT=ARCOS(COSDPH*CP*CL+SP*SL)
      GRLON=GRLON*DEGS
      GRLAT=GRLAT*DEGS
      ANGL=ABS(TLON-RLON)
      ANGT=PID2-TLAT
      ANGR=PID2-RLAT
C     USE THE COSINE LAW TO FIND THE ANGLE SUBTENDED AT THE EARTH CENTER
      COSTHE=COS(ANGT)*COS(ANGR)+SIN(ANGT)*SIN(ANGR)*COS(ANGL)
C     COMPUTE THE RANGE
      ANGTR=ARCOS(COSTHE)
      TRANSC=EARTHR+XMTRH
      TRANGE=TRANSC*ANGTR
C     COMPUTE TRANSMITTER COORDINATES
      COLAT=PID2-TLAT
      XT=TRANSC*COS(TLON)*SIN(COLAT)
      YT=TRANSC*SIN(TLON)*SIN(COLAT)
      ZT=TRANSC*COS(COLAT)
C     COMPUTE RECEIVER COORDINATES
      RECEIC=EARTHR+W(40)
      COLAT=PID2-RLAT
      XR=RECEIC*COS(RLON)*SIN(COLAT)
      YR=RECEIC*SIN(RLON)*SIN(COLAT)
      ZR=RECEIC*COS(COLAT)
C     COMPUTE THE MAGNITUDES OF CR, CT AND TR
      CT=SQRT(XT*XT+YT*YT+ZT*ZT)
      CR=SQRT(XR*XR+YR*YR+ZR*ZR)
      TR=SQRT((XR-XT)**2+(YR-YT)**2+(ZR-ZT)**2)
C     CALCULATE THE AZIMUTHAL ANGLE
      IF(ANGTR.NE.0.0)  GO TO 120
      AZ1=0.0
      IF(TLAT.GT.RLAT)  AZ1=PI
      GO TO 55
120   SINBTN=SIN(ANGL)*SIN(ANGR)/SIN(ANGTR)
      COSBTN=(COS(ANGR)-COS(ANGT)*COS(ANGTR))/(SIN(ANGT)*SIN(ANGTR))
      AZ1=ARSIN(SINBTN)
      IF(SINBTN.LT.0.0)  GO TO 45
      IF(COSBTN.LT.0.0)  AZ1=PI-AZ1
      GO TO 50
```

```
45      IF(COSBTN.LT.C.C)  AZ1=PI-AZ1
50      IF(TLCN.GT.RLCN)  AZ1=PI12-AZ1
C       COMPUTE THE ELEVATION ANGLE FROM TRIANGLE TCR
55      ARGUM=      (TR**2+CT**2-CR**2)/(2.*TR*CT)
        IF(ARGUM.GT.1.0)  ARGUM=1.0
        IF(ARGUM.LT.-1.C)  ARGUM=-1.0
        BETA=ARCCS(ARGUM)-PID2
75      NSOI=1
        APRCX(1,1,1)=BETA
        TLATD=TLAT*DEGS
        TLCND=TLCN*DEGS
        RLATD=RLAT*DEGS
        RLCND=RLCN*DEGS
        PH01=PH0*DEGS
        TH01=THC*DEGS
        PRINT 27
27      FORMAT(1H1)
        PRINT 30,TLATD,TLCND,XMTHR
30      FORMAT(1X,44HTHE TRANSMITTER GEOGRAPHIC COORDINATES ARE : ,4X,
      14HLAT=,F8.3,2X,5HLONG=,F8.3,2X,7HHEIGHT=,F8.2,3H KM)
        PRINT 31,RLATD,RLCND,W(4C)
31      FORMAT(1X,41HTHE RECEIVER GEOGRAPHIC COORDINATES ARE : , 4X,
      14HLAT=,F8.3,2X,5HLONG=,F8.3,2X,7HHEIGHT=,F8.2,3H KM)
        PRINT 19,PHC1,TH01
19      FORMAT(1X,44HTHE TRANSMITTER GEOMAGNETIC COORDINATES ARE:,2X,5HLON
      1G=,F12.5,3X,6HCOLAT=,F12.5)
        PRINT 21,GRLON,GRLAT
21      FORMAT(1X,41HTHE RECEIVER GEOMAGNETIC COORDINATES ARE:,2X,5HLONG=,
      1F12.5,3X,6HCOLAT=,F12.5)
        PRINT 32,TRANGE,TFREQ
32      FORMAT(1X,29HRANGE AT TRANSMITTER RADIUS = ,F12.2,2X,2HKM,5X,
      127HFREQUENCY OF TRANSMISSION = , F10.4,2X,3HMHZ)
        DBETA=BETA*DEGS
        AZMTH=AZ1*DEGS
        PRINT 37,DBETA,AZMTH
37      FORMAT(1X,28HTHE INITIAL ELEVATION ANGLE=,F8.3,2X,7HDEGREES,5X,
      125HAZIMUTH OF TRANSMISSION =, F12.6,2X,7HDEGREES)
        RETURN
        END
```

```
      SUBROUTINE HOME
C     JFLAG=0 MEANS WE CAN FIND AN APPROXIMATE ELEVATION ANGLE.
C          =1 MEANS WE CANNOT FIND AN APPROXIMATE ELEVATION ANGLE.
      DIMENSION DDMIN(21)
      DIMENSION JSOL(2),TEMPR(91,2),NRCINT(2),JTEME(21)
      REAL*8 A,B,C,DELT,DANS(52),HAIT     ,R1,R2,R3,R4,R5,R6,RE,RI,RJ,DET
      REAL*8 A1,B1,C1
      COMMON/CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)
      COMMON/DENSC/ DENS(52),HEIT(52),ELRANJ(91,3,2),APROX(8,2,2),FRATIO
     1,NMAX,NSTART,JFLAG,NSOL,DDMIN(21),JJJM1
      COMMON Y(6) /WW/ ID(10),WC,W(400)
      COMMON/DENX/ FH(19,52,5),THETA(19),HGT(52),PHI(5),FRES(4),HL
      COMMON/EG2/ IATMX,LONMX,LPTMX
      COMMON/COEFF/ A1(52),B1(52),C1(52),HAIT(52),RAPOGE,RAPOGM
      EQUIVALENCE(EARTHR,W(19)),(TICN,W(14)),(RICN,W(381)),(TLAT,W(16)),
     1(RLAT,W(382)),(BETA,W(17)),(TFREQ,W(3)),(AZ1,W(18))
      EQUIVALENCE (TRANGE,W(385)),(BETAI,W(389)),(DIFG,W(387))
      EQUIVALENCE (PRC,W(378)),(TH0,W(377))
      EQUIVALENCE (TLCN,W(13)),(TLAT,W(15)),(HCP,W(254)),(GROUP,W(394))
      DATA   INTERV/50/
C     W(380)=1 MEANS HOMING GROUND TO GROUND.
C     W(380)=2 MEANS HOMING GROUND TO SATELLITE OR SATELLITE TO GROUND
C     W(380)=3 MEANS FIND THE MINIMUM GROUP PATH
      NNN=W(380)+.1
      GO TO(1200,1300,1400),NNN
C     THE MINIMUM GROUP PATH ROUTINE
1400  CALL GROUPM
      RETURN
1300  CALL HOMES
      RETURN
1200  RE=EARTHR
C     SETUP THE NUMBER OF HOPS
      NHOP=HOP+0.1
C     SETUP THE FACTORS FOR THE MIDPOINT LOCATION OF EACH HOP DIVISION
      HOPF=1.0/(2.0*HCP)
      HOPI=-1.0
      DO 1 I=1,91
      DO 1 J=1,3
      DO 1 K=1,2
1     ELRANJ(I,J,K)=0.0
      DO 2 I=1,8
      DO 2 J=1,2
      DO 2 K=1,2
2     APROX(I,J,K)=0.0
      DO 3 I=1,NHOP
3     JSOL(I)=0
C     SET UP THE NUMBER OF INTERVALS FOR THE ELEVATION ANGLE.
      FRATIO=TFREQ**2
C     CHECK IF GROUP PATH HOMING IS NEEDED
      IF(W(394).EQ.0.0)  GO TO 111
      ANGT=PID2-TLAT
      ANGTR=GROUP/EARTHR
      TRANGE=GROUP
      RANGE=TRANGE/HOP
      AZ1=W(263)
      GO TO 5000
C     CALCULATE THE RANGE BETWEEN TRANSMITTER AND RECEIVER
111   ANGL=ABS(TLON-RLON)
      ANGT=PID2-TLAT
      ANGR=PID2-RLAT
```

```
C       USE THE COSINE LAW TO FIND ANGLE SUBTENDED AT EARTH CENTER
        COSTHE=COS(ANGT)*COS(ANGR)+SIN(ANGT)*SIN(ANGR)*COS(ANGI)
        ANGTR=ARCCS(COSTHE)
        TRANGE=ANGTR*(EARTHR+W(2C))
C       DIVIDE THE TOTAL RANGE BY THE NUMBER OF HOPS NEEDED.
        RANGE=TRANGE/HOP
C
C       THIS PORTION OF THE CODE SELECTS A DENSITY PROFILE THAT IS CLOSEST
C       TO THE MID POINT BETWEEN THE TRANSMITTER AND RECEIVER(AVLON,AVLAT)
C       COORDINATES
        IF(ANGTR.NE.0.0)  GO TO 4225
        RTN=0.0
        IF(TLAT.GT.RLAT)  RTN=PI
        GO TO 5000
4225    SINRTN=SIN(ANGI)*SIN(ANGR)/SIN(ANGTR)
        COSRTN=(COS(ANGR)-COS(ANGT)*COS(ANGTR))/(SIN(ANGT)*SIN(ANGTR))
        RTN=ARSIN(SINRTN)
        IF(SINRTN.LT.0.0)  GO TO 4500
        IF(COSRTN.LT.0.0)  RTN=PI-RTN
        GO TO 5000
4500    IF(COSRTN.LT.0.0)  RTN=-PI-RTN
5000    DO 8000 JHOP=1,NHOP
        NSOL=0
        HOPI=HOPI+2.0
        COSANG=COS(HOPI*HOPR*ANGTR)
        SINANG=SIN(HOPI*HOPR*ANGTR)
C       THIS PORTION IS FOR GROUP PATH RCMING
        IF(W(394).EQ.0.0)  GO TO 5005
        AZT=AZ1
        IF(AZT.GT.PI)  AZT=PIT2-AZT
        CCSNM=COS(ANGT)*CCSANG+SIN(ANGT)*SINANG*COS(AZT)
        AVLATG=PID2-ARCCS(CCSNM)
        TNM=ARSIN(SIN(AZT)*SINANG/SIN(ARCCS(CCSNM)))
        AVLCNG=TLCN+TNM
        IF(AZ1.GT.PI)  AVLCNG=TLCN-TNM
        GO TO 9
5005    COSNM=COS(ANGT)*CCSANG          +SIN(ANGT)*SINANG          *COS(RTN)
        AVLATG=PID2-ARCCS(CCSNM)
        TNM=ARSIN(SIN(RTN)*SINANG          /SIN(ARCCS(CCSNM)))
        IF(TLCN-RLCN)  4,4,5
4       AVLCNG=TLCN+TNM
        AZ1=RTN
        GO TO 6
5       AVLCNG=TLCN-TNM
        AZ1=PIT2-RTN
6       CONTINUE
C
C       TRANSFORM THE MIDPOINT TO GEOMAGNETIC COORDINATES
9       SP=SIN(PLAT)
        CP=SIN(PID2-PLAT)
        SINDPH=SIN(AVLCNG-PLCN)
        COSDPH=SIN(PID2-(AVLCNG-PLCN))
        SL=SIN(AVLATG)
        CL=SIN(PID2-AVLATG)
        AVLCN=ATAN2(SINDPH*CL,COSDPH*SP*CL-CP*SL)
        AVCLAT=ARCOS(COSDPH*CP*CL+SP*SL)
        IF(W(394).NE.0.0)  GO TO 91
C       TRANSFORM THE RECEIVER COORDINATES TO GEOMETRIC COORDINATES
        SINDPH=SIN(RLCN-PLCN)
        COSDPH=SIN(PID2-(RLCN-PLCN))
        SL=SIN(RLAT)
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      CL=SIN(PHI2-RLAT)
      GRLCN=ATAN2(SINDPH*CI,CCSDPH*SP*CL-CP*SI)
      GRLAT=ARCCS(COSDPH*CP*CI+SP*SI)
      GRICN=GRICN*DEGS
      GRLAT=GRIAT*DEGS
C     TEMPORARY PRINTING OF THE MID POINTY
91    GCLCNG=AVIONG*DEGS
      GOLATG=AVIATG*DEGS
      AVICND=AVLCN*DEGS
      AVLATD=AVCLAT*DEGS
      PRINT 9911
9911  FORMAT(2X,'MID POINT COORDINATES, GEOGRAPHIC AND GEOMAGNETIC' /)
      PRINT 9912,GCICNG,GCIATG,AVICND,AVIATD
9912  FORMAT(2X,'GEOGRAPHIC:',2X,'LONG=',F8.2,2X,'COLAT=',F8.2,7X,'GEOMA
     1GNETIC:',2X,'LONG=',F8.2,2X,'COLAT=',F8.2)
      DPHI=ABS(PHI(2)-PHI(1))
      KK=ICNMX
      DO 610 K=1,ICNMX
      DIFF1=ABS(AVLCN-PHI(K))
      IF(DIFF1-DPHI) 620,615,610
615   KK=K
      GO TO 630
620   KK=K
      IF(K.GE.ICNMX) GO TO 630
      DIFF2=ABS(AVLCN-PHI(K+1))
      IF(DIFF2.LT.DIFF1) KK=K+1
      GO TO 630
610   CONTINUE
C     WE CANNOT FIND A LONGITUDE CLOSE ENOUGH TO THE MID POINT.
      WRITE(6,625)
625   FORMAT(1X,82HTHE GIVEN DENSITY PROFILE DOES NOT INCLUDE THE MID PO
     1INT LONGITUDE BETWEEN T AND F)
      GO TO 300
630   DTHETA=ABS(THETA(2)-THETA(1))
      IL=IATMX
      DO 635 I=1,LATMX
      DIFF1=ABS(AVCLAT-THETA(I))
      IF(DIFF1-DTHETA) 645,640,635
640   LL=L
      GO TO 660
645   LL=L
      IF(L.GE.IATMX) GO TO 660
      DIFF2=ABS(AVCLAT-THETA(I+1))
      IF(DIFF2.LT.DIFF1) II=I+1
      GO TO 660
635   CONTINUE
C     CAN NOT FIND A LATITUDE CLOSE ENOUGH TO THE MID POINT
      WRITE(6,650)
650   FORMAT(1X,83HTHE GIVEN DENSITY   PROFILE DOES NOT INCLUDE THE MID
     1POINT LATITUDE BETWEEN T AND F)
      GO TO 300
C     EXTRACT THE DENSITY AT THE MID POINT AND THE CORRESPONDING HEIGHT
660   DO 665 I=1,IHTMX
      DENS(I)=PH(LL,I,KK)**2
      DANS(I)=DENS(I)
      HAIT(I)=HGT(L)
665   HEIT(I)=HGT(L)
C     FIND THE HEIGHT ON WHICH THE MAXIMUM DENSITY OCCURS
      DHEIT =HEIT(2)-HEIT(1)
      TEMP=DENS(1)
      NMAX=1
```

```
      DO 670 I=2,LHTMX
      IF(DENS(I).LE.TEMP) GO TO 670
      NMAX=L
      TEMP=DENS(L)
670   CONTINUE
      HMAX=HEIT(NMAX)
      FVMAX=SQRT(DENS(NMAX))
C     ASSUME AN INITIAL ELEVATION ANGLE OF ZERO
      BETAP=0.0
      IF(TFREQ-FVMAX) 330,320,320
330   BETAT=PI/2
      GO TO 340
C     COMPUTE THE ELEVATION ANGLE FOR A TRAPPED RAY
320   TEMPA=(EARTHR+HMAX)*SQRT(1.0-(FVMAX**2/FRATIO))/EARTHR
      IF(TEMPA-1.0) 321,321,400
321   BETAT=ARCOS(TEMPA)
340   TLATD=TLAT*DEGS
      TLOND=TLON*DEGS
      RLATD=RLAT*DEGS
      RLOND=RLON*DEGS
      PHO1=PHO*DEGS
      THO1=TEO*DEGS
      PRINT 29
29    FORMAT(1H1)
      PRINT 30,TLATD,TLOND,W(20)
30    FORMAT(1X,44HTHE TRANSMITTER GEOGRAPHIC COORDINATES ARE : ,4X,
     14HLAT=,F8.3,2X,5HLONG=,F8.3,2X,7HHEIGHT=,F8.2,3H KM)
      IF(W(394).NE.0.0)  GO TO 301
      PRINT 31,RLATD,RLOND,W(40)
31    FORMAT(1X,41HTHE RECEIVER GEOGRAPHIC COORDINATES ARE : , 4X,
     14HLAT=,F8.3,2X,5HLONG=,F8.3,2X,7HHEIGHT=,F8.2,3H KM)
301   PRINT 19,PHO1,THO1
19    FORMAT(1X,44HTHE TRANSMITTER GEOMAGNETIC COORDINATES ARE:,2X,5HLON
     1G=,F12.5,3X,6HCOLAT=,F12.5)
      IF(W(394).NE.0.0)  GO TO 3211
      PRINT 21,GRLON,GRLAT
21    FORMAT(1X,41HTHE RECEIVER GEOMAGNETIC COORDINATES ARE:,2X,5HLONG=,
     1F12.5,3X,6HCOLAT=,F12.5)
3211  PRINT 32,TRANGE,TFREQ
32    FORMAT(1X,29HRANGE AT TRANSMITTER RADIUS = ,F12.2,2X,2HKM,5X,
     127HFREQUENCY OF TRANSMISSION = , F10.4,2X,3HMHZ)
      NSTART=1
C     CALCULATE COEFFICIENTS AND STORE THEM IN ARRAYS A1,B1,C1
87    I=NSTART
      J=NSTART+1
      K=NSTART+2
      L=NSTART+3
      IF(NMAX-3) 1000,1000,10
C     THERE ARE MORE THAN THREE POINTS
10    R1=RE     +HAIT(I)
      R2=RE     +HAIT(J)
      R3=RE     +HAIT(K)
      R4=R3-R2
      R5=R3-R1
      R6=R1-R2
      DD2=2.0D0
C     CALCULATE THE COEFFICIENT FOR THE FIRST THREE POINTS
      DET=R4*R5*R6/(R1*R2*R3)
      A=(-R1*R1*R4*DANS(I)+R2*R2*R5*DANS(J)+R3*R3*R6*DANS(K))/(DET*R1*R2
     1*R3)
      B=(R1*R1*(R3*R3-R2*R2)*DANS(I)-R2*R2*(R3*R3-R1*R1)*DANS(J)-R3*R3*(
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```
     1R1*R1-R2*R2)*DANS(K))/(DET*R1*R2*R3)
      C=(-R1*R4*DANS(I)+R2*R5*DANS(J)+R3*R6*DANS(K))/DET
      DELT=-B/(R3*R3)-DD2*C/(R3**3)
      A1(1)=A
      B1(1)=B
      C1(1)=C
      M=1
      DO 73 I=1,NMAX
      M=M+1
      RJ=RB      +HAIT(I)
      RI=RB      +HAIT(I-1)
C     CALCULATE THE COEFFICIENT A,B,C
      DET=(RJ-RI)**2/((RI**2)*RJ)
      A=(RI*RI*(RI-RJ)*DELT+RI*(RI-DD2*RJ)*DANS(I-1)+RJ*RJ*DANS(I))/(DET
     1*RJ*RI**2)
      B=(RI*(RJ*RJ-RI*RI)*DELT-DD2*RJ*RJ*(DANS(I)-DANS(I-1)))/(DET*RI*RJ
     1)
      C=(RI*(RI-RJ)*DELT+RJ*(DANS(I)-DANS(I-1)))/DET
      DELT=-B/(RJ*RJ)-DD2*C/(RJ**3)
      A1(M)=A
      B1(M)=B
      C1(M)=C
73    CONTINUE
C     CALCULATE THE RANGE AND GROUP PATH
      CALL FITT(CRP,CGPP,BETAP)
86    CALL FITT(CRT,CGPT,BETAT)
      IF(JFLAG.EQ.0) GO TO 88
C     DECREMENT THE PENETRATION ANGLE.
      BETAT=BETAT-0.40*RAD
      GO TO 86
C     CHECK IF THE TRANSMISSION FREQUENCY IS GREATER THAN FC
88    IF(TFREQ-FVMAX) 92,92,89
C     CORRECT THE TRAPPING ANGLE BETAT
89    CRT1=CRT
      CGPT1=CGPT
      BETAT1=BETAT
C     INCREMENT THE ANGLE BETAT BY 0.1 DEGREE AND CHECK THE SLOPE.
      BETAT=BETAT+0.10*RAD
      CALL FITT(CRT,CGPT,BETAT)
      IF(JFLAG.EQ.0) GO TO 95
      BETAT=BETAT1
      CRT=CRT1
      CGPT=CGPT1
      GO TO 92
95    IF(CRT.LT.CRT1) GO TO 89
C     WE HAVE THE MAXIMUM AND MINIMUM ELEVATION ANGLES AND CORRESPONDING
C     RANGES. DIVIDE THE ELEVATION ANGLE RANGE INTO INTERV, THE NUMBER
C     OF INTERVALS.
92    NINT=INTERV+1
      ELRANJ(1,1,JHOP)=BETAP
      ELRANJ(1,2,JHOP)=CRP
      ELRANJ(1,3,JHOP)=CGPP
      ELRANJ(NINT,1,JHOP)=BETAT
      ELRANJ(NINT,2,JHOP)=CRT
      ELRANJ(NINT,3,JHOP)=CGPT
      DINT=(BETAT-BETAP)/INTERV
C
C     GENERATE THE RANGES CORRESPONDING TO THE INTERVAL VALUES OF THE
C     ELEVATION ANGLES.
      DO 110 I=2,INTERV
      BETAI=(I-1)*DINT
```

```
      CALL FITT(CR,CGE,BETAI)
      ELRANJ(I,1,JHCE)=BETAI
      ELRANJ(I,2,JHCE)=CR
      ELRANJ(I,3,JHCE)=CGE
110   CONTINUE
C     LOCATE THE MAXIMA AND MINIMA OF THE ELEVATION-RANGE CURVE
C     INITIALIZE THE VECTOR DBMIN
      DO 1110 I=1,21
1110  DBMIN(I)=0.0
      TEMP=ELRANJ(1,2,JHOP)**
      JJJ=1
      JJ=1
      DBMIN(JJJ)=ELRANJ(1,1,JHCE)
      JTEMP(1)=1
      DO 1111 I=2,KINT
      GO TO(1112,1113),JJ
1112  IF(ELRANJ(I,2,JHOE).LT.TEMP)  GO TO 1118
C     REPLACE AND STORE THE ELEVATION ANGLE VALUES
      JJ=2
      JJJ=JJJ+1
      DBMIN(JJJ)=ELRANJ(I-1,1,JHCE)
      JTEMP(JJJ)=I-1
      GO TO 1118
1113  IF(ELRANJ(I,2,JHOE).GT.TEMP)  GO TO 1118
C     REPLACE THE INDEX VALUE
      JJ=1
      JJJ=JJJ+1
      DBMIN(JJJ)=ELRANJ(I-1,1,JHCE)
      JTEMP(JJJ)=I-1
1118  TEMP=ELRANJ(I,2,JHOP)
1111  CONTINUE
      JJJ=JJJ+1
      DBMIN(JJJ)=BETAT
      JTEMP(JJJ)=KINT
      DO 1147 KQ=1,JJJ
1147  DBMIN(KQ)=DBMIN(KQ)*DEGS
      PRINT 8733, (DBMIN(KQ),KQ=1,JJJ)
C     REMOVE SMALL OSCILLATIONS FROM THE LAYERS OF THE D-B CURVE
C     ASSUME 150 KM OSCILLATIONS OR LESS TO BE REMOVED
      JJR=1
      JJ=1
      DO 1130 I=2,JJJ
C     RETRIEVE THE INDICES FOR COMPARISON
      J1=JTEMP(JJB)
      J2=JTEMP(I)
      DIFFR=ELRANJ(J1,2,JHCE)-ELRANJ(J2,2,JHCE)
      GO TO (1135,1140),JJ
C     D-B CURVE SLOPING DOWNWARDS
1135  IF(DIFFR.GT.0.0)  GO TO 1137
      IF(ELRANJ(J2,2,JHCE).LT.ELRANJ(J1,2,JHCE))  GO TO 1130
      DBMIN(JJR)=DBMIN(I)
      JTEMP(JJR)=JTEMP(I)
      GO TO 1130
1137  IF(ABS(DIFFR).LT.150.0)  GO TO 1130
      JJ=2
      JJR=JJR+1
      DBMIN(JJR)=DBMIN(I)
      JTEMP(JJR)=JTEMP(I)
      GO TO 1130
C     D-B CURVE SLOPING UPWARDS
1140  IF(DIFFR.LT.0.0)  GO TO 1143
```

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      IF(ELRANJ(J2,2,JHCP).GT.ELRANJ(J1,2,JHCP))  GO TO 1130
      DBMIN(JJP)=DBMIN(I)
      JTEMP(JJP)=JTEMP(I)
      GO TO 1130
1143  IF(ABS(DIFFR).LT.150.0)  GO TO 1130
      JJ=1
      JJR=JJR+1
      DBMIN(JJR)=DBMIN(I)
      JTEMP(JJP)=JTEMP(I)
1130  CONTINUE
      DO 1146  KQ=1,JJR
1146  DBMIN(KQ)=DBMIN(KQ)*DEGS
      PRINT 8733, (DBMIN(KQ),KQ=1,JJR)
      JJJM1=JJP-1
      PRINT 114
114   FORMAT(2X,21HTHE RANGE-BETA VALUES)
      PRINT 115
115   FORMAT(1X,9HPOINT NO.,10X,6HEL DEG,10X,8HRANGE KM,10X,9HG.PATH KM)
      L=2
      K=3
      IF(W(394).EQ.0.0)  GO TO 118
      L=3
      K=2
118   DO 116 I=1,KINT
      BETAD=ELRANJ(I,1,JHCP)*DEGS
116   PRINT 117,I,BETAD,ELRANJ(I,L,JHCP),ELRANJ(I,K,JHCP)
117   FORMAT(1X,I6,8X,F12.3,5X,F12.3,7X,F12.3)
C     CHECK THE GIVEN RANGE AGAINST THE CALCULATED RANGES TO FIND THE
C     NUMBER OF SOLUTIONS.
      KFLAG=0
C     SET A FLAG EQUAL TO ZERO   SO THAT IF THE INTERPOLATION DOES NOT
C     CONVERGE WITHIN TEN    TRIES, THERE IS NO SOLUTION FOR THAT ANGLE
C     (MFLAG)
      IF(ELRANJ(1,2,JHOP).LT.RANGE)  KFLAG=1
      DO 120 I=2,KINT
      IF(KFLAG.EQ.1) GO TO 180
C     CHECK THE NEXT RANGE IF LARGER THAN GIVEN RANGE
      IF(ELRANJ(I,2,JHOP).GT.RANGE)  GO TO 120
C     INCREMENT NSOL AND HOME THE SOLUTION TO FIND THE APPROXIMATE
C     ELEVATION ANGLE.
156   DB=(ELRANJ(I,1,JHCP)-ELRANJ(I-1,1,JHCP))/9.0
      BETAI=ELRANJ(I-1,1,JHCP)
      CR=ELRANJ(I-1,2,JHOP)
      MFLAG=0
      DO 127 J=1,10
      IF(KFLAG.EQ.1) GO TO 129
      IF(CR.LT.RANGE) GO TO 128
125   ELRANJ(I-1,1,JHCP)=BETAI
      ELRANJ(I-1,2,JHCP)=CR
      BETAI=BETAI+DB
      CALL FITT(CR,CGE,BETAI)
      GO TO 127
128   ELRANJ(I,1,JHOP)=BETAI
      ELRANJ(I,2,JHCP)=CR
      GO TO 126
129   IF(CR.GT.RANGE) GO TO 128
      GO TO 125
127   CONTINUE
C     WE CANNOT FIND A SOLUTION
175   DO 1275 JK=1,JJJM1
      JJK=JK
```

```
      IF(BETAI.GT.DBMIN(JK).AND.BETAI.LE.DBMIN(JK+1))  GO TO 1280
1275  CONTINUE
1280  GO TO (1281,1283,1285,1287,1289,1291,1293,1295),JJK
1281  PRINT 1282
1282  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR FIRST LAYER LOW
     1ANGLE      )
      GO TO 171
1283  PRINT 1284
1284  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR FIRST LAYER HIGH
     1 ANGLE     )
      GO TO 171
1285  PRINT 1286
1286  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR SECOND LAYER LOW
     1 ANGLE     )
      GO TO 171
1287  PRINT 1288
1288  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR SECOND LAYER HIG
     1H ANGLE    )
      GO TO 171
1289  PRINT 1290
1290  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR THIRD LAYER LOW
     1ANGLE      )
      GO TO 171
1291  PRINT 1292
1292  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR THIRD LAYER HIGH
     1 ANGLE     )
      GO TO 171
1293  PRINT 1294
1294  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR FOURTH LAYER LOW
     1 ANGLE     )
      GO TO 171
1295  PRINT 1296
1296  FORMAT(2X,62HCANNOT FIND APPROXIMATE SOLUTION FOR FOURTH LAYER HIG
     1H ANGLE    )
      GO TO 171
C     USE LINEAR INTERPOLATION RANGE=P+Q*BETA
126   IF(ELRANJ(I-1,1,JHOP).EQ.ELRANJ(I,1,JHOP))  GO TO 175
      Q=(ELRANJ(I-1,2,JHOP)-ELRANJ(I,2,JHOP))/(ELRANJ(I-1,1,JHOP)-
     1ELRANJ(I,1,JHOP))
      IF(Q.EQ.0.0) GO TO 175
      P=ELRANJ(I-1,2,JHOP)-Q*ELRANJ(I-1,1,JHOP)
      BETAI=(RANGE-P)/Q
      CALL FITT(CR,COF,BETAI)
      IF(JFLAG) 130,130,175
130   DIFF=RANGE-CR
      IF(DIFF) 135,160,145
135   IF(DIFF+DIFG) 140,160,160
140   IF(KFLAG.EQ.1) GO TO 146
136   ELRANJ(I-1,1,JHOP)=BETAI
      ELRANJ(I-1,2,JHOP)=CR
      MFLAG=MFLAG+1
      IF(MFLAG.GE.10) GO TO 175
      GO TO 126
145   IF(DIFF-DIFG) 160,160,150
150   IF(KFLAG.EQ.1) GO TO 136
146   ELRANJ(I,1,JHOP)=BETAI
      ELRANJ(I,2,JHOP)=CR
      MFLAG=MFLAG+1
      IF(MFLAG.GE.10) GO TO 175
      GO TO 126
C     WE HAVE FOUND THE APPROXIMATE ANGLE
```

```
160    NSCI=NSCI+1
       JSOL(JHCP)=NSCI
       APRCX(NSCI,1,JHCP)=BETAI
       APRCX(NSCI,2,JHCP)=I
       DBETA=BETAI*DEGS
       DO 1160  JK=1,JJJM1
       JJK=JK
       IF(BETAI.GT.DBMIN(JK).AND.BETAI.LT.DBMIN(JK+1))  GO TO 1170
1160   CONTINUE
1170   GO TO(1171,1173,1175,1177,1179,1181,1183,1185),JJK
1171   PRINT 1172,NSCI,DBETA,CR,CGP
1172   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER= ,I3,2X,22HFIRST LAYER LO
      1W ANGLE , 10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
       GO TO 171
1173   PRINT 1174,NSCI,DBETA,CR,CGP
1174   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER= ,I3,2X,23HFIRST LAYER HI
      1GH ANGLE ,10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
       GO TO 171
1175   PRINT 1176,NSCI,DBETA,CR,CGP
1176   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER= ,I3,2X,23HSECOND LAYER L
      1OW ANGLE ,10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
       GO TO 171
1177   PRINT 1178,NSCI,DBETA,CR,CGP
1178   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER=,I3,2X,24HSECOND LAYER HI
      1GH ANGLE ,10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
       GO TO 171
1179   PRINT 1180,NSCI,DBETA,CR,CGP
1180   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER= ,I3,2X,22HTHIRD LAYER LO
      1W ANGLE  ,10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
       GO TO 171
1181   PRINT 1182,NSCI,DBETA,CR,CGP
1182   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER= ,I3,2X,23HTHIRD LAYER HI
      1GH ANGLE ,10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
       GO TO 171
1183   PRINT 1184,NSCI,DBETA,CR,CGP
1184   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER= ,I3,2X,23HFOURTH LAYER L
      1OW ANGLE ,10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
       GO TO 171
1185   PRINT 1186,NSCI,DBETA,CR,CGP
1186   FORMAT(2X,28HAPPROXIMATE SOLUTION NUMBER=,I3,2X,24HFOURTH LAYER HI
      1GH ANGLE ,10H,EL.ANGLE=,F8.3,2X,6HRANGE=,F8.3,2X,7HG.PATH=,F8.3)
171    IF(KFLAG.EQ.0) III=1
       IF(KFLAG.EQ.1) III=0
       KFLAG=III
       GO TO 120
C      CHECK IF THE NEXT RANGE IS SMALLER THAN GIVEN RANGE
180    IF(ELRANJ(I,2,JEOP).LT.RANGE)  GO TO 120
       GO TO 150
120    CONTINUE
280    IF(NSCI.GT.0) GO TO 300
       PRINT 2805
2805   FORMAT(2X,34HTHERE ARE NO APPROXIMATE SOLUTIONS )
       GO TO 300
1000   PRINT 1005
1005   FORMAT(1X,47HTHERE ARE LESS THAN THREE POINTS IN THE DENSITY)
       GO TO 300
400    PRINT 410
410    FORMAT(1X,25HTHE FREQUENCY IS TOO HIGH)
300    CONTINUE
8000   CONTINUE
C      CHECK TO SEE IF IT IS ONE BOP MODE
```

```
      IF(NHOE.EQ.1)  GC TC 12000
C     CHECK IF THE HOES AFE SYMMETFIC WITHIN A CERTAIN TCLERANCE
      DO 8050  I=1,NHCE
      IF(JSOI(I).EQ.C)  GC TC 8050
C     THERE ARE SOLUTICNS
      GO TO 8060
8050  CONTINUE
C     SET THE CONTROI NSOI TO ZFRC DEACTING NC SCLUTICNS
      NSOI=C
      GO TC 12000
C     CHECK FOR SYMMETFIC HOFS
8060  KHCE=NHCE-1
      DO 8070 I=1,KECE
      IF(JSOI(I).EQ.JSOI(I+1))  GC TC 8070
C     THE HOES ARE NC1 SYMMETRICAL
      GO TO 8100
8070  CONTINUE
C     CHECK IF THE ELEVATICN ANGLES AFE CICSE TC EACH OTHER
      KPCINT=JSCL(1)
C     ONE DEGFEE SEFARATICN MAXIMUM
      XXJ=1.C*FAD
      DO 8080 I=1,KECINT
      HTEMP=AFFCX(I,1,1)
      DO 8085 J=2,NHCE
      IF(ABS(FTEMP-AFFCX(I,1,J)).LE.XXJ)  GO TC 8085
C     THE HOES ARE NOT SYMMETRIC SINCE THE ELEVATICN ANGLES ARE NCT
C     CLOSE TC EACH CTFER
      GC TC 8100
8085  CONTINUE
8080  CONTINUE
C     THE HOES ARE SYMMETRIC
      NSOI=JSCI(1)
      GC TO 12000
8100  DO 8120 J=1,NHCE
      JJ=1
      TEMPR(JJ,J)=ELRANJ(1,2,J)
      XJ=JJ*RAD
      DO 8110 I=2,KINT
8111  IF(ELRANJ(I,1,J).LT.XJ)  GC TO 8110
C     INTERECIATE LINEARLY CN THE RANGE
      SLCPE=(ELRANJ(I-1,2,J)-ELRANJ(I,2,J))/(ELRANJ(I-1,1,J)-
     1ELRANJ(I,1,J))
      CROSS=ELRANJ(I-1,2,J)-SLCPE*ELRANJ(I-1,1,J)
      TEMPR(JJ+1,J)=SLOPE*XJ+CROSS
      JJ=JJ+1
      XJ=JJ*RAD
      GC TO 8111
8110  CCNTINUF
      NPOINT(J)=JJ
8120  CONTINUE
C     FIND THE NUMBER CF SCLUTICNS EY CONFARING THE SUM CF THE RANGES/
C     HOP WITH THE GIVEN FANGE
      ITEMP=NFCINT(1)
      DO 8130 I=2,NHCE
      IF(NPCINT(I).GT.ITEMP)  GC TC 8130
      ITEMP=NFCINT(I)
8130  CONTINUE
C     RECOPY THE TEMFCRARY ARRAYS CNTC THE PERMANENT CNES
      DO 8140 I=1,ITEMP
      DO 8140 J=1,NHCE
      ELRANJ(I,1,J)=(I-1)*RAD
```

```
8140  ELRANJ(I,2,J)=TEMER(I,J)
      NSCL=0
      SUM1=0.0
      DO 8145 J=1,NECE
8145  SUM1=SUM1+TEMER(1,J)
      DO 8150 I=2,ITEMP
      SUM=0.0
      DO 8160 J=1,NHCE
8160  SUM=SUM+TEMER(I,J)
      IF(SUM.LT.TRANGE.AND.SUM1.LT.TRANGE)  GO TO 8149
      IF(SUM.GT.TRANGE.AND.SUM1.GT.TRANGE)  GO TO 8149
C     WE FOUND A SOLUTION
      NSCL=NSCL+1
C     INTERPOLATE TO FIND THE APPROXIMATE ELEVATION ANGLE
      SLOPE=(SUM-SUM1)/(ELRANJ(I,1,J)-ELRANJ(I-1,1,J))
      CROSS=SUM1-SLOPE*ELRANJ(I-1,1,J)
      APROX(NSCL,1,1)=(TRANGE-CROSS)/SLOPE
      APROX(NSCL,2,1)=I
8149  SUM1=SUM
8150  CONTINUE
      DO 8771 KQ=1,91
      PR1=ELRANJ(KQ,1,1)*DEGS
      PR2=ELRANJ(KQ,1,2)*DEGS
      PRINT 8734,PR1,ELRANJ(KQ,2,1),PR2,ELRANJ(KQ,2,2)
8734  FORMAT(1X,2F12.3,12X,2F12.3)
8771  CONTINUE
8733  FORMAT(1X,12F8.3)
12000 RETURN
      END
```

```
      SUBROUTINE FITT(RANG,GROUP,ANG)
      REAL*8 EARTHR,RE,AA,BB,CC,FF,A1,B1,C1,HAIT,ARGUM,X1,X2,X3,R1,R2,X,
     1R3,XI,RI,XJ,RJ,BETA,          CR,CB1,CB2,CB3,CGP,CGP1,CGP2,CGP3
      COMMON/CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)
      COMMON/DENSC/ DENS(52),HEIT(52),ELRADJ(91,3,2),APROX(8,2,2),FRATIO
     1,NMAX,NSTART,JFIAG,NSCL,DEMIN(21),JJJM1
      COMMON/COEFF/ A1(52),B1(52),C1(52),HAIT(52),RAPCGP,RAPCGM
      COMMON Y(6) /WW/ ID(10),WC,W(400)
      EQUIVALENCE (EARTH ,W(19))
C     CR IS THE CALCULATED RANGE
C     NMAX IS THE NUMBER OF POINTS UP TO THE HEIGHT OF MAXIMUM DENSITY
C     HEIT IS THE HEIGHT INCREMENTS CORRESPONDING TO THE DENSITY
C     DENS IS THE NORMALIZED DENSITY
C     CGP IS THE CALCULATED GROUP PATH
C     FRATIO IS THE TRANSMISSION FREQUENCY,F,SQUARED.
C     BETA IS THE ELEVATION ANGLE
C     NSTART IS THE INDEX AT WHICH THE DENSITY HAS A VALUE
      SQRT(X)=DSQRT(X)
      ALOG(X)= DLOG(X)
      ARSIN(X)=DARSIN(X)
      ARCOS(X)=DARCOS(X)
      COS(X)=DCOS(X)
      SIN(X)=DSIN(X)
      ABS(X)=DABS(X)
      DD1=1.0D0
      DD2=2.0D0
      DD4=4.0D0
      BETA=ANG
      FF=FRATIO
      EARTHR=EARTH
      JFIAG=0
      RE=EARTHR**2
      M=NSTART
C     CALCULATE THE RANGE AND GROUP PATH FROM EARTH SURFACE TO THE
C     BOTTOM OF THE ICNOSPHERE
      CR=ABS(EARTHR*COS(BETA)/(HAIT(M)+EARTHR))
      CR=DD2*EARTHR*(ARCOS(CR)-BETA)
      CGP=DD2*(SQRT((EARTHR+HAIT(M))**2-(EARTHR*COS(BETA))**2)-EARTHR*
     1SIN(BETA))
C     SET UP THE CONSTANTS AA,BB,CC FOR THE FIRST THREE POINTS
      JJ=1
      AA=DD1-A1(JJ)/FF
      BB=-B1(JJ)/FF
      CC=-C1(JJ)/FF-(EARTHR*COS(BETA))**2
      ARGUM=BB*BB-DD4*AA*CC
      I=M+3
      R1=EARTHR+HAIT(M)
      R2=EARTHR+HAIT(M+1)
      R3=EARTHR+HAIT(M+2)
      X1=AA*R1*R1+BB*R1+CC
      X2=AA*R2*R2+BB*R2+CC
      X3=AA*R3*R3+BB*R3+CC
      IF(X1) 400,400,52
52    IF(X2) 55,55,56
C     THE RAY REFLECTED
55    XI=X1
      RI=R1
      GO TO 305
56    IF(X3) 57,57,75
57    XI=X1
```

```
      RI=R1
      GO TO 305
C     CALCULATE THE RANGE
75    IF(CC) 70,60,60
70    CR1=DD2*RE*COS(BETA)/SQRT(-CC)
      CR2=(BE*R3+DD2*CC)/(ABS(R3)*SQRT(ARGUM))
      CR3=(BB*R1+DD2*CC)/(ABS(R1)*SQRT(ARGUM))
      CR=CR+CR1*(ARSIN(CR2)-ARSIN(CR3))
      GO TO 80
60    CR1=-DD2*RE*COS(BETA)/SQRT(CC)
      CR2=ABS(DD2*SQRT(CC*X3)/R3+DD2*CC/R3+BE)
      CR3=ABS(DD2*SQRT(CC*X1)/R1+DD2*CC/R1+BB)
      CR=CR+CR1*ALOG(CR2/CR3)
80    CGP1=DD2*(SQRT(X3)-SQRT(X1))/AA
C     CALCULATE THE GROUP PATH
      IF(AA) 90,100,100
90    CGP2=(DD2*AA*R3+BE)/(SQRT(ARGUM))
      CGP3=(DD2*AA*R1+BE)/(SQRT(ARGUM))
      CGP=CGP+CGP1+BB*(ARSIN(CGP2)-ARSIN(CGP3))/(AA*SQRT(-AA))
      GO TO 110
100   CGP2=ABS(DD2*SQRT(AA*X3)+DD2*AA*R3+BE)
      CGP3=ABS(DD2*SQRT(AA*X1)+DD2*AA*R1+BE)
      CGP=CGP+CGP1-BE*ALOG(CGP2/CGP3)/(AA**1.5)
C     START THE CALCULATION FOR THE REST OF THE DENSITY PROFILE
110   DO 20 I=1,NMAX
      JJ=JJ+1
C     SET UP THE CONSTANTS AA,BB,CC
120   AA=DD1-A1(JJ)/EE
      BB=-B1(JJ)/EE
      CC=-C1(JJ)/EE-(EARTHR*COS(BETA))**2
      ARGUM=BE*BB-DD4*AA*CC
      RJ=EARTHR+HAIT(I)
      RI=EARTHR+HAIT(I-1)
      XJ=AA*RJ*RJ+BB*RJ+CC
      XI=AA*RI*RI+BB*RI+CC
      IF(XI) 300,300,172
172   IF(XJ) 300,300,171
171   IF(CC) 170,160,160
C     CALCULATE THE RANGE OF TC REFLECTION HEIGHT
170   CR1=DD2*RE*COS(BETA)/SQRT(-CC)
      CR2=(BE*RJ+DD2*CC)/(ABS(RJ)*SQRT(ARGUM))
      CR3=(BB*RI+DD2*CC)/(ABS(RI)*SQRT(ARGUM))
      CR=CR+CR1*(ARSIN(CR2)-ARSIN(CR3))
      GO TO 180
160   CR1=-DD2*RE*COS(BETA)/SQRT(CC)
      CR2=ABS(DD2*SQRT(CC*XJ)/RJ+DD2*CC/RJ+BB)
      CR3=ABS(DD2*SQRT(CC*XI)/RI+DD2*CC/RI+BB)
      CR=CR+CR1*ALOG(CR2/CR3)
180   CGP1=DD2*(SQRT(XJ)-SQRT(XI))/AA
C     CALCULATE THE GROUP PATH OF TC REFLECTION HEIGHT
      IF(AA) 190,200,200
190   CGP2=(DD2*AA*RJ+BE)/(SQRT(ARGUM))
      CGP3=(DD2*AA*RI+BB)/(SQRT(ARGUM))
      CGP=CGP+CGP1+BE*(ARSIN(CGP2)-ARSIN(CGP3))/(AA*SQRT(-AA))
      GO TO 20
200   CGP2=ABS(DD2*SQRT(AA*XJ)+DD2*AA*RJ+BE)
      CGP3=ABS(DD2*SQRT(AA*XI)+DD2*AA*RI+BE)
      CGP=CGP+CGP1-BB*ALOG(CGP2/CGP3)/(AA**1.5)
20    CONTINUE
130   JFLAG=3
      GO TO 400
```

```
300   CONTINUE
      RADCGP=(-BB+SQRT(ARGUM))/(DD2*AA)
      RADCGM=(-BB-SQRT(ARGUM))/(DD2*AA)
305   IF(CC) 310,320,320
310   CR1=DD2*EE*COS(BETA)/SQRT(-CC)
      CR2=PID2
      CR3=(BE*BI+DD2*CC)/(ABS(BI)*SQRT(ARGUM))
      CR=CR+CR1*(CR2-ARSIN(CR3))
      GO TO 350
320   CR1=-DD2*BE*COS(BETA)/SQRT(CC)
      CR2=ABS(SQRT(ARGUM))
      CR3=ABS(DD2*SQRT(CC*XI)/BI+DD2*CC/BI+EE)
      CR=CR+CR1*ALOG(CR2/CR3)
350   CGP1=-DD2*SQRT(XI)/AA
      IF(AA) 360,370,370
360   CGP2=-PID2
      CGP3=(DD2*AA*BI+BB)/(SQRT(ARGUM))
      CGP=CGP+CGP1+EE*(CGP2-ARSIN(CGP3))/(AA*SQRT(-AA))
      GO TO 400
370   CGP2=ABS(SQRT(ARGUM))
      CGP3=ABS(DD2*SQRT(AA*XI)+DD2*AA*BI+EE)
      CGP=CGP+CGP1-EE*ALOG(CGP2/CGP3)/(AA**1.5)
400   RANG=CR
      GROUP=CGP
      IF(W(394).EQ.0.0)  GO TO 500
      RANG=CGP
      GROUP=CR
500   RETURN
      END
```

```
      SUBROUTINE GRCDEM
C     A SUBROUTINE TO COMPUTE THE MINIMUM GROUP PATH
C     FIRST WE FIND THE PENETRATION ANGLE AND THEN USE CHECKING
C     AND CHOPPING TECHNIQUE TO LOCATE THE MINIMUM WITHIN A GIVEN
C     TOLERANCE IN GROUP PATH
      COMMON/CONST/ PI,PIT2,PID2,DEGS,PAD,IUM(3)
      COMMON/FLAGS/IFLAG,IGRT
      COMMON Y(12),T/WW/II(10),NC,W(400)
      COMMON/DENSC/ DENS(52),HEIT(52),FLRANJ(91,3,2),APROX(8,2,2),FRATIO
     1,NMAX,NSTART,JFLAG,NSCI,DPMIN(21),JJJM1
      COMMON/DENY/ FH(19,52,5),TEETA(19),EGT(52),PEI(5),FREQ(4),HI
      COMMON/EG2/ LATMX,LONMX,LHTMX
      COMMON/FR/N,SIZE,MODE,DCMY(5),ESTART
      COMMON/FIN/XN2,XMUX,PN2(8),ECLAP(4),SPACE,CCIL,FIELD
      COMMON/NICPT/ NPRINT,NCPT,MPRT
      EQUIVALENCE (EARTHR,W(19)),(F,W(3)),(AZ1,W(18)),(BETA,W(17)),
     1 (ONLY,W(371)),(AZA,W(263))
      EQUIVALENCE  (PHO,W(378)),(THO,W(377))
      NSCI=0
C     SELECT THE LONGITUDE COORDINATE OF DENSITY
      DPHI=ABS(PHI(2)-PHI(1))
      KK=LONMX
      DO 110 K=1,LONMX
      DIFF1=ABS(PHO-PHI(K))
      IF(DIFF1-DPHI)  120,115,110
115   KK=K
      GO TO 130
120   KK=K
      IF(K.GE.LONMX)  GO TO 130
      DIFF2=ABS(PHO-PHI(K+1))
      IF(DIFF2.LT.DIFF1)  KK=K+1
      GO TO 130
110   CONTINUE
C     PRINT THE ERROR MESSAGE
      PRINT 125
125   FORMAT(2X,'CANNOT FIND A LONGITUDE CLOSE TO THE TRANSMITTER')
      GO TO 400
C     SELECT THE COLATITUDE COORDINATE OF DENSITY
130   ITHETA=ABS(THETA(2)-THETA(1))
      LL=LATMX
      DO 135 I=1,LATMX
      DIFF1=ABS(THO-THETA(I))
      IF(DIFF1-ITHETA)  145,140,135
140   II=I
      GO TO 160
145   LL=I
      IF(I.GE.LATMX)  GO TO 160
      DIFF2=ABS(THO-TEETA(I+1))
      IF(DIFF1.LT.DIFF2)  II=I+1
      GO TO 160
135   CONTINUE
C      PRINT THE ERROR MESSAGE
      PRINT 150
150   FORMAT(2X,'CANNOT FIND A COLATITUDE CLOSE TO THE TRANSMITTER')
      GO TO 400
C     EXTRACT THE DENSITY
160   DO 165 I=1,LHTMX
165   DENS(I)=FH(LL,I,KK)
      TEMP=DENS(1)
      DO 170 I=2,LHTMX
```

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      IF(IENS(I).LE.TEMP)  GO TO 170
      NMAX=L
      TEMP=DENS(L)
170   CONTINUE
      HMAX=EGT(NMAX)
      FC=DENS(NMAX)
      WRITE(6,24) F
24    FORMAT(1X,'THE TRANSMISSION FREQUENCY=',F12.5,'   MHZ')
      TOLERR=1.0
C     TEMPORARY ALLOCATION OF AZIMUTH IN W(263)
      AZ1=AZA
C     COMPUTE THE ELEVATION ANGLE FOR A TRAPPED RAY
      TEMPA=(EARTHR+HMAX)*SQRT(1.0-(FC/F)**2)/EARTHR
      IF(TEMPA-1.0) 10,10,900
10    BETAR=ARCOS(TEMPA)
20    BETA1=BETAR
C    INITIALIZE THE RAY PARAMETERS AND TRACE THE RAY
      CALL RAYINT(AZ1,BETA1)
      BETA=BETA1
      EL=BETA*DEGS
      AZM=AZ1*DEGS
      WRITE(6,25) EL,AZM
25    FORMAT(1X,'ELEVATION ANGLE OF TRANSMISSION=',F12.6,'  DEG',5X,
     1'AZIMUTH ANGLE OF TRANSMISSION=',F12.6,'  DEG')
      CALL TRACE
      MPRT=0
      IF(IGRT.NE.0)  GO TO 950
C     CHECK FOR PENETRATION
      IF(ONLY.EQ.0.0) GO TO 30
      BETAR=0.99*BETAR
      GO TO 20
C     STORE THE VALUES
30    GROUP1=T
C     TAKE 95%OF INITIAL TRAPPING ANGLE
      BETA2=0.95*BETAR
      CALL RAYINT(AZ1,BETA2)
      BETA=BETA2
      EL=BETA*DEGS
      WRITE(6,25) EL,AZM
      CALL TRACE
      MPRT=0
      IF(IGRT.NE.0)  GO TO 950
      GROUP2=T
C     TAKE 90% OF INITIAL TRAPPING ANGLE
      BETA3=0.90*BETAR
      BETA1=BETA3
      CALL RAYINT(AZ1,BETA3)
      BETA=BETA3
      EL=BETA*DEGS
      WRITE(6,25) EL,AZM
      CALL TRACE
      MPRT=0
      IF(IGRT.NE.0)  GO TO 950
      GROUP3=T
C     CHECK FOR THE SHAPE OF THE CURVE
40    IF(GROUP3.LT.GROUP2) GO TO 80
C     SLOPING TOWARDS THE RIGHT TO GROUP 2
50    IF(GROUP2.LT.GROUP1) GO TO 200
C     SLOPING TOWARDS THE RIGHT;FIND A NEW PENETRATION ANGLE
C     INCREMENT BETAR BY 5%
      BETAR=1.05*BETAR
```

```
60      CALL RAYINT(AZ1,BETAR)
        BETA=BETAR
        EL=BETA*DEGS
        WRITE(6,25) EL,AZM
        CALL TRACE
        MPRT=0
        IF(IGRT.NE.0)  GO TO 950
C       CHECK FOR PENETRATION
        IF(CNLY.GT.0.0) GO TO 70
C       INTERCHANGE THE VALUES
        BETA3=BETA2
        BETA2=BETA1
        BETA1=BETAR
        GROUP3=GROUP2
        GROUP2=GROUP1
        GROUP1=1
        GO TO 50
C       RAY PENETRATED DECREMENT THE ANGLE BY 1%
70      BETAR=0.99*BETAR
        GO TO 60
C       SLOPING TOWARDS THE LEFT
80      BETAL=0.99*BETAI
        CALL RAYINT(AZ1,BETAI)
        BETA=BETAL
        EL=BETA*DEGS
        WRITE(6,25) EL,AZM
        CALL TRACE
        MPRT=0
        IF(IGRT.NE.0)  GO TO 950
        BETA1=BETA2
        BETA2=BETA3
        BETA3=BETAL
        GROUP1=GROUP2
        GROUP2=GROUP3
        GROUP3=1
        GO TO 40
C       WE FOUND A SMALL VALUE GROUP2 INCLUDED BETWEEN TWO LARGER VALUES
C       GROUP1 AND GROUP3
C       USE THE CHOP METHOD ON EITHER SIDE OF GROUP2
200     BETA5=BETA3+0.5*(BETA2-BETA3)
        BETA4=BETA2+0.5*(BETA1-BETA2)
        CALL RAYINT(AZ1,BETA4)
        BETA=BETA4
        EL=BETA*DEGS
        WRITE(6,25) EL,AZM
        CALL TRACE
        MPRT=0
        IF(IGRT.NE.0)  GO TO 950
        GROUP4=1
        CALL RAYINT(AZ1,BETA5)
        BETA=BETA5
        EL=BETA*DEGS
        WRITE(6,25) EL,AZM
        CALL TRACE
        MPRT=0
        IF(IGRT.NE.0)  GO TO 950
        GROUP5=1
C       CHECK IF IT IS WITHIN THE TOLERANCE
        IF(ABS(GROUP5-GROUP2).GT.TOLGRP) GO TO 210
        IF(GROUP4.LT.GROUP5)  GO TO 350
        GO TO 300
```

```
C       CHECK THE LEFT SIDE
210     IF(GROUP5.GT.GROUP2) GO TO 220
C       REPLACE THE LEFT SIDE VALUES
        BETA1=BETA2
        BETA2=BETA5
        GROUP1=GROUP2
        GROUP2=GROUP5
        GO TO 200
C       CHECK IF IT IS WITHIN THE TOLERANCE
220     IF(ABS(GROUP4-GROUP2).LE.TOLGRP) GO TO 350
C       CHECK THE RIGHT SIDE
        IF(GROUP4.GT.GROUP2) GO TO 240
C       REPLACE THE RIGHT SIDE VALUES
        BETA3=BETA2
        BETA2=BETA4
        GROUP3=GROUP2
        GROUP2=GROUP4
        GO TO 200
C       REPLACE THE MIDDLE PORTION
240     BETA3=BETA5
        BETA1=BETA4
        GROUP3=GROUP5
        GROUP1=GROUP4
        GO TO 200
C       WE FOUND THE LEFT SIDE MINIMUM VALUE OF GROUP PATH
300     IF(GROUP5.GT.GROUP2)  GO TO 305
        GROUP2=GROUP5
        BETA2=BETA5
305     WRITE(6,307)
307     FORMAT(//1X,'***'//)
        WRITE(6,310)
310     FORMAT(1X,'THE LEFT SIDE     ,MINIMUM GROUP PATH AND ITS ANGLE ARE
      1.')
        BETAD=BETA2*DEGS
        WRITE(6,320) BETAD,GROUP2
320     FORMAT(1X,'ELEVATION ANGLE=',F9.4,2X,'MINIMUM GROUP PATH=',F9.4)
        WRITE(6,307)
        GO TO 400
C       WE FOUND THE RIGHT SIDE MINIMUM VALUE GROUP PATH
350     IF(GROUP4.GT.GROUP2)  GO TO 355
        GROUP2=GROUP4
        BETA2=BETA4
355     WRITE(6,307)
        WRITE(6,360)
360     FORMAT(1X,'THE RIGHT SIDE     ,MINIMUM GROUP PATH AND ITS ANGLE ARE
      1.')
        BETAD=BETA2*DEGS
        WRITE(6,320) BETAD,GROUP2
        WRITE(6,307)
400     RETURN
900     WRITE(6,910)
910     FORMAT(1X,'THE ARGUMENT OF ARCOS IS GREATER THAN ONE')
        GO TO 400
950     IGRT=0
        GO TO 400
        END
```

```
      SUBROUTINE PROFIL
C     A ROUTINE TO GENERATE A NEW DENSITY PROFILE
      DIMENSION AZT(5),RBMPCD(20,5,4),RR(2,3),R(3),NP(5)
      COMMON /DENX/ PP(19, 52, 5),THETA(19),HGT( 52),PHI( 5),PRES(4),HL
      COMMON /PG1/ DTH(2),DPH(2),DHT(2),DITH,DIPH,DIHT,THZC,PHIZC,HTZC
      COMMON /PG2/  IATMX,ICNMX,IHTMX
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)
      COMMON Y(12),T,STP,DPDT(12) /WW/ ID(10),W0,W(400)
      EQUIVALENCE (EARTHR,W(19)),(PLOD,W(13)),(PLAT,W(15)),(TLON,W(14)),
     1(TLAT,W(16)),(AIPH,W(376)),(TCLAT,W(377)),(TLONG,W(378))
C     AZIMUTH TOLERANCE
      AZTOL=10.*RAD
C     CONVERT THE ANGLES TO RADIANS
      DO 40 I=1,LATMX
40    THETA(I)=THETA(I)*RAD
      DO 50 I=1,LCNMX
50    PHI(I)=PHI(I)*RAD
C     READ THE NUMBER OF AZIMUTH AND THE NUMBER OF POINTS IN EACH
C     AZIMUTH
      READ(5,100) N,(NP(I),I=1,N)
100   FORMAT(6I10)
C     READ THE AZIMUTHAL ANGLES
      READ(5,110)  (AZT(I),I=1,N)
110   FORMAT(8F10.0)
C     READ RB,RM,RC,AND D FOR EACH AZIMUTH
      DO 130 J=1,N
      NP1=NP(J)
      READ(5,120)  ((RBMPCD(I,J,K),K=1,4),I=1,NP1)
120   FORMAT(4F10.0)
130   CONTINUE
C     LOCATE THE CLOSEST RANGE TO THE TRANSMITTER
      TEST=RBMPCD(1,1,4)
      K1=1
      K2=1
      DO 160 I=1,N
      NP1=NP(I)
      DO 165 I1=1,NP1
      IF(RBMPCD(I1,I,4).GE.TEST)  GO TO 165
      K1=I1
      K2=I
      TEST=RBMPCD(I1,I,4)
165   CONTINUE
160   CONTINUE
C     GENERATE THE NEW DENSITY PROFILE IN ALL THE SPACE
      RB=RBMPCD(K1,K2,1)
      RM=RBMPCD(K1,K2,2)
      RC=RBMPCD(K1,K2,3)
      YM=RM-RB
      DO 170 K=1,LCNMX
      DO 170 J=1,LATMX
      DO 170 I=1,LHTMX
      PH(J,I,K)=0.0
      RT=EARTHR+HGT(I)
      FACT=((RT-RM)*RB/(YM*RT))**2
      IF(FACT.GT.1.0)  GO TO 170
      PH(J,I,K)=SQRT(RC**2*(1.0-FACT))
170   CONTINUE
C     FIND THE MAXIMUM RANGE AND SET THE LIMIT
      TEST=RBMPCD(1,1,4)
      DO 145 I=1,N
```

```
      NP1=NP(I)
      DO 140 I1=1,NE1
      IF(REMFCD(I1,1,4).LE.TEST) GO TO 140
      TEST=REMFCD(I1,I,4)
140   CONTINUE
145   CONTINUE
      RANGEL=TEST+100.0
C     PRINT INPUT DATA
      PRINT 1230,N,(NE(I),I=1,N)
1230  FORMAT(1X,'N=',I3,5('  NE=',I4))
      PRINT 1231,(AZT(I),I=1,N)
1231  FORMAT(1X,'AZIMUTHS=',5(F8.2,3X))
      DO 1233 J=1,N
      NP1=NP(J)
      PRINT 1232,AZT(J)
1232  FORMAT(1X,'AZIMUTH=',F8.2)
      PRINT 1234,((REMFCD(I,J,K),K=1,4),I=1,NE1)
1234  FORMAT(1X,'RR=',F8.2,'  RM=',F8.2,'  PC=',F8.2,'  D=',F8.2)
1233  CONTINUE
      PRINT 1235, RANGEL,ALPH
1235  FORMAT(1X,'MAXIMUM RANGE=',F8.2,'   ALPH=',E14.7)
C     GO THROUGH THE GRID POINTS AND TRY TO INTERPOLATE THE GIVEN DATA
      DO 400 K=1,LONMX
      TNP=ABS(TLONG-PHI(K))
      DO 380 J=1,LATMX
C     FIND THE AZIMUTH OF ONE POINT P(J,K)
      ANGTCP=COS(TCLAT)*COS(THETA(J))+SIN(TCLAT)*SIN(THETA(J))*COS(TNP)
      ALPHA=ARCOS(ANGTCP)
      SINPTN=SIN(TNP)*SIN(THETA(J))/SIN(ALPHA)
      COSPTN=(COS(THETA(J))-COS(TCLAT)*COS(ALPHA))/(SIN(TCLAT)*
     1SIN(ALPHA))
      AZP=ARSIN(SINPTN)
      IF(SINPTN.LT.0.0) GO TO 210
      IF(COSPTN.LT.0.0) AZP=PI-AZP
      GO TO 220
210   IF(COSPTN.LT.0.0) AZP=-PI-AZP
220   IF(TLONG.GT.PHI(K))  AZP=PIT2-AZP
      RANGEP=EARTHR*ALPHA
      P1=PHI(K)*DEGS
      P2=THETA(J)*DEGS
      P3=AZP*DEGS
      PRINT 145C,RANGEP,P3,P2,P1
145C  FORMAT(1X,'RANGE=',F8.2,'   AZIMUTH=',F8.2,'   COLAT=',F8.2,
     1'   LONG=',F8.2)
C     CHECK IF THE AZIMUTH AZP OF P IS WITHIN THE LIMITS OF GIVEN
C     AZIMUTHS PLUS TOLERANCE
      AZ1=AZT(1)*RAD-ALPH-AZTOL
      AZN=AZT(N)*RAD-ALPH+AZTOL
      IF(AZP.GE.AZ1.AND.AZP.LE.AZN)  GO TO 240
C     THE AZIMUTH OF P IS OUT OF RANGE
      GO TO 380
C     CHECK IF THE RANGE IS OUTSIDE THE MAXIMUM GIVEN RANGE + TOLERANCE
C     OF 100NM
240   IF(RANGEP.GT.RANGEL)  GO TO 380
C     LOCATE TWO AZIMUTHAL ANGLES FOR P
      KK=N
      DO 260 I=1,N
      AZT1=AZT(I)*RAD-ALPH
      IF(AZP.GT.AZT1)  GO TO 260
      KK=I
      GO TO 265
```

```
260   CONTINUE
265   KKK=KK-1
      IF(KK.EQ.1)  KKK=KK+1
C     COMPUTE THE GEOMAGNETIC AZIMUTHS
      AZT1= AZT(KK)*RAD-ALPH
      AZT2= AZT(KKK)*RAD-ALPH
C     SEARCH FOR THE CLOSEST TWO   VALUES OF GROUND RANGES TO GROUND
C     RANGE OF P
      II=1
      TEST=RANGEP
      I=KK
270   NP1=NP(I)
      K1=NP1
      DO 290 I1=1,NP1
      IF(REMFCD(I1,I,4).LT.TEST)  GO TO 290
      K1=I1
      GO TO 295
290   CONTINUE
C     INTERPOLATE LINEARLY FOR THE VALUES OF RE, RM, RC
295   K2=K1-1
      IF(K1.EQ.1)  K2=K1+1
      DO 300 M=1,3
      SLOPE=(REMFCD(K1,I,M)-REMFCD(K2,I,M))/(REMFCD(K1,I,4)-
     1REMFCD(K2,I,4))
      CROSS=REMFCD(K1,I,M)-SLOPE*REMFCD(K1,I,4)
      PR(II,M)=CROSS+SLOPE*RANGEP
300   CONTINUE
      II=II+1
      I=KKK
      IF(II.LE.2) GO TO 270
C     FIND THE DIFFERENCE IN AZIMUTHAL ANGLES AND COMPUTE THE VALUES OF
C     RB, RM, RC AT THE POINT P
      ANG1P=AZT1-AZE
      ANG2P=AZE-AZT2
      ANG12=AZT1-AZT2
      DO 310 M=1,3
310   R(M)=(ANG2P*RR(1,M)+ANG1E*RR(2,M))/ANG12
C     R(1) CONTAINS RE,R(2) CONTAINS RM,R(3) CONTAINS RC
C     COMPUTE VALUES OF RC AT DIFFERENT HEIGHTS
      YM=R(2)-R(1)
      DO 340 I=1,LHTMX
      RH(J,I,K)=0.0
      RT=EARTHR+HGT(I)
      FACT=((RT-R(2))*R(1)/(YM*RT))**2
      IF(FACT.GT.1.0)  GO TO 340
      RH(J,L,K)=SQRT(R(3)**2*(1.0-FACT))
340   CONTINUE
380   CONTINUE
400   CONTINUE
C     PREPARE ANGLES IN DEGREES FOR PRINTING
      DO 420 I=1,LONPX
420   PHI(I)=PHI(I)*DEGS
      DO 440 I=1,LATMX
440   THETA(I)=THETA(I)*DEGS
      RETURN
      END
```

```
      SUBROUTINE  ELECTY                                                    ELE     1
C                                                                           ELE     2
C        COMPUTES THE ELECTRON DENSITY AND ITS DERIVATIVES AT ANY POINT     ELE     3
C        (R,PHI,TH) ALONG THE RAY IN A THREE DIMENSIONAL IONOSPHERE .       ELE     4
C                                                                           ELE     5
C                                                                           ELE     6
C     DESCRIPTION OF PARAMETERS                                             ELE     7
C                                                                           ELE     8
C     GRID PARAMETER CARD:       FORMAT (3I5,A5,6F10.3)                     ELE     9
C        IONMX  - MAXIMUM = OF IONGITUDE PLANES                             ELE    10
C        IATMX  - MAXIMUM = OF  IATITUDE SECTORS                            ELE    11
C        IHTMX  - MAXIMUM = OF  HEIGHT LAYERS                               ELE    12
C        NAME   - I.D. (ONLY)  OF THE DATA GRID.                            ELE    13
C        THZO   - LEAST COLATITUDE  SECTOR IN DEG.                          ELE    14
C        PHIZO  - LEAST   LONGITUDE PLANE  IN DEG.  (WESTMOST)              ELE    15
C        HTZO   - LOWEST  HEIGHT LAYER                                      ELE    16
C        DLTH   - INTERVAL IN THETA DIRECTION                               ELE    17
C        DLPH   - INTERVAL IN PHI DIRECTION                                 ELE    18
C        DLHT   - INTERVAL IN HEIGHT                                        ELE    19
C                                                                           ELE    20
C        PH     - GRID DATA OF PLASMA FREQ.  ARRANGED IN (TH,HT) PLANES.    ELE    21
C                                                                           ELE    22
C     SUBROUTINE REQUIRED :    INTER(LI,KK,JJ)                            ELE    23
C        FRES   - OUTPUT RETURNED BY SUBROUTINE INTER                       ELE    24
C                                                                           ELE    25
      COMMON /FLAGS/ IFLAG,IGRT
C                                                                           ELE    31
C     NOTE:                                                                 ELE    26
C                                                                           ELE    27
C        WHEN CHANGING TO DIFFERENT SIZE GRID CHECK PROPER DIMENSIONS.      ELE    28
C                                                                           ELE    29
C        ARCON CORP., WAKEFIELD, MASS., 01880     KRISHNA  VANGURI.         ELE    30
C                                                                           ELE    33
      COMMON /XX/      X,PYER,PXETH,PXEPH,PXPT,HMAX                         ELE    35
      COMMON /DENX/ PH(19, 52, 5),THETA(19),HGT( 52),PHI( 5),FRES(4),HL     ELE***36
      COMMON /EG1/ DTE(2),DPH(2),DHT(2),DLTH,DLPH.DLHT,THZO,PHIZO,HTZO      ELE    37
      COMMON /EG2/  IATMX,IONMX,IHTMX                                       ELE    38
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)                           ELE    39
      COMMON R(12),T,SIP,DRDT(12) /WW/ ID(10),NO,W(400)                     ELE    40
      EQUIVALENCE(TH,R(2)),(PH,R(3))                                        ELE    41
      EQUIVALENCE (F,W(3)) , (EARTHR,W(19))                                 ELE    42
      DATA  IREAD / 0 /                                                     ELE    43
C                                                                           ELE    44
C     ----------------------------------------------------------------      ELE    45
C                                                                           ELE    46
      IF(IREAD-1) 100,300,100                                               ELE    47
  100 IFLAG = 0                                                             ELE    48
      IGRT=0
      IREAD = 1                                                             ELE    49
C                                                                           ELE    50
C     I/O TAPE DESIGNATIONS:   IX=INPUT UNIT,    IY=OUTPUT TAPE UNIT.       ELE    51
C                                                                           ELE    52
      IX = 5                                                                ELE    53
      IY = 6                                                                ELE    54
C                                                                           ELE    55
C     READ GRID PARAMETERS.     SIZE AND INITIAL VALUES.                    ELE    56
C                                                                           ELE    57
      READ(IX,120) IONMX,IATMX,IHTMX,NAME,PHIZO,THZO,HTZO,DLPH,DLTH,DLHTELE    58
  120 FORMAT (3I5,A4,6F10.3)                                                ELE    59
C                                                                           ELE    60
```

```
C      EXIT IF LIMITS EXCEED.                                                ELE   61
C                                                                            ELE   62
       IF (LONMX - 5)  125,125,135                                           ELE***63
  125  IF (LATMX -19)  130,130,135                                           ELE***64
  130  IF (LHTMX - 52) 150,150,135                                           ELE***65
  135  WRITE (IY,140) LONMX,LATMX,LHTMX                                      ELE   66
  140  FORMAT (1X,35H PARAMETERS OUTSIDE THE LIMITS:    ,/                   ELE   67
     1 10X,41H MAXIMUM NUMBER OF LONGITUDE PLANES  =   ,I5 , /              ELE   68
     2 10X,41H MAXIMUM NUMBER OF  LATITUDE SECTORS =   ,I5 , /              ELE   69
     3 10X,41H MAXIMUM NUMBER OF  HEIGHT   LAYERS  =   ,I5 //)              ELE   70
       STOP                                                                  KV - II
  150  CONTINUE                                                              ELE   72
C                                                                            ELE   73
C      SETUP UNIFORM SPHERICAL GRID.                                         ELE   74
C                                                                            ELE   75
       DO 151 J = 1,LHTMX                                                    ELE   76
  151  HGT(J) =  HTZO + FLOAT(J-1) * DLHT                                    ELE   77
       DO 152 L = 1,LATMX                                                    ELE   78
  152  THETA(L)  = THZO + FLOAT(L-1) * DLTH                                  ELE   79
C                                                                            ELE   80
C      READ FREQ. DATA PLANES OF SIZE=(LATMX X LHTMX) FOR EACH LONGITUDE     ELE   81
C                                                                            ELE   82
       DO 169 KPH= 1,LONMX                                                   ELE   83
       PHI(KPH)= PHIZO + FLOAT(KPH-1) * DLPH                                 ELE   84
C                                                                            ELE   85
       READ(IX,160)  ((FH(I,J,KPH),I=1,LATMX),J=1,LHTMX)                     ELE   86
  160  FORMAT(15F5.2,5X)                                                     ELE   87
  169  CONTINUE                                                              ELE   88
       IF(W(393).GT.0.0)  CALL PROFIL
C
C                                                                            ELE   89
C      WRITE OUT DATA PLANES:  (ICOSPHERE GRID MODEL)                        ELE   90
C                                                                            ELE   91
       WRITE (IY,202)                                                        ELE   92
  202  FORMAT( 1H1,47H PREPARED BY ARCON CORP. WAKEFIELD, MASS. 01880,       ELE   93
     4  3X, 21HTEL: (617) 245-3404  ,'   AUG.   1973. '/)                   NC5   9
       WRITE(IY,203)LONMX,LATMX,LHTMX,NAME,PHIZO,THZO,HTZO,DLPH,DLTH,DLHTELE   95
  203  FORMAT (/1X,48H UNIFORM GRID PARAMETERS :                    ,/ ELE   96
     11X,8HLONMX = ,I6,4X,8HLATMX = ,I6,4X,8HLHTMX = ,I6,4X,5HNAME=,A5,/ELE   97
     21X,8HPHIZO = ,F9.2,1X,8HTHZO  = ,F9.2,1X,8HHTZO  = ,F9.2,/        ELE   98
     31X,8HDLPHO = ,F9.2,1X,8HDLTH  = ,F9.2,1X,8HDLHT  = ,F9.2,/  )     ELE   99
       WRITE (IY,202)                                                      ELE  100
       WRITE (IY,210)    (THETA(I),I=1,LATMX)                              ELE  101
  210  FORMAT (    2X, 12HLONG.  HGT. , 20F6.1 ,//)                       ELE  102
       WRITE (IY,235)                                                      ELE  103
C                                                                          ELE  104
C                                                                          ELE  105
       DO 240 K = 1,LONMX                                                  ELE  106
       DO 230 J = 1,LHTMX                                                  ELE  107
       WRITE (IY,220)  PHI(K), HGT(J)  ,(FH(L,J,K),I=1,LATMX)              ELE  108
  220  FORMAT (1X, 2F6.1, 1X, 20F6.2)                                      ELE  109
  230  CONTINUE                                                            ELE  110
       WRITE (IY,235)                                                      ELE  111
  235  FORMAT (1X)                                                         ELE  112
  240  CONTINUE                                                            ELE  113
C                                                                          ELE  114
C      CONVERT ALL ANGLES INTO RADIANS                                     ELE  115
C                                                                          ELE  116
       DO 245 K = 1,LONMX                                                  ELE  117
  245  PHI(K) = PHI(K) * RAD                                               ELE  118
C                                                                          ELE  119
```

```
      DO 250 L = 1,LATMX                                                    BLE  120
  250 THETA(L) = THETA(L) * RAD                                             BLE  121
C                                                                           BLE  122
      DLPH  = DLPH  * RAD                                                   BLE  123
      DLTH  = DLTH  * RAD                                                   BLE  124
      PHIZO = PHIZO * RAD                                                   BLE  125
      THZO  = THZO  * RAD                                                   BLE  126
C                                                                           BLE  127
C     CALCULATE  PHI,THETA,HEIGHT DIFFERENCES                               BLE  128
C                                                                           BLE  129
      DTH(1) =  0.5 / (DLTH*DLTH)                                           BLE  130
      DPH(1) =  0.5 / (DLPH*DLPH)                                           BLE  131
      DHT(1) =  0.5 / (DLHT*DLHT)                                           BLE  132
      DTH(2) = -2.0 * DTH(1)                                                BLE  133
      DPH(2) = -2.0 * DPH(1)                                                BLE  134
      DHT(2) = -2.0 * DHT(1)                                                BLE  135
C                                                                           BLE  136
C     ADDED FOR THE HOMING FEATURE ONLY                                     UOPT 1
      IF(W(380).NE.0.0) RETURN                                              UOPT 2
  300 CONTINUE                                                              BLE  137
C                                                                           BLE  138
      HL = R(1) - EARTHR                                                    BLE  139
      FREQS = I * F                                                         BLE  140
      X = 0.0                                                               BLE  141
      PXPR  = 0.0                                                           BLE  142
      PXPTH = 0.0                                                           BLE  143
      PXPPH = 0.0                                                           BLE  144
C                                                                           BLE  145
C     FIND THETA SECTOR AND INDEX LL                                        BLE  146
C                                                                           BLE  147
      LL =  IFIX (((TH - THZO) / DLTH) + 1.00001)                           BLE  148
      IF((TH.LT.(THZO-1.E-06)).OR.(TH.GT.(THETA(LATMX)+1.E-06)))GOTO320     BLE  149
      IF (LL - 1)    320,310,310                                            BLE  150
  310 IF (LL - LATMX)  330,325,320                                          BLE  151
  320 CONTINUE                                                              BLE  152
C                                                                           BLE  153
      LL = LL+100                                                           BLE  154
      GO TO 4000                                                            BLE  155
  325 LL = LATMX - 1                                                        BLE  156
  330 CONTINUE                                                              BLE  157
C                                                                           BLE  158
C     FIND PHI PLANE AND INDEX KK                                           BLE  159
C                                                                           BLE  160
      KK =  IFIX (((PH - PHIZO)/ DLPH) + 1.00001)                           BLE  161
      IF((PH.LT.(PHIZO-1.E-06)).OR.(PH.GT.(PHI(LONMX)+1.E-06)))GOTO350      BLE  162
      IF (KK - 1)    350,340,340                                            BLE  163
  340 IF (KK - LONMX)  360,355,350                                          BLE  164
  350 CONTINUE                                                              BLE  165
C                                                                           BLE  166
      KK = KK+100                                                           BLE  167
      GO TO 4000                                                            BLE  168
  355 KK = LONMX - 1                                                        BLE  169
  360 CONTINUE                                                              BLE  170
C                                                                           BLE  171
C     FIND HEIGHT LAYER  AND INDEX JJ                                       BLE  172
C                                                                           BLE  173
      JJ =  IFIX (((HI - HTZO) / DLHT) + 1.00001)                           BLE  174
      IF (JJ - 1)    380,370,370                                            BLE  175
  370 IF (JJ - LHTMX)  510,380,380                                          BLE  176
  380 CONTINUE                                                              BLE  177
C                                                                           BLE  178
```

```
      HMAX = HGT(IHTMX)                                                 ELE  179
      GO TO 999                                                         ELE  180
C                                                                       ELE  181
C     PRINT ERROR MESSAGES BEFORE EXIT.                                 ELE  182
C                                                                       ELE  183
 4000 ANG1 = TH * DEGS                                                  ELE  184
      ANG2 = PH * DEGS                                                  ELE  185
      X    = 1.0                                                        ELE  186
      EXPH = 1.0                                                        ELE  187
      EXPTH = 1.0                                                       ELE  188
      HMAX=HGT(IHTMX)                                                   ELE**189
      R(1)=RADTHR+HMAX                                                  ELE**190
      IFLAG = 1                                                         ELE  191
      IGRT=1
      WRITE (IY,4444) ANG1,ANG2,II,KK,IFLAG                             ELE  192
 4444 FORMAT (//1X,36HANGLES OUTSIDE RANGE OF ICNOSPHERE  /16X,         ELE  193
     16HTHETA=,F7.1,3X,4HPHI=, F7.1,  / 1X,                             ELE  194
     2 15HINDICES ARE    ,6H II = ,I7,3X,4H KK=,I7,6HIFLAG=, I5)        ELE  195
C                                                                       ELE  196
      GO TO 999                                                         ELE  197
C                                                                       ELE  198
  510 CONTINUE                                                          ELE  199
C                                                                       ELE  200
C     INTERPOLATE FOR F(TH,HL,PH) USING THE INDICES  II,KK,JJ           ELE  201
C                                                                       ELE  202
      CALL INTER(II,KK,JJ)                                              ELE  203
C                                                                       ELE  204
C     COMPUTE X FROM   X=(PLASMA FREQ/F) **2, FL. DEN.= 12400 (X)       ELE  205
C                                                                       ELE  206
      X    = PRES(1) / FREQS                                            ELE  207
      EXPH = PRES(2) / FREQS                                            ELE  208
      EXPTH = PRES(3) / FREQS                                           ELE  209
      EXPPH = PRES(4) / FREQS                                           ELE  210
C                                                                       ELE  211
      IFLAG = 0                                                         ELE  212
C                                                                       ELE  213
  999 RETURN                                                            ELE  214
      END                                                               ELE  215
```

APPENDIX 2

This appendix contains expressions for the partial derivatives of the ground range (R) and the group path (P') with respect to the elevation angle of transmission (β) and the quasi-parabolic layer parameters (f_c, r_m, r_b), required in the inversion of point-to-point oblique ionograms and backscatter leading edges:

$$\frac{\partial R}{\partial \beta} = 2r_o \left(\frac{r_o \sin \beta}{r_b \sin \gamma} - 1 \right) + \frac{F r_o^2 \cos \beta}{\sqrt{C}} \left[\frac{4\sqrt{C}\, F\, r_o \sin \beta \cot \gamma}{W} + F^2 r_o^2 \sin 2\beta \left(\frac{4}{W} + G \right) + \tan \beta \; \ell n \left(\frac{U r_b^2}{W^2} \right) \right] \tag{A.1}$$

$$\frac{\partial R}{\partial f_c} = \frac{F^2 r_o^2 \cos \beta}{\sqrt{C}\, f_c} \left[2 F r_o^2 \cos^2 \beta \left(\frac{4}{W} + G \right) - \frac{8CF}{U} + \frac{1}{F} \ell n \left(\frac{U r_b^2}{W^2} \right) - \frac{4\sqrt{C}\, r_b \sin \gamma}{W} \right] \tag{A.2}$$

$$\frac{\partial R}{\partial r_m} = \frac{F r_o^2 \cos \beta}{\sqrt{C}\, r_m\, y_m} \left[B r_b r_m G - 2 B y_m \left(\frac{B}{U} - \frac{r_b}{W} \right) + \frac{4B}{U} \left(B r_m + C \right) \right] \tag{A.3}$$

$$\frac{\partial R}{\partial r_b} = \frac{2r_o^2 \cos\beta}{r_b^2 \sin\gamma}$$

$$+ \frac{Fr_o^2 \cos\beta}{\sqrt{C}\, r_b\, y_m} \left[\frac{4\sqrt{C}\, F\, r_b y_m \operatorname{cosec}\gamma}{W} - \frac{4B}{U}(Br_m + C) \right.$$

$$\left. - \frac{2By_m}{W}(r_m + y_m) - Br_m^2 G - 2y_m \right] \qquad \text{(A.4)}$$

In Eqs. (A.1) - (A.4),

$$W = 2\sqrt{C}\, Fr_b \sin\gamma + 2C + Br_b$$

$$G = \frac{4A}{U} + \frac{1}{2C} \ell n \left(\frac{Ur_b^2}{W^2} \right) + \frac{2Fr_b \sin\gamma}{\sqrt{C}\, W}$$

$$\frac{\partial P'}{\partial f_c} = \left[4 - \frac{4F^2}{A} - \frac{2B(F^2 + A)}{AV} \right] \frac{F^2 r_b \sin\gamma}{A f_c}$$

$$+ \frac{BF}{f_c A^{3/2}} \left[\frac{1}{2}\left(1 - \frac{3F^2}{A}\right) \ln \frac{U}{V^2} \right.$$

$$\left. - \frac{4F^2 (C - A r_o^2 \cos^2\beta)}{U} - \frac{4F^2 r_b}{V} \right] \qquad \text{(A.5)}$$

$$\frac{\partial P'}{\partial r_m} = \frac{BF}{r_m y_m A^{3/2}} \left[\left(2 + \frac{B}{V} \right) \frac{F r_b \sin \gamma}{\sqrt{A}} \right.$$

$$+ B \left(\frac{3}{4A} \ln \frac{U}{V^2} + \frac{2(A r_m r_b + C)}{U} + \frac{2 r_b}{V} \right)$$

$$\left. + (r_m + r_b) \left(\frac{1}{2} \ln \frac{U}{V^2} + \frac{B^2}{U} - \frac{B}{V} \right) \right] \qquad (A.6)$$

$$\frac{\partial P'}{\partial r_b} = 2 \left(1 - \frac{F^2}{A} + \frac{BF^2}{AV} \right) \operatorname{cosec} \gamma$$

$$- \frac{BF}{r_b y_m A^{3/2}} \left[\left(2 + \frac{B}{V} \right) \frac{F r_b \sin \gamma}{\sqrt{A}} + \left(r_m + \frac{3B}{4A} \right) \ln \frac{U}{V^2} \right.$$

$$\left. + \frac{2B (A r_m^2 + B r_m + C)}{U} - \frac{2 y_m (A r_b + B)}{V} \right] \qquad (A.7)$$

APPENDIX 3

The computer program for the inversion of backscatter ionograms by utilizing the quasi-parabolic model is listed in the subsequent pages. In addition, this appendix contains a complete set of flow charts (with the exception of subroutine FMFP, which was written from the standard IBM scientific subroutine package), and a description of the input/output formats.

```
PAGE   001

4       NAME MAIN
4       EQUIF=CARDRE,PRINTE,CARDPU;
        DIMENSION F(20),RBC(20),RMC(20),FCC(20),RANGEC(20),GPATHC(20),
       1FC(5,2 ),RB(5,20),RM(5,20),RANGE(5,20),GPATH(5,20),N(5),AZ(10)
        DIMENSION FMID(20)
  202 CONTINUE
        READ 1 ,TLAT,TLONG,FCS,HO,YM,NAZ,IND
   10 FORMAT(5F10.2,I1,9X,I1)
   40 FORMAT(10X,6F10.2)
        RO=637 .
        RBS=RO+HC
        RMS=RBS+YM
        I=1
   25 FORMAT(I2)
   50 READ 2 ,AZ(I),NUMTOT,NPTS
   20 FORMAT(F10.2,I2,8XI2)
        CALL BACKS3(NUMTOT,NPTS,RBS,RMS,FCS,RBC,RMC,FCC,FMID,RANGEC,
       1GPATHC,NC,1)
   30 FORMAT(5F10.2)
        PRINT 35,(FCC(J),RBC(J),RMC(J),RANGEC(J),GPATHC(J),J=1,NC)
   35 FORMAT(5X,5F10.4)
        N(I)=NC
        DO 60 J=1,NC
        FC(I,J)=FCC(J)
        RB(I,J)=RBC(J)
        RM(I,J)=RMC(J)
        RANGE(I,J)=RANGEC(J)
   60 GPATH(I,J)=GPATHC(J)
        I=I+1
        IF (I-NAZ) 70,70,80
   70 FCS=FCC(1)
        RBS=RBC(1)
        RMS=RMC(1)
        GO TO 50
80      IF(IND)180,85,180
85      READ 1 ,FW,AZM,GPM
        IF(FW)201,202,203
  203 CONTINUE
        CALP=(6370.*6370.+6470.*6470-GPM*GPM/4.)/(2.*6370.*6470.)
        DEST=ACOSF(CALP)*6370.*2.
        PRINT 4 ,FW,AZM,GPM,DEST
        IF(NAZ-1)204,204,205
  205 CONTINUE
        DO 100 K=1,NAZ
        IF (AZM-AZ(K)) 110,110,100
  100 CONTINUE
  110 FACTOR=(AZM-AZ(K-1))/(AZ(K)-AZ(K-1))
        N1=N(K)
        N2=N(K-1)
  112 DO 120 J=1,N1
        IF (DEST-RANGE(K,J)) 130,130,120
  120 CONTINUE
  130 FACT=(DEST-RANGE(K,J-1))/(RANGE(K,J)-RANGE(K,J-1))
        FC1=FC(K,J-1)+FACT*(FC(K,J)-FC(K,J-1))
        RB1=RB(K,J-1)+FACT*(RB(K,J)-RB(K,J-1))
        RM1=RM(K,J-1)+FACT*(RM(K,J)-RM(K,J-1))
        PRINT 4 ,FC1,RB1,RM1
        DO 140 J=1,N2
```

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```
      IF (DEST-RANGE(K-1,J)) 150,150,140
  140 CONTINUE
  150 FACT=(DEST-RANGE(K-1,J-1))/(RANGE(K-1,J)-RANGE(K-1,J-1))
      FC2=FC(K-1,J-1)+FACT*(FC(K-1,J)-FC(K-1,J-1))
      RB2=RB(K-1,J-1)+FACT*(RB(K-1,J)-RB(K-1,J-1))
      RM2=RM(K-1,J-1)+FACT*(RM(K-1,J)-RM(K-1,J-1))
      PRINT 4 ,FC2,RB2,RM2
      FC1=FC2+FACTOR*(FC1-FC2)
      RB1=RB2+FACTOR*(RB1-RB2)
      RM1=RM2+FACTOR*(RM1-RM2)
  208 CONTINUE
      PRINT 4 ,FC1,RB1,RM1
      DTOL=1.
      GPTOL=.1
      CALL DCAL(FC1,RB1,RM1,FW,GPM,D,GRTOL)
      PRINT 4 ,FW,AZM,GPM,DEST,D
      IF (ABSF(D-DEST)-DTOL) 170,170,160
  160 DEST=D
      IF(NAZ-1)204,204,112
  170 PRINT 4 ,FW,AZM,GPM,D
      GO TO 8
  180 CONTINUE
      READ 2 , KGO
  200 FORMAT(I10)
      IF(KGO)201,202,201
  201 CALL EXIT
  204 N1=N(NAZ)
      DO 206 J=1,N1
      IF(DEST-RANGE(1,J))207,207,206
  206 CONTINUE
  207 FACT=(DEST-RANGE(1,J-1))/(RANGE(1,J)-RANGE(1,J-1))
      FC1=FC(1,J-1)+FACT*(FC(1,J)-FC(1,J-1))
      RB1=RB(1,J-1)+FACT*(RB(1,J)-RB(1,J-1))
      RM1=RM(1,J-1)+FACT*(RM(1,J)-RM(1,J-1))
      GO TO 208
      END
```

```
PAGE   001

H       NAME A2
H       EQUIP=CARDRE,PRINTE,CARDPU
        FUNCTION FOGX(ARG)
C******SUBR FOGX CHECKS TO MAKE SURE WE ARE NOT TAKING THE NATURAL LOG OF A
C******NEG NUMBER...IF WE ARE IT FLAGS THE ERROR BY SETTING FOGX = 9999.
C******THE INVALID ARGUMENT IS PRINTED OUT WITH AN ERROR MSG.
        IF(ARG)10,20,20
10      CONTINUE
        PRINT 15,ARG
15      FORMAT(29H LOGF ENTERED WITH NEG ARG = , E13.7)
        FOGX=9999.
        GO TO 30
20      FOGX=LOGF(ARG)
30      RETURN
        END
```

```
PAGE   001

H      NAME AS
H      EQUIP=CARDRE,PRINTE,CARDPU)
       SUBROUTINE FSTDFS(FW,FC,HB,YM,DGRT,GP,DGPDFC,DGPDRM,DGPDRB,RANGE,
      $DDFG)
C******THIS SUBR COMPUTES THE MIN TIME DELAY TRACE(MIN GROUP PATH VS FREQ) OF A
C******EARTH-CONCENTRIC Q-P IONOSPHERE LAYER AS WELL AS THE DERIVATIVES OF THE
C******THE GROUP PATH WRT RB,RM,AND FC.  IT IS A HOPEFULLY MORE RAPID VERSION OF
C******SUBR DIFFS, WHICH DID NOT USE NEWTONS METHOD TO FIND THE ELEVATION ANGLE B
C******BETA WHICH MINIMIZES THE GROUP PATH FUNCTION
C
C******CALLING SEQUENCE
C******    FW=(INPUT) FREQ OF TRANSMISSION (MHZ)
C******    FC=(INPUT) CRITICAL FREQUENCY OF Q-P LAYER (IN MHZ)
C******    HB=(INPUT) BASE HEIGHT OF Q-P LAYER (IN KM)
C******    YM= (INPUT) SEMITHICKNESS OF Q-P LAYER (KM)
C******DGRT=(INPUT) TOLERANCE OF GROUP PATH WRT ELEVATION ANGLE , BETA USED IN
C******FINDING MINIMUM GROUP PATH IN KM/RAD
C******    GP=(OUTPUT) MIN GROUP PATH (KM)
C******    DGPDFC=(OUTPUT) DERIV OF MIN GROUP PATH WRT FC
C******    DGPDRM=(OUTPUT) DERIV MIN GROUP PATH WRT RM
C******    DGPDRB=(OUTPUT) DERIV MIN GROUP PATH WRT RB
C******    RANGE=(OUTPUT) GROUND RANGE OF BACKSCATTER PT OF MIN GROUP RAY
C
       R0=6370.
       R02=R0*R0
       RB=R0+HB
       RB2=RB*RB
       RM=RB+YM
       RM2=RM*RM
       YM2=YM*YM
       F=FW/FC
       F2=F*F
       RBYM2=RB2/YM2
       A=F2-1.+RBYM2
       SQA=SQRDD(A)
       IF(SQA-9999.)50,950,50
C*****************************
950    NERPT=1
C*****************************
       GO TO 900
50     CONTINUE
       B=-2.*RM*RBYM2
C******NOW SET SOME CONSTANTS UP FOR FURTHER USE IN THE ITERATION LOOP
C******THESE CONSTS HAVE NO SIGNIFICANCE IN THE 1974 PAPER BY NNR
       CON1=2.*A*RB+B
       CON2=2.*RB*F*SQA
       B2=B*B
       F2A=F2/A
       CON3=2.*R0*(1.-F2A )
       CON4=(B+F2A )*2.*R0
       CON5=4.*B*F2*F*R02/SQA
       CON6=R0/RB
       CON7=8.*A*F2*R02
C******BETPEN IS THE PENETRATION ELEVATION ANGLE (BETA MUST BE BETWEEN 0
C******AND BETPEN)
       SQRDDX=SQRDD(RM2*RBYM2-B2/(4.*A))
       IF(SQRDDX-9999.)75,960,75
C*****************************
```

```
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960    NERPT=2
C*****************************
       GO TO 900
75     CONTINUE
       BETPEN=      ((1./(F*R0))*SQROOX)
       BETPEN = ACOSCK(BETPEN)
       IF(BETPEN-9999.)90,970,90
C*****************************
970    NERPT=3
C*****************************
       GO TO 900
90     CONTINUE
C*****INITIALIZE BETA, THE ELEVATION ANGLE OF THE GROUP RAY PUT THE INITIAL GUES
C*****RIGHT IN THE MIDDLE OF THE RANGE OF POSSIBLE BETA VALUES,
       BETA=BETPEN*.5
C*****ITERATE HERE FOR NEW VALUES OF BETA (AT STATEMENT 100) WHICH MINIMIZE
C*****THE GROUP PATH AT A GIVEN OPERATING FREQ FW (USING NEWTONS METHOD)
       NTRY=100
       ITERCT=0
100    SINBET=SIN(BETA)
       ITERCT=ITERCT+1
C*****IF WE HAVE TRIED TO FIND BETA SUCH THAT GP IS MINIMIZED TO WITHIN
C*****DGPT OF ZERO 100 TIMES LETS GIVE UP
       IF(ITERCT-NTRY)150,150,140
140    PRINT 145,DGPT,NTRY
145    FORMAT(///,48H FSTDIF ROUTINE CANNOT MINIMIZE DGPDB TO WITHIN  ,
      $F5.3,10H KM AFTER ,I3,6H TRIES ,//)
       PRINT 147 ,BETA,DGPDB
147    FORMAT(8H BETA = ,F5.2,9H DGPDB = ,F10.5)
       RETURN
150    CONTINUE
       COSBET=COS(BETA)
       COSBE2=COSBET*COSBET
       C=RBYM2*RM2-F2*R02*COSBE2
       COSGAM=R0*COSBET/RB
       GAMMA=ACOSCK(COSGAM)
       IF(GAMMA-9999.)165,971,165
C*****************************
971    NERPT=4
C*****************************
       PRINT 972,BETA,BETPEN,ITERCT
972    FORMAT(8H BETA = ,E12.5,5X,10H BETPEN = ,E12.5,5X,10H ITERCT = ,I4)
      $)
       GO TO 900
165    CONTINUE
       SINGAM=SINF(GAMMA)
       COTGAM=COSGAM/SINGAM
       V=CON1+CON2*SINGAM
       V2=V*V
       U=B2-4.*A*C
       U2=U*U
       F1=(CON3+CON4/V)
       G2=2.*R0*CON5*SINBET/U
       DGPDB=F1*SINBET*COTGAM-G2*COSBET
C*****BRANCH OUT OF NEWTON ITERATION LOOP WHERE WHEN BETA IS FOUND WHICH
C*****MINIMIZES DGPDB TO WIHIN TOLERANGE DGPT (IN KM/RAD )
       IF(ABSF(DGPDB)-DGPT)800,350,350
350    CONTINUE
```

```
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      SQR00Y=SQR00(1.-(CON6*COSBET)**2.)
      IF(SQR00Y-9999.)375,973,375
C*************************
973   NERPT=5
C*************************
      PRINT 972,BETA,BETPEN,ITERCT
      GO TO 900
375   CONTINUE
      DGAMDB=SINBET*CON6/SQR00Y
      DCOTDB=-DGAMDB/(SINGAM*SINGAM)
      DVDB=CON2*COSGAM*DGAMDB
      DF1DB=-(CON4/V2)*DVDB
      DUDB=-CON7*COSBET*SINBET
      DF2DB=-CON5*(COSBET/U-(SINBET/U2)*DUDB)
      DGPDB2=DF1DB*SINBET*COTGAM+F1*(COSBET*COTGAM+SINBET*DCOTDB)
     $+DF2DB*COSBET+G2*SINBET
      BETA=BETA-DGPDB/DGPDB2
C*****IF BETA HAS BEEN INCREMENTED TO A NEG VALUE BY NEWTONS METHOD WE BETTER SE
C*****BETTER SET BETA BACK TO A LITTLE ABOVE 0 OR WE MIGHT HAVE TROUBLES LATER O
      IF(BETA)400,400,500
400   BETA=.0001
500   IF(BETPEN-BETA)600,600,700
600   BETA=BETPEN-.0001
C*****CHECK TO ENSURE NEWTONS METHOD IS NOT CAUSING US TO OVERSTEP THE
C*****PENETRATION ELEVATION ANGLE BOUNDARY.  IF SO SET BETA BACK TO LITTLE BELOW
C*****BETPEN
700   GO TO 100
800   CONTINUE
C*****AT STATEMENT 800 THE ELEVATION ANGLE BETA CORRESPONDING TO THE MINIMUM GRO
C*****GROUP PATH HAS BEEN COMPUTED BY SOLVING FOR THE VALUE OF BETA WHICH FORCES
C*****DGPDB TO 0.  NOW WE CAN COMPUTE THE MINIMUM GROUP PATH GPM
      UV2LOG=FOGX(U/V2)
      IF(UV2LOG-9999.)850,974,850
C*************************
974   NERPT=6
C*************************
      GO TO 900
850   CONTINUE
      BFA32=B*F/(A*SQA)
      GPM=CON3/CON6*SINGAM-2.*R0*SINBET
     $*(BFA32*.5)*UV2LOG
      GP=GPM
      DPDFC1=2.*CON3/R0-2.*B*(F2+A)/(A*V)
      DPDFC2=F2*RB*SINGAM/(A*FC)
      DPDFC3=BFA32/FC
      DPDFC4=.5*(1.-3.*F2A )*UV2LOG-4.*F2*(C+A*R02*COSBE2)/U
     $-4.*F2*RB/V
      DGPDFC=DPDFC1*DPDFC2+DPDFC3*DPDFG4
      DPDRM2=.75/A*UV2LOG+2.*(A*RM*RB+G)/U+2.*RB/V
      DPDRM1=BFA32/(RM*YM)
      DPDRM3=(2.+B/V)*F*RB*SINGAM/SQA+B*DPDRM2+(RM+RB)*(.5*UV2LOG+B2/U-B/V)
     $/V)
      DGPDRM=DPDRM1*DPDRM3
      DPDRB1=(CON3+CON4/V)/(R0*SINGAM)
      DPDRB2=-BFA32/(RB*YM)*((2.+B/V)*F*RB*SINGAM/SQA+(RM+.75*B/A)*UV2LO
     $G +2.*B*(A*RM2+B*RM+C)/U-2.*YM*(A*RB+B)/V)
      DGPDRB=DPDRB1+DPDRB2
      SQC=SQR00(C)
```

```
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      IF(SQC-9999.)875,975,875
C*************************
975   NERPT=7
C*************************
      GO TO 900
875   CONTINUE
      R1=2.*SQC*RB*F*SINGAM+2.*C+B*RB
      R12=R1*R1
      W=FOGX(U*RB2/R12)
      R3=.5*F*R0*COSBET*W/SQC
      RANGE=2.*R0*(GAMMA-BETA-R3)
      BDEG=BETA*57.2985
      RETURN
900   FC=-ABSF(FC)
      PRINT 980,NERPT
980   FORMAT(1X,100(1H*),/,30H FATDFS ERROR AT POINT NUMBER ,I2,/,1X,
     $100(1H*))
      PRINT 985,FW,FC,HB,YM,DGPT
985   FORMAT(1X,5E15.7)
      RETURN
      END
```

```
PAGE   001

H       NAME A4
H       EQUIP=CARDRE,PRINTE,CARDPU;
        SUBROUTINE FUNCT(N,ARG,VAL,GRAD)
        DIMENSION ARG(3),GRAD(3),FBSMES(50),PBSMES(50),GPC(50),DGPC(50,3)
        DIMENSION RAN(50),BTA(50)
        COMMON FBSMES,PBSMES,DT,NPTS,RAN,BTA,GPCMID
C
C******SUBROUTINE FUNCT TO BE USED BY BACKS3 BACKSCATTER IONOGRAM INVERSION
C******ROUTINE. IT EVALUATES THE SUM SQUARED ERROR AND ITS GRADIENT AS A
C******FUNCTION OF O-P LAYER PARAMETERS RB, RM, FC.
C
C******CALLING SEQUENCE.....
C******N=(INPUT) CONTROL PARAMETER
C******    IF N=3 FUNCTION NORMALLY, RETURN VALUE OF ERROR FUNCTION AND ITS
C******    GRADIENT
C******ARG=(INPUT) 3 ELEMENT ARRAY OF LAYER PARAMETERS
C******    ARG(1)=RB
C******    ARG(2)=RM
C******    ARG(3)=FC
C******VAL=(OUTPUT)VALUE OF ERROR FUNCTION E CORRESPONDING TO CURRENT VALUE OF
C******    ARG
C******GRAD=(OUTPUT) 3 ELEMENT ARRAY WITH
C******    GRAD(1)=DE/DRB
C******    GRAD(2)=DE/DRM
C******    GRAD(3)=DE/DFC
C******END OF INPUT SEQUENCE*********
C
        FC=ARG(3)
        HB=ARG(1)-6370.
        YM=ARG(2)-ARG(1)
C******HERE N=3 AND WE WILL EVALUATE E AND GRAD OF E
C******FIRST INITIALIZE VALUES FOR SUBSEQUENT SUMMING OPERATION
40      SUMF=0
        SUMDRB=0
        SUMDRM=0
        SUMDFC=0
        NMID=NPTS/2+1
        DO 50 I=1,NPTS
C******FSTDFS CALLED TO EVALUATE MIN GROUP PATH AND ITS DERIVS WRT RB,RM,FC AS A
C******FUNCTION OF FBSMES(I), THE ITH TRANSMISSION FREQ
        FBSM=FBSMES(I)
        ITRY=0
        CALL FSTDFS(FBSM,FC,HB,YM,DT,GPC(I),DGPC(I,1),DGPC(I,2),
       $DGPC(I,3),RA,BT)
        RAN(I)=RA
        BTA(I)=BT
C******CATCH AND STORE THE FINAL VALUE OF THE GROUP PATH CORRESPONDING TO THE MID
C******BACKSCATTER LEADING EDGE POINT
        IF(I-NMID)46,47,46
47      GPCMID=GPC(I)
46      CONTINUE
C       NUMERICAL ERROR FLAGGED IF FC NEGATIVE .  IF SO ABORT JOB BY MAKING N
C       NEGATIVE AND RETURNING TO MAINLINE VIA SUBR FMFP
        IF(FC)42,45,45
42      N=-3
        ARG(1)=A1SV
        ARG(2)=A2SV
        ARG(3)=A3SV
```

```
PAGE   002

      VAL=VALSV
      G1SV=GRAD(1)
      G2SV=GRAD(2)
      G3SV=GRAD(3)
      RETURN
45    CONTINUE
      ERR=GPC(I)-PBSMES(I)
      ERSQ=ERR*ERR
      SUMF=SUMF+ERSQ
      SUMDRB=SUMDRB+ERR*DGPC(I,3)
      SUMDRM=SUMDRM+ERR*DGPC(I,2)
50    SUMDFC=SUMDFC+ERR*DGPC(I,1)
C*****NOTE SUMF NOW CONTAINS SUM SQUARED ERROR
C*****GRAD CONTAINS THE GRADIENT OF THE SUM SQUARED ERROR
      VAL=SUMF
      GRAD(1)=2.*SUMDRB
      GRAD(2)=2.*SUMDRM
      GRAD(3)=2.*SUMDFC
C
C*****
C***** NOTE THAT AT THIS POINT IT MEAY BE DESIRABLE TO INSERT PRINT OUT STATEMEN
C*****TO OUTPUT ARG, VAL,GRAD,RANGE,AND BDEG TO OBSERVE CONVERGENCE OF THE ERROR
C*****FUNCTION TO A MINIMUM AS FMFP DOES ITS STUFF.....
C*****ALSO PRINTING OUT RANGE AND BDEG GIVES US SOME INDICATION OF WHERE WE ARE
C*****SAMPLING THE IONOSPHERE IN THE CASE OF HORIZONTAL GRADIENTS
C*****
C
      A1SV=ARG(1)
      A2SV=ARG(2)
      A3SV=ARG(3)
      VALSV=VAL
      G1SV=GRAD(1)
      G2SV=GRAD(2)
      G3SV=GRAD(3)
200   RETURN
      END
```

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H      NAME AS
H      EQUIP=CARDRE,PRINTE,CARDPU;
       SUBROUTINE FMFP(N,X,F,G,EST,EPS,LIMIT,IER,H)
C                                                                      FMFP  20
C        DIMENSIONED DUMMY VARIABLES                                   FMFP  30
       DIMENSION H(1),X(1),G(1)                                        FMFP  40
C                                                                      FMFP  50
C        COMPUTE FUNCTION VALUE AND GRADIENT VECTOR FOR INITIAL ARGUMENTFMFP  60
       CALL FUNCT(N,X,F,G)                                             FMFP  70
       IF(N)56,100,100
100    CONTINUE
C                                                                      FMFP  80
C        RESET ITERATION COUNTER AND GENERATE IDENTITY MATRIX          FMFP  90
       IER=0                                                           FMFP 100
       KOUNT=0                                                         FMFP 110
       N2=N+N                                                          FMFP 120
       N3=N2+N                                                         FMFP 130
       N31=N3+1                                                        FMFP 140
     1 K=N31                                                           FMFP 150
       DO 4 J=1,N                                                      FMFP 160
       H(K)=1.                                                         FMFP 170
       NJ=N-J                                                          FMFP 180
       IF(NJ)5,5,2                                                     FMFP 190
     2 DO 3 L=1,NJ                                                     FMFP 200
       KL=K+L                                                          FMFP 210
     3 H(KL)=0.                                                        FMFP 220
     4 K=KL+1                                                          FMFP 230
C                                                                      FMFP 240
C        START ITERATION LOOP                                          FMFP 250
     5 KOUNT=KOUNT+1                                                   FMFP 260
C                                                                      FMFP 270
C        SAVE FUNCTION VALUE, ARGUMENT VECTOR AND GRADIENT VECTOR      FMFP 280
       OLDF=F                                                          FMFP 290
       DO 9 J=1,N                                                      FMFP 300
       K=N+J
       H(K)=G(J)                                                       FMFP 320
       K=K+N                                                           FMFP 330
       H(K)=X(J)                                                       FMFP 340
C                                                                      FMFP 350
C        DETERMINE DIRECTION VECTOR H                                  FMFP 360
       K=J+N3                                                          FMFP 370
       T=0.                                                            FMFP 380
       DO 8 L=1,N                                                      FMFP 390
       T=T-G(L)*H(K)                                                   FMFP 400
       IF(L-J)6,7,7                                                    FMFP 410
     6 K=K+N-L                                                         FMFP 420
       GO TO 8                                                         FMFP 430
     7 K=K+1                                                           FMFP 440
     8 CONTINUE                                                        FMFP 450
     9 H(J)=T                                                          FMFP 460
C                                                                      FMFP 470
C        CHECK WHETHER FUNCTION WILL DECREASE STEPPING ALONG H,        FMFP 480
       DY=0.                                                           FMFP 490
       HNRM=0.                                                         FMFP 500
       GNRM=0.                                                         FMFP 510
C                                                                      FMFP 520
C        CALCULATE DIRECTIONAL DERIVATIVE AND TESTVALUES FOR DIRECTION FMFP 530
C        VECTOR H AND GRADIENT VECTOR G.                               FMFP 540
```

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      DO 10 J=1,N                                                       FMFP 550
      HNRM=HNRM+ABSF(H(J))                                              FMFP 560
      GNRM=GNRM+ABSF(G(J))                                              FMFP 570
   10 DY=DY+H(J)*G(J)                                                   FMFP 580
C                                                                       FMFP 590
C        REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTIONAL  FMFP 600
C        DERIVATIVE APPEARS TO BE POSITIVE OR ZERO.                     FMFP 610
      IF(DY)11,51,51                                                    FMFP 620
C                                                                       FMFP 630
C        REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTION    FMFP 640
C        VECTOR H IS SMALL COMPARED TO GRADIENT VECTOR G.               FMFP 650
   11 IF(HNRM/GNRM-EPS)51,51,12                                         FMFP 660
C                                                                       FMFP 670
C        SEARCH MINIMUM ALONG DIRECTION H                               FMFP 680
C                                                                       FMFP 690
C        SEARCH ALONG H FOR POSITIVE DIRECTIONAL DERIVATIVE             FMFP 700
   12 FY=F                                                              FMFP 710
      ALFA=2.*(EST-F)/DY                                                FMFP 720
      AMBDA=1.                                                          FMFP 730
C                                                                       FMFP 740
C        USE ESTIMATE FOR STEPSIZE ONLY IF IT IS POSITIVE AND LESS THAN FMFP 750
C        1. OTHERWISE TAKE 1. AS STEPSIZE                               FMFP 760
      IF(ALFA)15,15,13                                                  FMFP 770
   13 IF(ALFA-AMBDA)14,15,15                                            FMFP 780
   14 AMBDA=ALFA                                                        FMFP 790
   15 ALFA=0.                                                           FMFP 800
C                                                                       FMFP 810
C        SAVE FUNCTION AND DERIVATIVE VALUES FOR OLD ARGUMENT           FMFP 820
   16 FX=FY                                                             FMFP 830
      DX=DY                                                             FMFP 840
C                                                                       FMFP 850
C        STEP ARGUMENT ALONG H                                          FMFP 860
      DO 17 I=1,N                                                       FMFP 870
   17 X(I)=X(I)+AMBDA*H(I)                                              FMFP 880
C                                                                       FMFP 890
C        COMPUTE FUNCTION VALUE AND GRADIENT FOR NEW ARGUMENT           FMFP 900
      CALL FUNCT(N,X,F,G)                                               FMFP 910
      FY=F                                                              FMFP 920
      IF(N)54,101,101
  101 CONTINUE
C                                                                       FMFP 930
C        COMPUTE DIRECTIONAL DERIVATIVE DY FOR NEW ARGUMENT.  TERMINATE FMFP 940
C        SEARCH, IF DY IS POSITIVE. IF DY IS ZERO THE MINIMUM IS FOUND  FMFP 950
      DY=0.                                                             FMFP 960
      DO 18 I=1,N                                                       FMFP 970
   18 DY=DY+G(I)*H(I)                                                   FMFP 980
      IF(DY)19,36,22                                                    FMFP 990
C                                                                       FMFP1000
C        TERMINATE SEARCH ALSO IF THE FUNCTION VALUE INDICATES THAT     FMFP1010
C        A MINIMUM HAS BEEN PASSED                                      FMFP1020
   19 IF(FY-FX)20,22,22                                                 FMFP1030
C                                                                       FMFP1040
C        REPEAT SEARCH AND DOUBLE STEPSIZE FOR FURTHER SEARCHES         FMFP1050
   20 AMBDA=AMBDA+ALFA                                                  FMFP1060
      ALFA=AMBDA                                                        FMFP1070
C        END OF SEARCH LOOP                                             FMFP1080
C                                                                       FMFP1090
C        TERMINATE IF THE CHANGE IN ARGUMENT GETS VERY LARGE            FMFP1100
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      IF(HNRM*AMBDA-1.E10)16,16,21                                      FMFP1110
C                                                                       FMFP1120
C        LINEAR SEARCH TECHNIQUE INDICATES THAT NO MINIMUM EXISTS       FMFP1130
   21 IER=2                                                             FMFP1140
      RETURN                                                            FMFP1150
C                                                                       FMFP1160
C        INTERPOLATE CUBICALLY IN THE INTERVAL DEFINED BY THE SEARCH    FMFP1170
C        ABOVE AND COMPUTE THE ARGUMENT X FOR WHICH THE INTERPOLATION   FMFP1180
C        POLYNOMIAL IS MINIMIZED                                        FMFP1190
   22 T=0.                                                              FMFP1200
   23 IF(AMBDA)24,36,24                                                 FMFP1210
   24 Z=3.*(FX-FY)/AMBDA+DX+DY                                          FMFP1220
      ALFA=AMAX1F(ABSF(Z),ABSF(DX),ABSF(DY))                            FMFP1230
      DALFA=Z/ALFA                                                      FMFP1240
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA                                 FMFP1250
      IF(DALFA)51,25,25                                                 FMFP1260
   25 W=ALFA*SQRT(DALFA)                                                FMFP1270
      ALFA=(DY+W-Z)*AMBDA/(DY+2.*W-DX)                                  FMFP1280
      DO 26 I=1,N                                                       FMFP1290
   26 X(I)=X(I)+(T-ALFA)*H(I)                                           FMFP1300
C                                                                       FMFP1310
C        TERMINATE, IF THE VALUE OF THE ACTUAL FUNCTION AT X IS LESS    FMFP1320
C        THAN THE FUNCTION VALUES AT THE INTERVAL ENDS. OTHERWISE REDUCEFMFP1330
C        THE INTERVAL BY CHOOSING ONE END-POINT EQUAL TO X AND REPEAT   FMFP1340
C        THE INTERPOLATION.  WHICH END-POINT IS CHOOSEN DEPENDS ON THE  FMFP1350
C        VALUE OF THE FUNCTION AND ITS GRADIENT AT X                    FMFP1360
C                                                                       FMFP1370
      CALL FUNCT(N,X,F,G)                                               FMFP1380
      IF(N) 96,102,102
102   CONTINUE
      IF(F-FX)27,27,28                                                  FMFP1390
   27 IF(F-FY)36,36,28                                                  FMFP1400
   28 DALFA=0.                                                          FMFP1410
      DO 29 I=1,N                                                       FMFP1420
   29 DALFA=DALFA+G(I)*H(I)                                             FMFP1430
      IF(DALFA)30,33,33                                                 FMFP1440
   30 IF(F-FX)32,31,33                                                  FMFP1450
   31 IF(DX-DALFA)32,36,32                                              FMFP1460
   32 FX=F                                                              F-FP1 70
      DX=DALFA                                                          FMFP1480
      T=ALFA                                                            FMFP1490
      AMBDA=ALFA                                                        FMFP1500
      GO TO 23                                                          FMFP1510
   33 IF(FY-F)35,34,35                                                  FMFP1520
   34 IF(DY-DALFA)35,36,35                                              FMFP1530
   35 FY=F                                                              FMFP1540
      DY=DALFA                                                          FMFP1550
      AMBDA=AMBDA-ALFA                                                  FMFP1560
      GO TO 22                                                          FMFP1570
C                                                                       FMFP1580
C        COMPUTE DIFFERENCE VECTORS OF ARGUMENT AND GRADIENT FROM       FMFP1590
C        TWO CONSECUTIVE ITERATIONS                                     FMFP1600
   36 DO 37 J=1,N                                                       FMFP1610
      K=N+J                                                             FMFP1620
      H(K)=G(J)-H(K)                                                    FMFP1630
      K=N+K                                                             FMFP1640
   37 H(K)=X(J)-H(K)                                                    FMFP1650
C                                                                       FMFP1660
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C        TERMINATE, IF FUNCTION WAS NOT DECREASED DURING LAST ITERATION FMFP1670
      IF(OLDF-F+EPS)51,38,38                                           FMFP1680
C                                                                      FMFP1690
C        TEST LENGTH OF ARGUMENT DIFFERENCE VECTOR AND DIRECTION VECTOR FMFP1700
C        IF AT LEAST N ITERATIONS HAVE BEEN EXECUTED, TERMINATE, IF    FMFP1710
C        BOTH ARE LESS THAN  EPS                                       FMFP1720
   38 IER=0                                                            FMFP1730
      IF(KOUNT-N)42,39,39                                              FMFP1740
   39 T=0.                                                             FMFP1750
      Z=0.                                                             FMFP1760
      DO 40 J=1,N                                                      FMFP1770
      K=N+J                                                            FMFP1780
      W=H(K)                                                           FMFP1790
      K=K+N                                                            FMFP1800
      T=T+ABS(F(H(K))                                                  FMFP1810
   40 Z=Z+W*H(K)                                                       FMFP1820
      IF(HNRM-EPS)41,41,42                                             FMFP1830
   41 IF(T-EPS)55,55,42
C                                                                      FMFP1850
C        TERMINATE, IF NUMBER OF ITERATIONS WOULD EXCEED  LIMIT        FMFP1860
   42 IF(KOUNT-LIMIT)43,50,50                                          FMFP1870
C                                                                      FMFP1880
C        PREPARE UPDATING OF MATRIX H                                  FMFP1890
   43 ALFA=0.                                                          FMFP1900
      DO 47 J=1,N                                                      FMFP1910
      K=J+N3                                                           FMFP1920
      W=0.                                                             FMFP1930
      DO 46 L=1,N                                                      FMFP1940
      KL=N+L                                                           FMFP1950
      W=W+H(KL)*H(K)                                                   FMFP1960
      IF(L-J)44,45,45                                                  FMFP1970
   44 K=K+N-L                                                          FMFP1980
      GO TO 46                                                         FMFP1990
   45 K=K+1                                                            FMFP2000
   46 CONTINUE                                                         FMFP2010
      K=N+J                                                            FMFP2020
      ALFA=ALFA+W*H(K)                                                 FMFP2030
   47 H(J)=W                                                           FMFP2040
C                                                                      FMFP2050
C        REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF RESULTS     FMFP2060
C        ARE NOT SATISFACTORY                                          FMFP2070
      IF(Z+ALFA)48,1,48                                                FMFP2080
C                                                                      FMFP2090
C        UPDATE MATRIX H                                               FMFP2100
   48 K=N31                                                            FMFP2110
      DO 49 L=1,N                                                      FMFP2120
      KL=N2+L                                                          FMFP2130
      DO 49 J=L,N                                                      FMFP2140
      NJ=N2+J                                                          FMFP2150
      H(K)=H(K)+H(KL)*H(NJ)/Z-H(I)*H(J)/ALFA                           FMFP2160
   49 K=K+1                                                            FMFP2170
      GO TO 9                                                          FMFP2180
C        END OF ITERATION LOOP                                         FMFP2190
C                                                                      FMFP2200
C        NO CONVERGENCE AFTER  LIMIT  ITERATIONS                       FMFP2210
   50 IER=1                                                            FMFP2220
      RETURN                                                           FMFP2230
C                                                                      FMFP2240
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C        RESTORE OLD VALUES OF FUNCTION AND ARGUMENTS                    FMFP2250
   51 DO 52 J=1,N                                                        FMFP2260
      K=N2+J                                                             FMFP2270
   52 X(J)=H(K)                                                          FMFP2280
      CALL FUNCT(N,X,F,G)                                                FMFP2290
      IF(N)56,103,103
103   CONTINUE
C                                                                        FMFP2300
C        REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DERIVATIVE    FMFP2310
C        FAILS TO BE SUFFICIENTLY SMALL                                  FMFP2320
      IF(GNRM-EPS)55,55,53                                               FMFP2330
C                                                                        FMFP2340
C        TEST FOR REPEATED FAILURE OF ITERATION                          FMFP2350
   53 IF(IER)56,54,54                                                    FMFP2360
   54 IER=-1                                                             FMFP2370
      GOTO 1                                                             FMFP2380
   55 IER=0                                                              FMFP2390
      PRINT 57,KOUNT
   57 FORMAT(///,22H FMFP CONVERGED AFTER ,I4,11H ITERATIONS ,///)
   56 RETURN                                                             FMFP2400
      END                                                                FMFP2410
```

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H      NAME A6
H      EQUIP=CARDRE,PRINTE,CARDPU;
       SUBROUTINE BACKS3(NUMTOT,NPTX,RBSTRT,RMSTRT,FCSTRT,RBB,RMM,FCC,
      $FMID,RANMID,GPTHMD,NUSED,PRNTSW)
       DIMENSION X(3),G(3),H(32)
       DIMENSION FBSMES(50),PBSMES(50)
       DIMENSION RAN(50),BTA(50),RBB(20),RMM(20),FCC(20),FMID(20)
       DIMENSIONRANMID(20),GPTHMD(20)
       COMMON FBSMES,PBSMES,DT,NPTS,RAN,BTA,GPCMID
C******
C******THIS IS SUBROUTINE FOR THE MIN SUM SQUARED ERROR NOISY BACKSCATTER LEADING
C******EDGE INVERSION PROCEDURE...
C******DESCRIBED IN QUARTERLY REPORTS 3,4,5 OF DR. YEH AND DR. RAO S AIR FORCE
C******(AFCRL) CONTRACT NUMBER F 19628-75-C-0088, 1975-1976
C
C
C
C
C      SUBROUTINE INPUT PARAMETERS FOLLOW.............
C
C      NUMTOT=TOTAL NUMBER OF POINTS SCALED FROM IONOGRAM BACSCATTER LEADING
C           EDGE WHICH ARE TO BE INVERTED IN GROUPS OF NPTS
C      NPTS=NUMBER OF POINTS IN EACH GROUP OF DATA THAT WILL BE SUCCESSIVELY INV
C
C           FOR EXAMPLE...TO PROCESS 12 BACKSCATTER IONOGRAM PTS BY INVERTING
C           OVERLAPPING GROUPS OF 4 POINTS AT A TIME, WE LET NUMTOT=12AND
C           NUMGRP=4.
C
C           NOTE THAT THE STARTING SET OF ESTIMATED LAYER PARAMETERS IS USED
C           IN INVERTING THE FIRST SET OF BACKSCATTER LEADING EDGE POINTS.  EACH
C           SUBSEQUENT SET USES THE IMMEDIATELY PRECEEDING SOLUTION AS THE
C           STARTING POINT.
C
C      RBSTRT=STARTING VALUE RB IN KM
C      RMSTRT=STARTING VALUE RM IN KM
C      FCSTRT=STARTING VALUE FC IN MHZ
C
C      OUTPUT PARAMETER DEFNS.............
C      RBB=VECTOR OF RB PARAMETERS OBTAINED FROM THE SUCCESSIVE INVERSION OF THE
C           SETS OF NPTS POINTS
C      RMM=VECTOR OF RM PARAMETERS
C      FCC=VECTOR OF FC PARAMETERS
C      FMID=VECTOR OF THE FREQUENCIES CORRESPONDING TO THE MIDDLE BACKSCATTER
C          LEADING EDGE POINT IN THE SUCCESSIVE SETS OF NPTS POINTS
C      RANMID=VECTOR OF RANGES IN KM
C      GPTHMD=VECTOR OF GROUP PATHS IN KM
C      NUSED=NUMBER OF SETS OF NPTS POINTS SUCCESSFULLY INVERTED FROM
C           BACKSCATTER LEADING EDGE
C      PRNTSW=0=DO NOT PRINT INTERMEDIATE RESULTS
C      PRNTSW=1=PRINT INTERMEDIATE RESULTS
C
C
C******DESCRIPTION OF DATA DECK TO BE READ IN BY SUBROUTINE
C
C******PARAMETER CARD DESCRIPTION   IN 4F10.5 FORMAT
C******DT IS THE TOLERANCE IN THE DERIV OF THE MINIMUM GROUP PATH WITH RESPECT
C******     TO BETA, TYPICALLY SET TO .05 KM/RAD.
C******   EST = ESTIMATED MIN VALUE OF SUM-SQUARED ERROR FUNCTION E (IN SQ KM)
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C******   UPON WHICH THE MINIMIZATION PROCEDURE BASES ITS STEPSIZE
C******   EPS=TOL TO WHICH MINIMUM IS TO BE COMPUTED (SUBR FMFP USES IT TO DETERM
C******   WHEN TO TERMINATE MINIMIZATION ITERATIONS)
C******   LIMIT=MAXIMUM NUMBER OF ITERATIONS ALLOWED IN MINIMIZATION ITERATION
C******   LOOP BEFORE GIVING UP
C
      NPTS=NPTX
      X(1)=RBSTRT
      X(2)=RMSTRT
      X(3)=FCSTRT
C******   X(1)=STARTING VALUE RB IN KM
C******   X(2)=STARTING VALUE RM IN KM
C******   X(3)=STARTING VALUE FC IN MHZ
      READ 10,DT,EST,EPS,XLIMIT
10    FORMAT(4F10.5)
      LIMIT=XLIMIT
      NUMSET=NUMTOT-NPTS+1
      NUSED=NUMSET
      JARRAY=1
      DO 101 ICT=1,NUMSET
C******SAVE STARTING VALUES IN CASE WE WANT TO ABORT ATTEMPT AND TRY AGAIN
      XSV1=X(1)
      XSV2=X(2)
      XSV3=X(3)
C******KT=0 INDICATES FIRST ATTEMPT AT MINIMIZATION
      KT=0
      ESTSVE=EST
C******FIRST INITIALIZE FUNCT BY READING IN
C******IN AND PRINT OUT THE NUMPT POINTS SCALED OFF LEADING EDGE OF BS IONOGRAM
C******AS WELL AS READ IN THE TOLERANCE TO WHICH THE MIN GROUP PATH IS FOUND IN
C******SUBROUTINE FSTDIF AND THE NUMBER OF IONOGRAM PTS USED IN DEFINING THE ERRO
C******ERROR FUNCTION E. IN SUBROUTINE FUNCT....
      IF(PRNTSW)119,121,119
119   CONTINUE
      PRINT 12, DT
C******DT IS THE TOLERANCE OF DERIV OF GROUP PATH WRT BETA (KM/RAD)
C******FOR USE IN FSTDIF ROUTINE WHEN IT COMPUTES THE MIN GROUP PATH
12    FORMAT(//,54H1TOLERANCE OF DERIV OF GROUP PATH WRT BETA (KM/RAD) =
     $   ,F5.3//)
C******NPTS IS THE NUMBER OF POINTS SCALED FROM LEADING EDGE OF B S IONOGRAM
C******FOR USE IN EVALUATING THE SUM SQUARED ERROR FCN
      PRINT 13,NPTS
121   CONTINUE
13    FORMAT(//, 61H NUMBER OF SCALED IONOGRAM POINTS TO BE USED IN ERROR
     $ FCN =  ,I2)
      IF(ICT   -1)130,125,130
125   READ 16,(FBSMES(I),PBSMES(I),I=1,NPTS)
C******HERE THE LEADING EDGE POINTS ARE READ IN
16    FORMAT (2F10.5)
      GO TO 150
130   DO 145 IND=2,NPTS
      FBSMES(IND-1)=FBSMES(IND)
145   PBSMES(IND-1)=PBSMES(IND)
      READ 16,FBSMES(NPTS),PBSMES(NPTS)
150   CONTINUE
      IF (PRNTSW)151,201,151
151   CONTINUE
      PRINT 17
```

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17    FORMAT(53H MEASURED POINT ON BACKSCATTER IONOGRAM LEADING EDGE    ,
     $/,9X,15HFREQUENCY (MHZ)  ,4X,6X,15HGROUP PATH(KM)  )
      PRINT 19,(FBSMES(I),PBSMES(I),I=1,NPTS)
19    FORMAT(/,11X,F8.3,19X,F8.3,/)
      PRINT 20,X(1),X(2),X(3),EST,EPS,LIMIT
20    FORMAT(    26H STARTING VALUE RB (KM) = ,F8.4,8X,
     $25H STARTING VALUE RM(KM) = ,F8.4,8X,
     $27H STARTING VALUE FC (MHZ) = ,F8.6,/,
     $37H ESTIMATE OF MIN SUM SQUARED ERROR = ,F10.5,/,
     $27H EXPECTED ABSOLUTE ERROR = ,F10.8,/,
     $32H MAXIMUM NUMBER OF ITERATIONS = ,I4////)
      N=3
      CALL FUNCT(N,X,VAL,G)
      PRINT 21,VAL,G
21    FORMAT(33H INITIAL SUM SQUARED ERROR (E) =  ,  E12.5,/,
     $9H DEDRB = ,E12.5,5X,
     $9H DEDRM = ,E12.5,5X,
     $9H DEDFC = ,E12.5,//)
201   CONTINUE
      N=3
      OLDMIN=99999.
C*****THIS BUSINESS ABT OLDMIN HAS TO DO WITH NUMERICAL ERROR PROB IN FSTDIF SUB
C*****SEE STATEMENT 2600 TO SEE WHAT I AM TALKING ABOUT
C*****FMFP IS THE NONLINEAR MINIMIZATION ROUTINE FOUND IN IBM SCIENTIFIC SUBR
C*****SUBROUTINE PACKAGE.  IT INTERNALLY CALLS FUNCT TO PERFORM SUCCESSIVE
C*****EVALUATIONS OF THE SUM SQUARED ERROR FUNCTION AND ITS GRADIENT
22    CALL FMFP(N,X,FMIN,G,EST,EPS,LIMIT,IER,H)
C*****IF NUMERICAL ERRORS SUCH AS SQRT (NEG NUMBER), LOG(LESS THAN 0), OR
C*****ACOS(GREATER THAN 1) WERE FLAGGED IN THE EVALUATION OF FUNCT, N IS
C*****NEGATED AND SO WE TEST FOR THIS ERROR CONDITION UPON EXIT FROM FMFP
      IF(N)25,27,27
25    PRINT 26,KT,EST
26    FORMAT(///30H NUMERICAL ERRORS ON TRY NR =   ,I2,16H WITH EST MIN =
     $   ,F7.5,///)
      KT=KT+1
C*****SET N BACK TO +3
      N=-N
      CALL FUNCT(N,X,FMIN,G)
C*****EVALUATE FMIN ON BASIS OF PARTIALLY ITERATED PARAMETERS JUST BEFORE NUMERI
C*****NUMERICAL PROBLEMS WERE ENCOUNTERED
      IF(OLDMIN-FMIN)2610,2610,2600
2600  SOLN1=X(1)
      SOLN2=X(2)
      SOLN3=X(3)
      SVMIN=FMIN
      GSV1=G(1)
      GSV2=G(2)
      GSV3=G(3)
      OLDMIN=FMIN
2610  CONTINUE
C*****IF THIS IS 7TH TRY GIVE UP ENTIRELY
      IF(KT-7)260,270,270
C*****IF NUMERICAL ERRORS, AFTER EST, WHICH WILL DECREASE THE STEP SIZE AND TRY
C*****TRY AGAIN.  FIRST GO BACK TO ORIGHINAL STARTING SOLUTION
260   IF(FMIN-EST)262,262,261
C*****IF OUR ESTIMATE WAS TOO SMALL INCREASE IT
261   EST=EST+10.
      GO TO 263
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C*****IF OUR ESTIMATE WAS TOO LARGE DECREASE IT
262     EST=EST/10.
263     CONTINUE
        X(1)=XSV1
        X(2)=XSV2
        X(3)=XSV3
C*****NOW GO BACK TO 22 AND TRY AGAIN
        GO TO 22
270     PRINT 271
271     FORMAT(///,57H WILL GIVE UP TRYING TO INVERT THIS DATA AFTER 7 ATTE
       $MPTS,//)
        PRINT 275
275     FORMAT(100(1H$),/,106H WILL USE LAST VALUES OF ITERATED PARAMETERS
       $ BEFORE NUMERICAL PROBLEM OCCURRED--NOTE FMFP DID NOT CONVERGE,/,
       $,100(1H$),///)
        X(1)=SOLN1
        X(2)=SOLN2
        X(3)=SOLN3
        FMIN=SVMIN
        G(1)=GSV1
        G(2)=GSV2
        G(3)=GSV3
        GO TO 38
27      CONTINUE
C
C*****IER=0, A VALID MINIMUM WITHIN THE SPECIFIED TOLERANCE (EPS) WAS FOUND BY F
C*****FMFP
C*****IER=1, A MINIMUM WAS NOT FOUND IN THE SPECIFIED (LIMIT) NUMBER OF ITERATIO
C*****IER=-1, IMPLIES ERROR IN GRADIENT CALCULATION
C*****IER=2, LINEAR SEARCH CANNOT FIND MINIMUM
C
        IF(PRNTSW)301,401,301
301     CONTINUE
        IGOTO=IER+2
        GO TO(30,32,34,36),IGOTO
32      PRINT 33
33      FORMAT(///,60H VALID MINIMUM FOUND BY FMFP WITHIN SPECIFIED TOLERANCE EPS
       $NCE EPS  )
        GO TO 38
30      PRINT 33
31      FORMAT(///,35H ERROR IN FMFP GRADIENT CALCULATION )
        GO TO 38
34      PRINT 35,LIMIT
35      FORMAT(///,86H ERR FCN COULD NOT BE MINIMIZED WITHIN THE SPECIFIED
       $ NR (LIMIT) OF ITERATIONS LIMIT = ,I5)
        GO TO 38
36      PRINT 37
37      FORMAT(///,45H LINEAR SEARCH IN FMFP COULD NOT FIND MINIMUM)
38      CONTINUE
C*****PRINT INVERTED LAYER PARAMETERS AS WELL AS MINIMUM VALUE OF SUM-SQUARED
C*****ERROR FUNCTION
        PRINT 40,X,FMIN ,G
40      FORMAT(///,29H ITERATED VALUE OF RB (KM) = ,F6.2, 5X,
       $29H ITERATED VALUE OF RM (KM) = ,F6.2, 5X,
       $30H ITERATED VALUE OF FC (KHZ) = ,F8.4,/,
       $25H MIN SUM SQUARED ERROR = ,F10.5,/
       $9H DEDRB = ,E12.5,5X,9H DEDRM = ,E12.5,5X,9H DEDFC = ,E12.5)
        PRINT 51
```

```
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*0    FORMAT(///104H RANGE AND ELEVATION ANGLES CORRESPONDING TO THE NPTS B
     $ BACKSCATTER LEADING EDGE POINTS USED IN INVERSION  ,/)
60    PRINT7 ,(I,FBSMES(I),RAN(I),BTA(I),I=1,NPTS)
70    FORMAT(3H F(,I2,4H) = ,F7.4,4H MHZ,5X,23HMIN GROUP RANGE (KM) =  ,
     $F9.3,5X,30HMIN GROUP ELEV ANGLE (DEG) =  ,F6.3)
401   CONTINUE
C*****HERE WE SAVE THE RESULTS OF THE RRESENT NPTS INVERSION
      RBB(JARRAY)=X(1)
      RMM(JARRAY)=X(2)
      FCC(JARRAY)=X(3)
      NMIDL=NPTS/2+1
      FMID(JARRAY)=FBSMES(NMIDL)
      RANMID(JARRAY)=RAN(NMIDL)
      GPTHMD(JARRAY)=GPCMID
      JARRAY=JARRAY+1
100   CONTINUE
      EST=ESTSVE
101   CONTINUE
C*****PRINT OUT SUMMARY OF THE INVERTED BACKSCATTER IONOGRAM POINTS
      PRINT 700,NUSED,NPTS
700   FORMAT(1H1,//,100(1H*),/,50H SUMMARY OF THE BACKSCATTER LEADING EDG
     $GE INVERSION,/,1X,I2,8H SETS OF,I2,44H OVERLAPPING SETS WERE SUCCE
     $SSIVELY INVERTED,///)
      PRINT 710
710   FORMAT(84H     RB (KM)     RM(KM)     FC (MHZ)     FREQ (MHZ)     RANGE (KM)
     $ (KM)     MIN GROUP PATH (KM)   /)
      PRINT 720,(RBB(J),RMM(J),FCC(J),FMID(J),RANMID(J),
     $GPTHMD(J),J=1,NUSED)
720   FORMAT(2X,F9.3,1X,F10.3,F10.3,3X,F10.3,4X,F10.3,9X,F10.3)
      RETURN
      END
```

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```
H       NAME A7
H       EQUIP=CARDRE,PRINTE,CARDPU;
        SUBROUTINE DCAL(FC,RB,RM,FW,GP,D,GPTOL)
        RO=637 .
        SINBET=(RB*RB-RO*RO-GP*GP/4.)/(RO*GP)
        BETA=ASINF(SINBET)
        BETA=0.0001
   20   COSBET=COSF(BETA)
        SINBET=SINF(BETA)
        F=FW/FC
        FF=F*F
        YM=RM-RB
        RBYM2=RB*RB/(YM*YM)
        A=FF-1.+RBYM2
        B=-2.*RM*RBYM2
        C=RBYM2*RM*RM-FF*RO*RO*COSBET*COSBET
        U=B*B-4.*A*C
        GAM=ACOSF((RO/RB)*COSBET)
        SINGAM=SINF(GAM)
        COSGAM=COSF(GAM)
        COTGAM=COSGAM/SINGAM
        SQA=SQRTF(A)
        V=2.*A*RB+B+2.*RB*F*SQA*SINGAM
        GPC=2.*(1.-FF/A)*RB*SINGAM-2.*RO*SINBET-(B*F/(2.*A*SQA))*LOGF
       1(U/(V*V))
        IF (ABSF(GPC-GP)-GPTOL) 100,100,50
   50   DGPDB=2.*(1.-FF/A+B*FF/(A*V))*RO*SINBET*COTGAM
       1-2.*(1.-(2.*B*F*FF*RO*SINBET)/(SQA*U))*RO*COSBET
        PRINT 40,FW,GP,BETA,GPC,DGPDB,FC,RB,RM
   40   FORMAT(6X,8F10.2)
        BETA=BETA+(GP-GPC)/DGPDB
        SINBET=SINF(BETA)
        GO TO 20
  100   SQC=SQRTF(C)
        W=2.*SQC*F*RB*SINGAM+2.*C+B*RB
        D=(GAM-BETA)-(F*RO*COSBET/(2.*SQC))*LOGF(U*RB*RB/(W*W))
        D=2.*RO*D
        RETURN
        END
```

```
PAGE   001

H      NAME AA
H      EQUIP=CARDRE,PRINTE,CARDPU:
       FUNCTION SQROO(ARG)
C
C*****SUBRCUTINE SQROO CHECKS TO SEE IF WE ARE TRYING TO TAKE THE SQRT OF A NEG
C*****NUMBER.  IF WE SO THEN AN ERROR CONDX IS FLAGGED AND THE ARG IS PRINTED OU
C*****WITH A MESSAGE... THE FLAGGING IS DONE BY SETTING SQROO = 9999.
C*****WITHCUT SUCH A SUBROUTINE THE G20 WILL SIGN YOU OFF IN A HURRY
C*****KEEPING US FROM GETTING TO ANY GOOD DATA
C
       IF(ARG)10,20,20
10     CONTINUE
       PRINT 15, ARG
15     FORMAT(29H SQRT ENTERED WITH NEG ARG =      ,E13.7)
       SQROO=9999.
       GO TO 30
20     SQROO=SQRT(ARG)
30     RETURN
       END
```

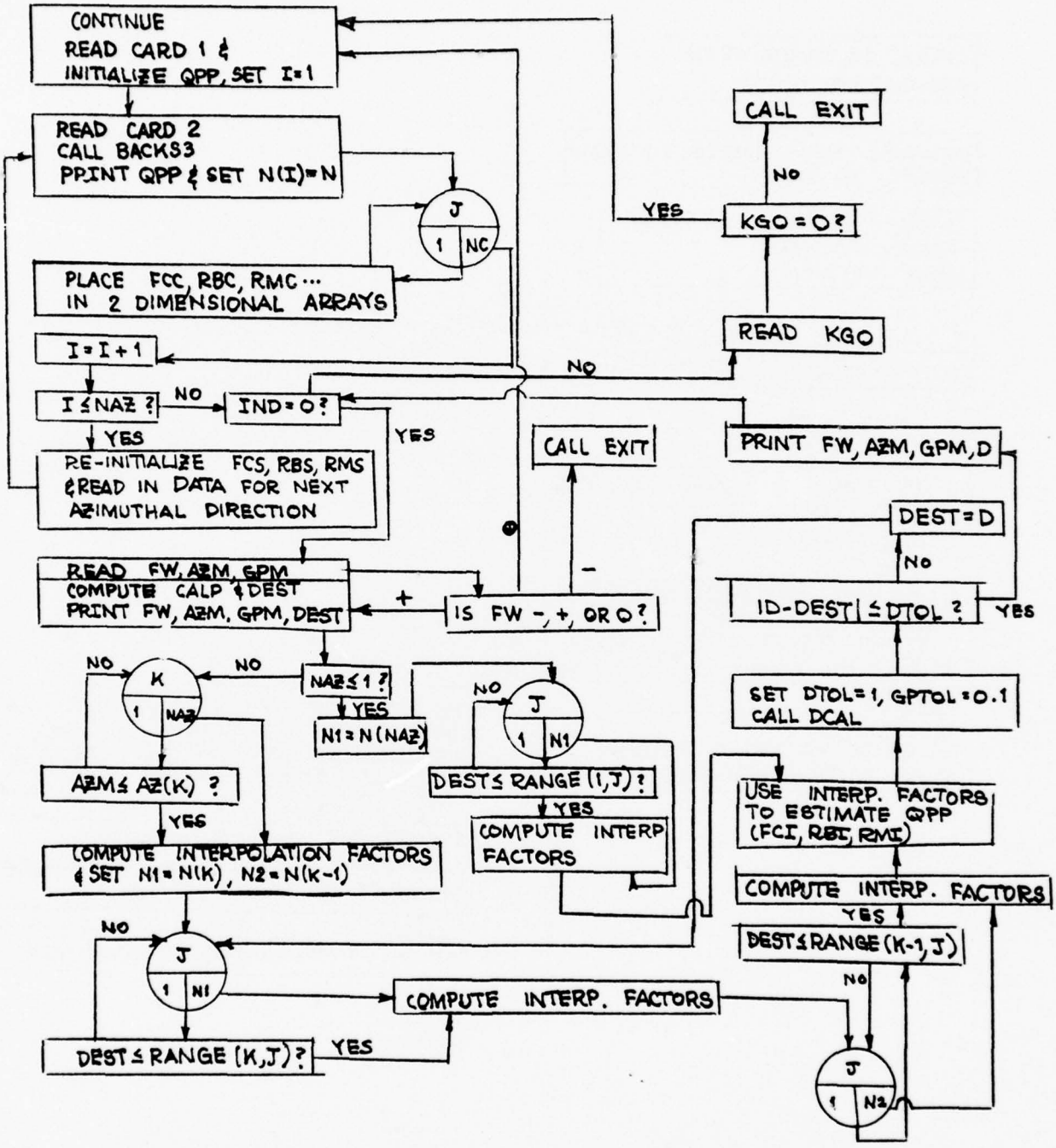

Figure A.3.1. Flow chart for main program.

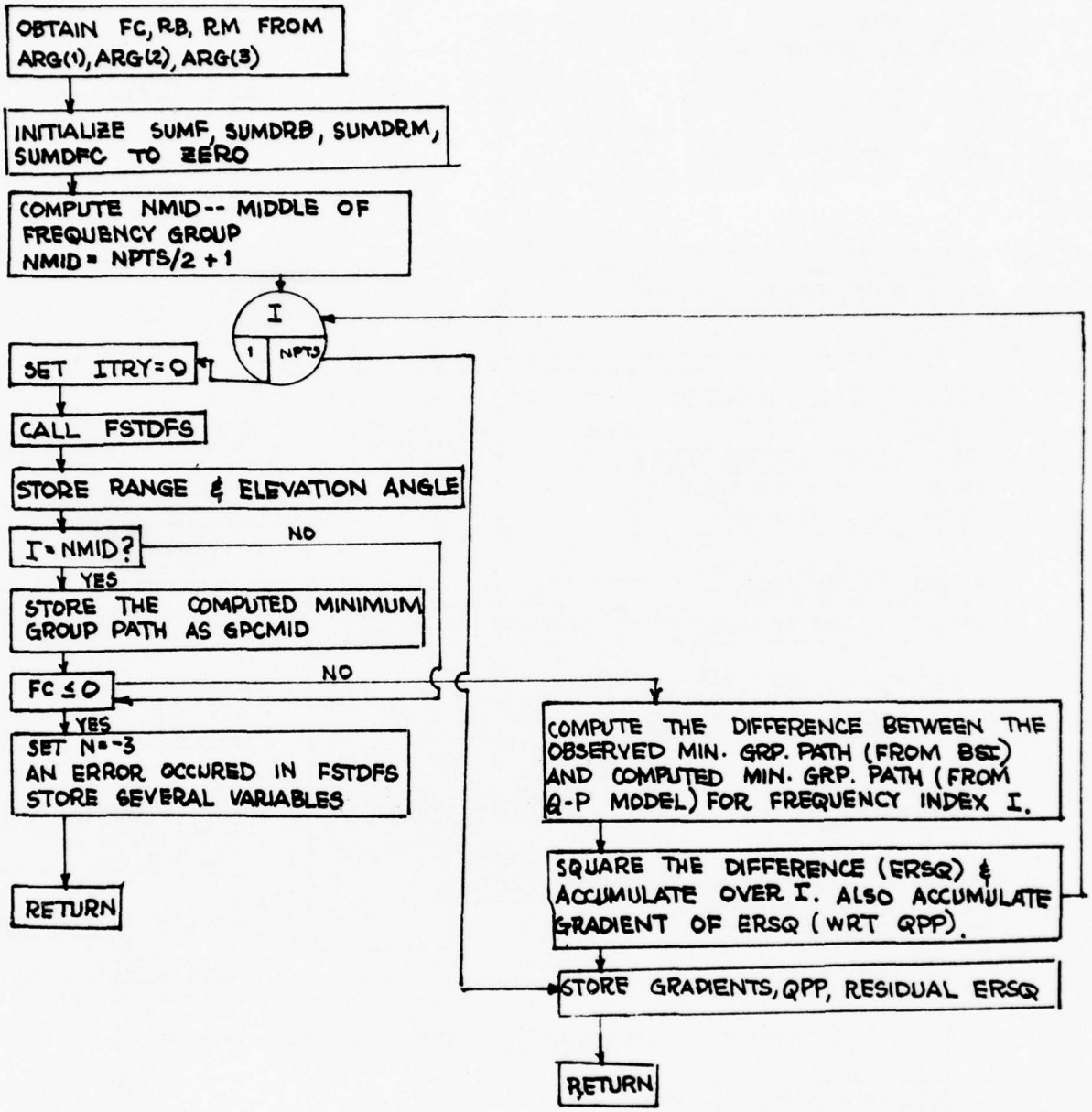

Figure A.3.2. Flow chart for subroutine FUNCT.

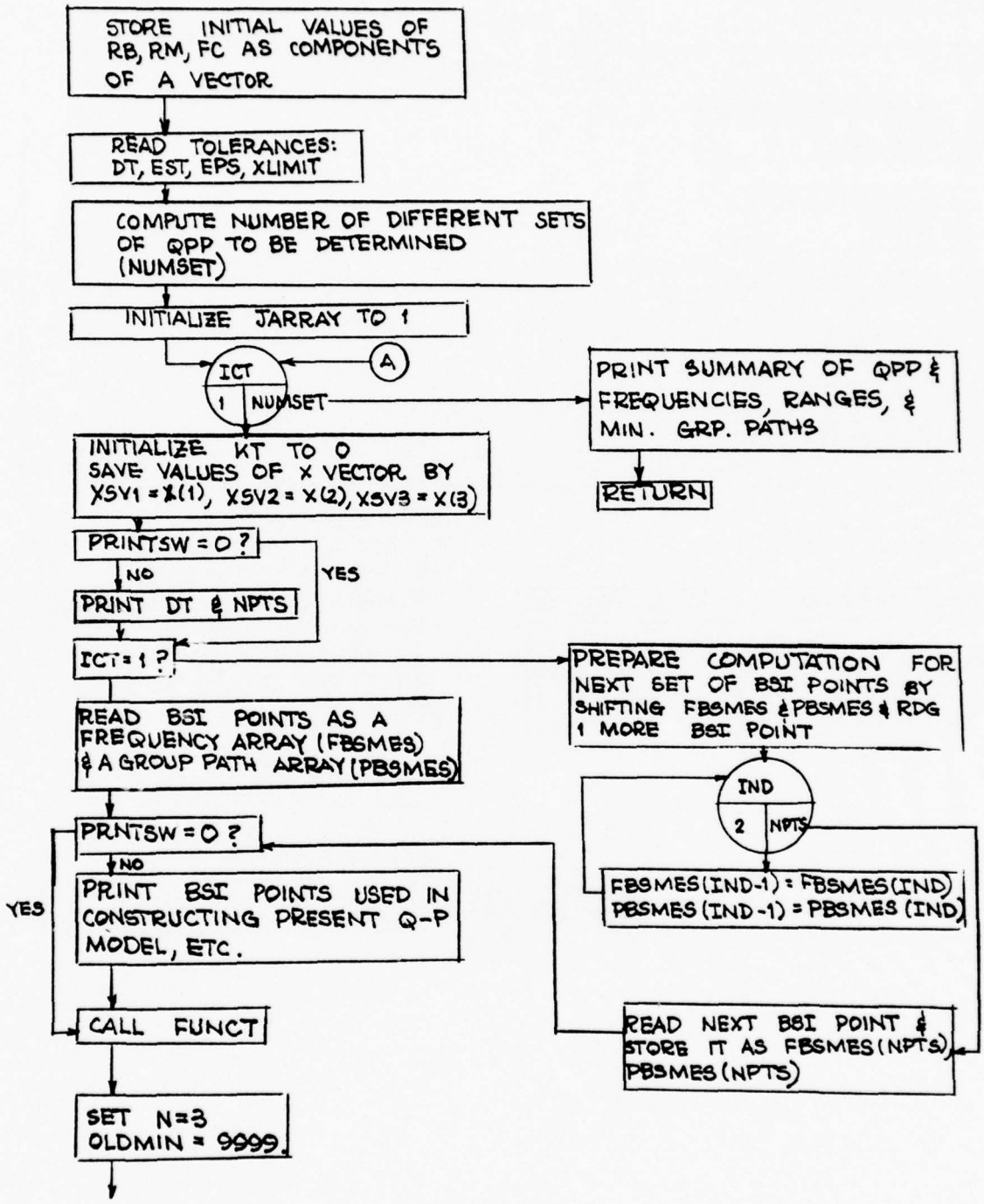

Figure A.3.3.a. Flow chart for subroutine BACKS3 (continued on next figure).

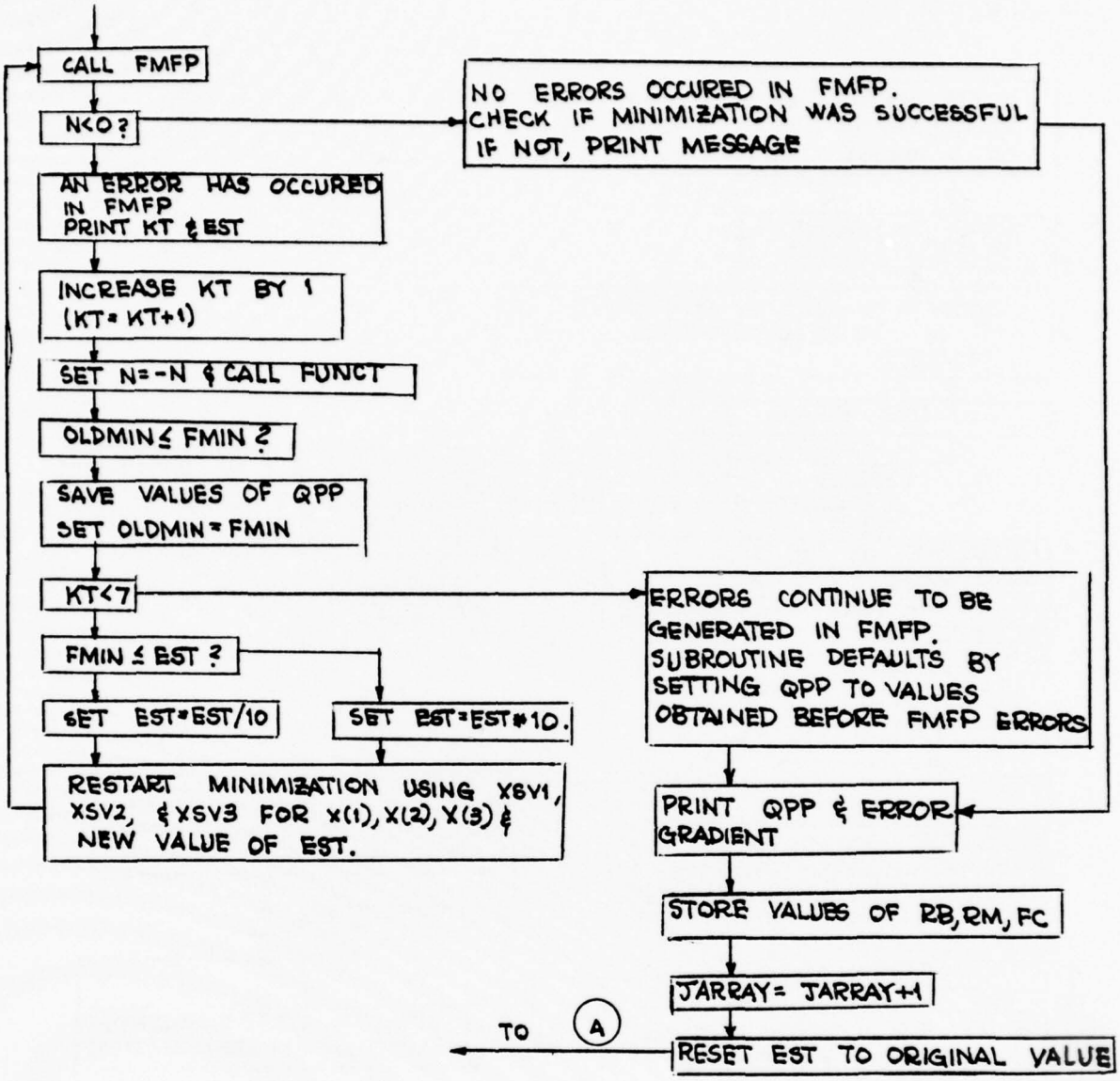

Figure A.3.3.b. Flow chart for subroutine BACKS
continuation of Figure A.3.3.a).

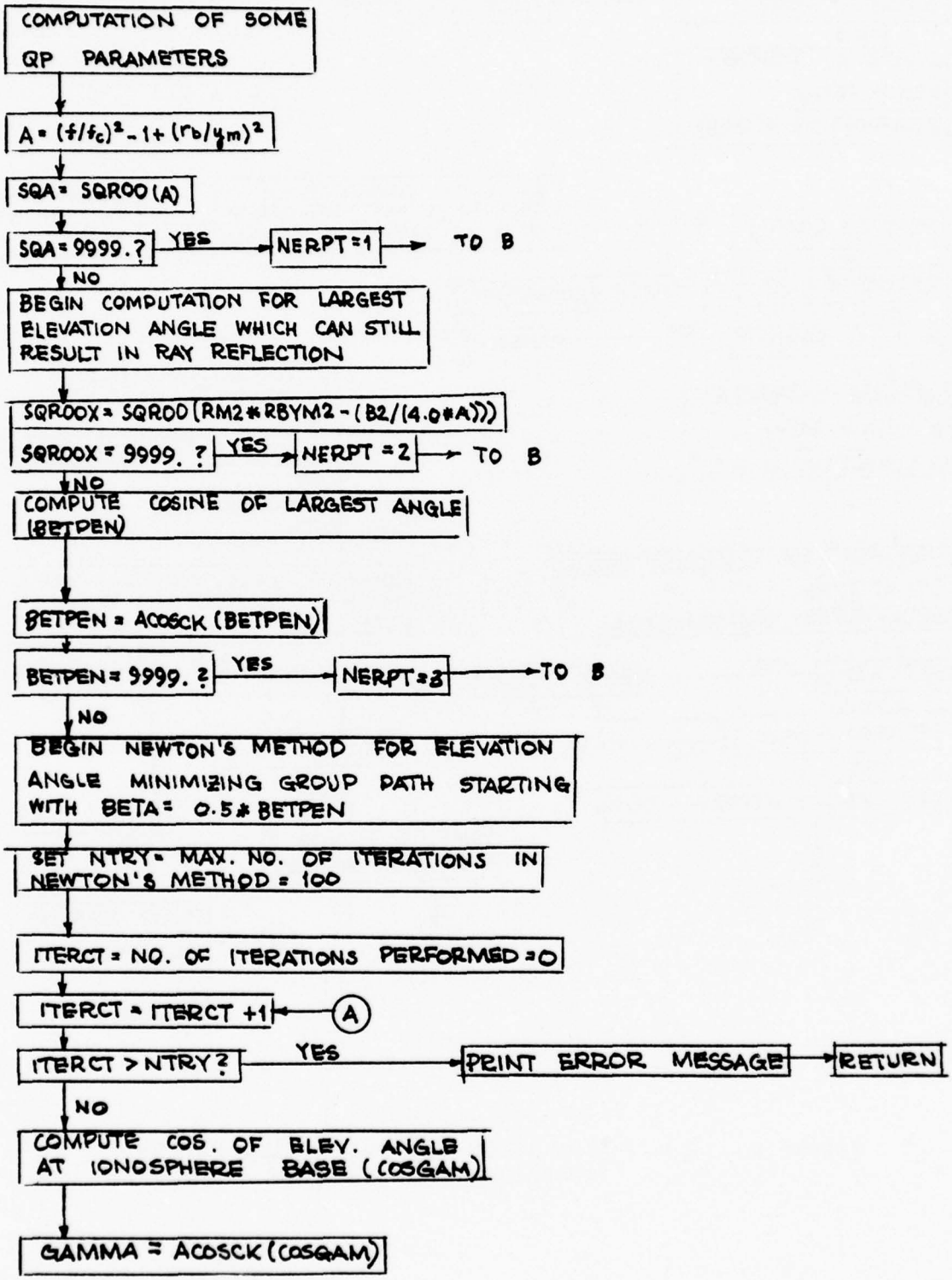

Figure A.3.4.a. Flow chart for subroutine FSTDFS (continued on next figure).

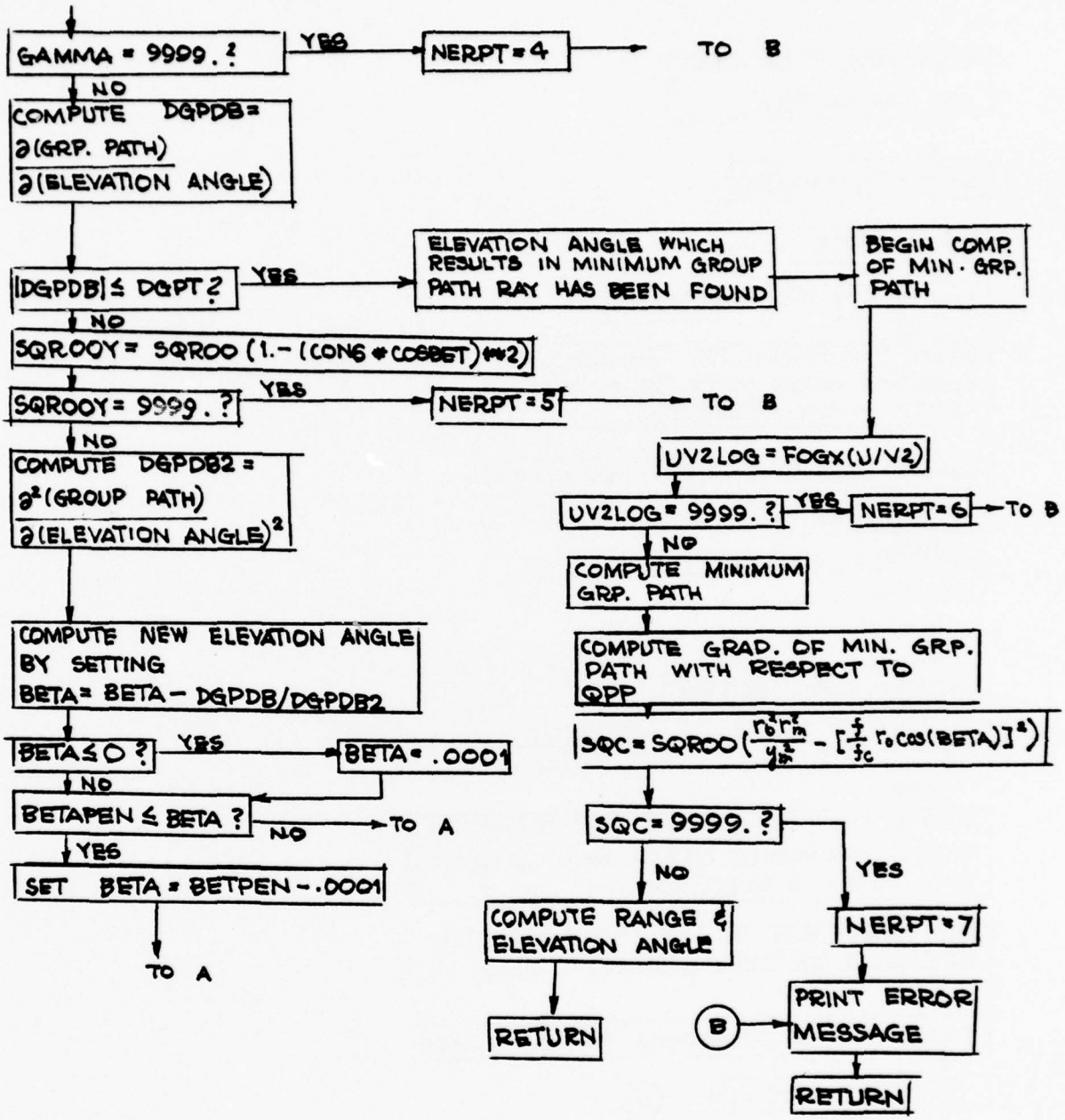

Figure A.3.4.b. Flow chart for subroutine FSTDFS (continuation of Figure A.3.4.a).

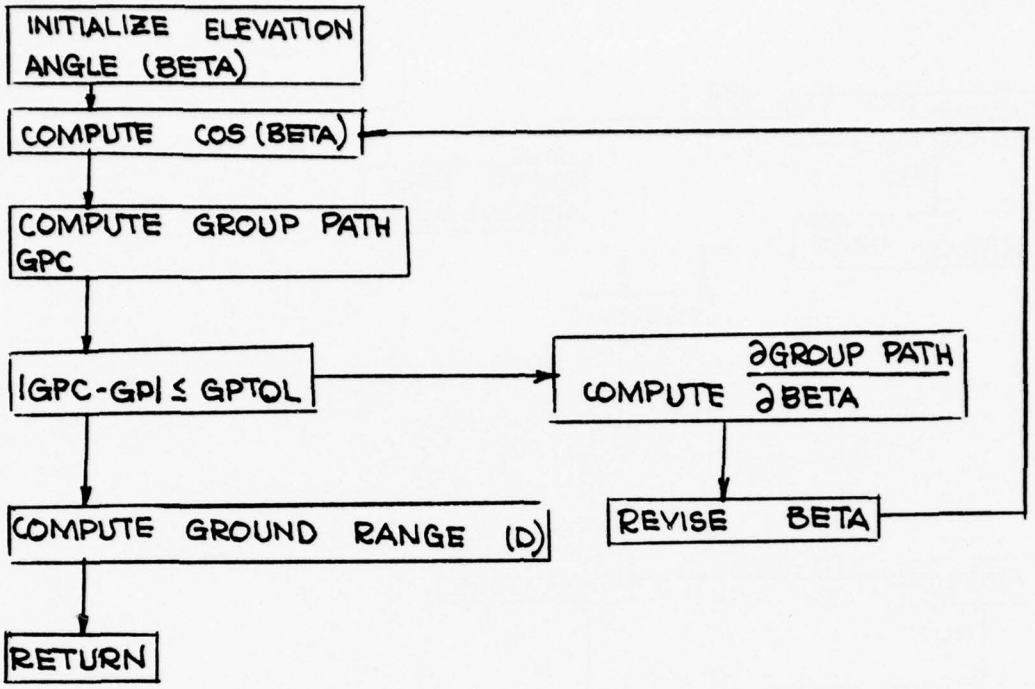

Figure A.3.5. Flow chart for subroutine DCAL.

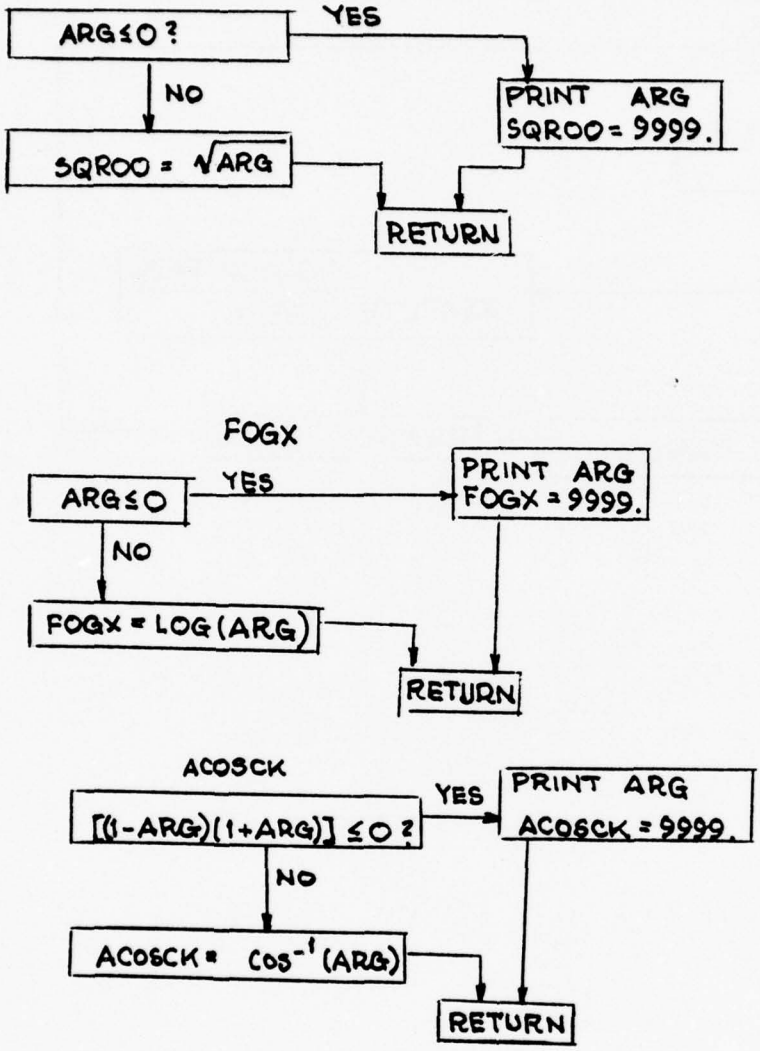

Figure A.3.6. Flow chart for special functions (square root, log, and $\cos^{-1}$).

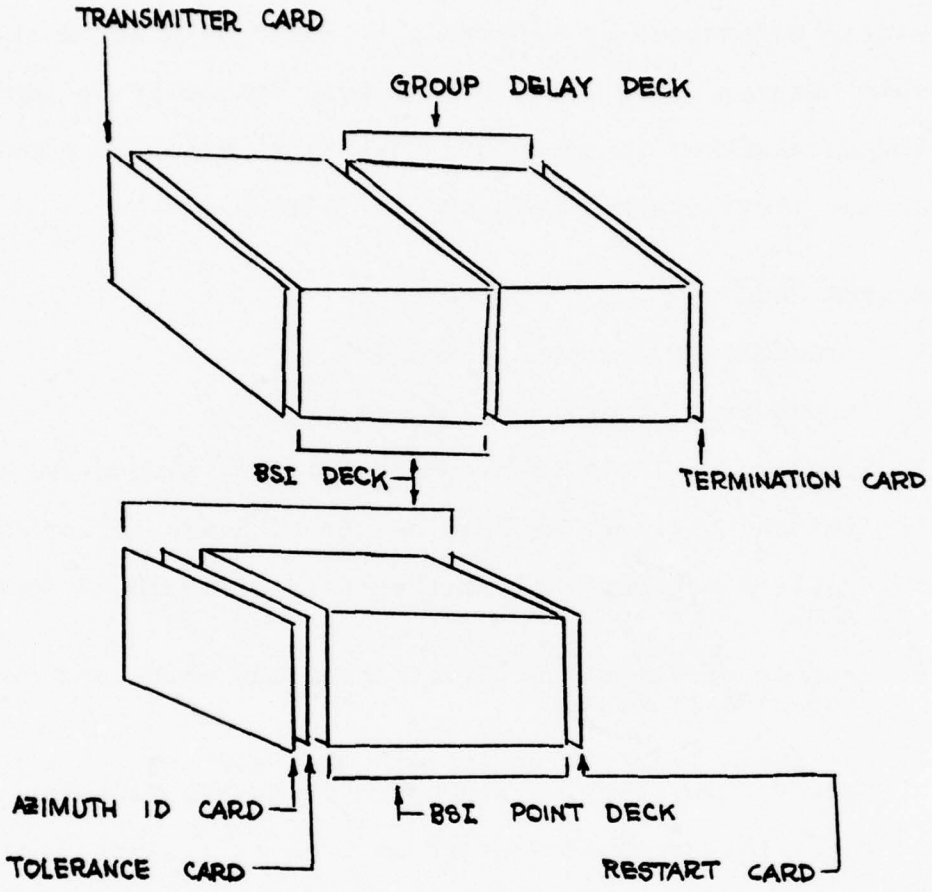

Figure A.3.7. Sequence of data cards.

The program formats are as follows

INPUT:

The input format shown in Figure A.3.7 can, perhaps, be more easily understood by referring to the flow diagram of the main program. The first card serves primarily to identify the backscatter ionogram sounder. The variables appearing on the first control card are as follows.

TRANSMITTER CARD:

1. transmitter latitude (degrees)
2. transmitter longitude (degrees)
3. initial estimate of overhead critical frequency (MHz)
4. initial estimate of base height of overhead ionosphere (km)
5. initial estimate of semithickness of overhead ionosphere (km)
6. number of azimuthal directions along which BSI are obtained (integer)
7. an index (ind) which is zero when the user is supplying group delay deck, and nonzero otherwise (integer)

The format of the transmitter card is 5F10.2, I1, 9X, I1 respectively.

The next set of cards comprise the BSI deck. The first card in this set is the azimuth identification card which contains

AZIMUTH ID CARD:

1. azimuth along which the following set of backscatter ionogram points have been obtained. (degrees)
2. the total number of backscatter ionogram points along the azimuth which will be read in. (integer)
3. the number of consecutive BSI points (less than or equal to the total number of BSI points) which will be considered simultaneously in obtaining a single net of QPP. (integer)

The format of the azimuth card is F10.2, I2, 8X, I2

TOLERANCE CARD:

1. tolerance of the deviation of the group path with respect to elevation angle (typically .05 km/rad.)
2. final step size control for minimization (typically .01)
3. desired final value of error in fitting QPP to measured BSI (typically .05 km/rad.)
4. maximum number of iterations which may be performed in fitting QPP to measured BSI (floating point)

The format of this card is 4F10.5.

BSI POINT DECK (SINGLE AZIMUTH)

There is one card for each measured point on the BSI along the given azimuthal direction. Each card contains:

1. frequency of BSI point (MHz)
2. group path of BSI point (km)

The format is 2F10.0.

USING THE RESTART CARD:

The restart card (format I10) is used only if IND≠0. When the user sets IND≠0 (referring to the main program flow chart) the program will not use the QPP to compute the ground range corresponding to a given group path. In fact, the program will go no further than the computation of QPP as performed by subroutine BACKS3. The only information on the restart card is the integer KGO. Setting KGO to zero (or,equivalently, using a blank card) tells the program to read in another batch of cards consisting of

1. TRANSMITTER CARD
2. AZIMUTH ID CARD
3. TOLERANCE CARD
4. BSI POINT DECK

and, if the new transmitter card has IND=0

5. GROUP DELAY DECK
6. TERMINATION CARD

Setting KGO < 0 will terminate the execution by telling the program that there is no more data to be processed.

NOT USING THE RESTART CARD:

The restart card should not be used when IND=0. Also, even when IND≠0 the restart card should not be used between BSI decks corresponding to the same transmitter card, but different azimuthal angles (multi-azimuth case). In this case, the last card of the BSI point deck is followed immediately by the:

1. AZIMUTH ID CARD
2. TOLERANCE CARD
3. BST POINT DECK

corresponding to the next azimuthal direction, with the data from the first transmitter card being used for all subsequent azimuthal angles. After the BSI point deck corresponding to the last azimuthal angle, the restart card is used to signify end of data (KGO < 0), or a new transmitter card (KGO = 0).

GROUP DELAY DECK:

The group delay deck provides the program with a specific value of group path from which to compute the ground range.

There is one card for each group path measurement to be used in this manner. Each card in the group delay deck contains:

GROUP DELAY DECK CARD:

1. frequency at which measurement is made (MHz)
2. azimuthal angle at which measurement is made (degrees)
3. measured value of group path (km)

The format is 3F10.0.

TERMINATION CARD:

After the last group delay deck card there must be a termination card, containing the parameter FW in F10.0 format. FW < 0 signifies end of data and terminates the program. FW=0 tells the program that more data will be read in, starting from the transmitter card and continuing in order to either a restart card or a new termination card.

SUMMARY OF THE BACKSCATTER LEADING EDGE INVERSION
3 SETS OF 3 OVERLAPPING SETS WERE SUCCESSIVELY INVERTED

RB (KM)	RM(KM)	FC (MHZ)	FREQ (MHZ)	RANGE (KM)		MIN GROUP PATH (KM)
6618.919	6710.406	9.193	14.000	1061.038		1302.247
6632.230	6690.685	8.558	17.000	1449.758	A	1644.938
6601.425	6690.132	8.713	20.000	1877.891		2046.869

9.1927	6618.9188	6710.4062	1061.0383	1302.2466	
8.5580	6632.2304	6690.6846	1449.7578	1644.9381	B
8.7130	6601.4253	6690.1316	1877.8905	2046.8686	

16.00	52.00	1700.00	1676.31	C
8.64	6615.93	6690.39	D	

16.00	1700.00	0.00	3730.98	-12739.52		8.64	6615.93	6690.39
16.00	1700.00	0.16	2271.71	-6004.27		8.64	6615.93	6690.39
16.00	1700.00	0.25	1829.23	-3462.60	E	8.64	6615.93	6690.39
16.00	1700.00	0.29	1714.51	-2700.21		8.64	6615.93	6690.39
16.00	1700.00	0.30	1700.28	-2597.05		8.64	6615.93	6690.39

16.00	52.00	1700.00	1676.31	1557.09	F
8.60	6624.51	6690.55	D		

16.00	1700.00	0.00	3780.11	-12739.53		8.60	6624.51	6690.55
16.00	1700.00	0.16	2286.07	-6003.18		8.60	6624.51	6690.55
16.00	1700.00	0.26	1832.05	-3471.20	E	8.60	6624.51	6690.55
16.00	1700.00	0.30	1714.65	-2715.10		8.60	6624.51	6690.55
16.00	1700.00	0.30	1700.27	-2613.88		8.60	6624.51	6690.55

16.00	52.00	1700.00	1557.09	1552.33	F
8.60	6624.85	6690.55	D		

16.00	1700.00	0.00	3782.03	-12739.53		8.60	6624.85	6690.55
16.00	1700.00	0.16	2286.62	-6003.28		8.60	6624.85	6690.55
16.00	1700.00	0.26	1832.14	-3471.84	E	8.60	6624.85	6690.55
16.00	1700.00	0.30	1714.65	-2716.14		8.60	6624.85	6690.55
16.00	1700.00	0.30	1700.27	-2615.06		8.60	6624.85	6690.55

16.00	52.00	1700.00	1552.33	1552.15 F
16.00	52.00	1700.00	1552.15	G

$$N(z) = N[1 - (z - Y)^2/Y^2] \qquad 0 < z < 2Y \qquad (30)$$

OUTPUT FORMAT:

Referring to the sample output which resulted from a single azimuth measurement, subroutine BACKS3 causes block A to be printed. Each row of block A contains RB, RM, FC, frequency, range, and minimum group path. Block B contains the same information as block A (except for frequency) and shows that the variables have been transferred from BACKS3 to the main program. Block C contains frequency, azimuth, measured group path and the first approximation to ground range in the subsequent attempt to compute the ground range from a given value of group path. Block D contains the interpolated values of QPP at the reflection point Block E is printed out in subroutine DCAL and represents the progress of subroutine DCAL in homing a ray through the QP ionosphere model based upon group path. Each row of block E contains frequency, measured group path, initial elevation angle (β), computed group path, the derivative of group path with respect to β, and the three QPP describing the ionosphere. In the last row of block E, it can be seen that the computed and measured group paths are very close, so DCAL returns to the main program. Upon returning to the main program block F is printed and contains frequency, azimuth, measured group path, estimated ground range, and ground range based upon the homed in value of the initial ray elevation angle. Since these two ground ranges are so different a new interpolation procedure is initiated in the main program to compute the values of QPP at the reflection point of the homed in ray. This interpolation is summarized in block D, and comparing the first block D with

the second block D, we can see that the QPP at the reflection point are indeed different when computed for the homed in ray. This leads to an iterative process which alternates between subroutine DCAL, and the recomputation of the interpolated QPP. The final block F shows, however that the most recent computed ground range, and the ground range from the previous iteration are very close. Thus, block G is printed in the main program and contains the frequency, azimuth, group path and final solution for the ground range.

6. CONCLUSION

APPENDIX 4

A Method for Inverting Oblique Sounding Data in the Ionosphere

by S. I. Chuang and K. C. Yeh

This paper has been submitted to Radio Science.

REFERENCES

A METHOD FOR INVERTING OBLIQUE SOUNDING DATA IN THE IONOSPHERE

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Department of Electrical Engineering
National Taiwan University

We have developed in this paper a method of inverting backscatter leading edge ionograms and fixed distance oblique ionograms to obtain ionization profiles. The problem of ionogram inversion is first formulated in such a way that the Backus-Gilbert technique can be applied. This inversion technique starts by assuming an initial profile based on available knowledge. Oblique group paths are computed by using this initial profile and are compared with the experimental data. The discrepancy in group path is used as a basis for improving the profile and thus an iterative procedure is developed. This iteration is continued until an optimum solution is found. As is well-known, when ionograms are given at discrete frequencies there may exist an infinite number of profiles that will yield the observed group path data. Among this infinite number of solutions there is an optimum one in the sense that its fractional departure from the initially assumed profile is a minimum in terms of value at boundaries and in terms of the profile curvature. The method is then applied to the special vertical incidence case and reasonably fast convergence is obtained. This success demonstrates the promise of this method in inverting oblique sounding data.

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1. INTRODUCTION

In ionospheric sounding one usually measures the wave parameters, such as transit time, frequency shift, polarization, power returned, etc. One wishes then to make use of these measured wave parameters to deduce the physical parameters of the ionosphere, such as electron density, temperature, motion, etc. In order to deduce these physical parameters one must make use of a theory, known as the inversion theory, to relate these desired physical parameters to wave parameters. Sometimes the inversion is quite simple; sometimes the inversion can be quite complex. A case in point is the inversion of vertical ionograms, e.g., [Budden, 1961, p. 161], where the virtual height h'(f) as a function of frequency f is measured and the true ionization profile N(h) is desired. From wave propagation studies one realizes that h' is a nonlinear functional of N and as such it is in general very difficult to solve. Fortunately for the case of vertical ionograms with magnetoionic effects ignored, the integral can be put in the form of a convolution type which can then be solved, for example, by Laplace transform, provided that N(h) increases monotonically with height. Now if one wishes to sound obliquely in order to obtain the profile some distance away from the local vertical, the integral is no longer of the convolution type. In this case, since the group delay is a nonlinear functional of N(h), the inversion becomes very difficult. Current techniques are largely numerical and are designed to obtain a model profile that will yield data approximating the measured data.

However, ionospheric investigations have been carried out for several decades. The accumulated knowledge enables us to make a zeroth-order guess about the profile. If this zeroth-order profile does not depart appreciably from the true profile, it is possible to reformulate the problem by a linearization procedure. This reformulated problem is much simpler to deal with

because it is now linear. Many mathematical techniques applicable to linear problems can be used in order to arrive at a solution. Our approach is similar to that used in geophysics for inverting seismic data [Backus and Gilbert, 1967].

2. FORMULATION

For simplicity we will ignore the magnetoionic effects completely. The ionosphere is assumed to be spherically stratified so that the Bouguer's rule applies at any point along the ray, i.e.,

$$r_0 \cos \phi_0 = rn \cos \phi = r_t n_t = \text{constant} \tag{1}$$

where r is the geocentric distance of a point along the ray, $n = n(r)$ is the refractive index and $\phi = \phi(r)$ is the elevation angle of the ray. The earth radius is r_0 at which $\phi_0 = \phi(r_0)$. The ray is reflected with a geocentric distance r_t at which $n_t = n(r_t)$ and $\phi(r_t) = 0$. The group path of an ionospherically reflected signal is

$$P = 2 \int_{r_0}^{r_b} ds + 2 \int_{r_b}^{r_t} ds/n \quad . \tag{2}$$

This integral is integrated along the ray from r_0 to the bottom of the ionosphere at r_b in free space and then from r_b to r_t in the ionosphere. In the ionosphere the refractive index is related to the electron density profile N(r) through

$$n^2 = 1 - kN/f^2 \tag{3}$$

where f is the radio frequency and $k = e^2/4\pi^2 m\varepsilon_0$ has a numerical value 80.6 in S.I. units. It is easily seen from (2) that P is a nonlinear functional of N. We are not aware of any analytic method that can be used to solve for N.

Therefore, we will adopt a perturbation scheme to linearize the problem. Our approach is similar to that used in inverting teleseismic ray data [Johnson and Gilbert, 1972].

Perturbations in P can come from two sources: perturbations in n and perturbations in ϕ_0. Taking the variation in (2) and linearizing, we obtain

$$\delta P = 2 \int_{r_b}^{r_t} \frac{-\delta n}{n^2} \, ds + \frac{\partial P}{\partial \phi_0} \, \delta \phi_0 \quad . \tag{4}$$

In practice two kinds of oblique sounding data can be obtained: backscatter leading edge and oblique ionogram. For a thorough review on the nature of these data, the reader is referred to Davies [1965]. We discuss both of these cases for our purpose in the following.

(i) Backscatter leading edge.

The backscattered leading edge has a minimum time delay. In this case $\partial P/\partial \phi_0 = 0$ and the variation in group path is caused entirely by variations in the refractive index. The variation in n can be related to the variation in N by using (3). Substituting (3) and (1) in (4) yields

$$\delta P = \int_{r_b}^{r_t} K_1' x \, dr \tag{5}$$

where

$$x = \delta N/N \tag{6}$$

and

$$K_1' = krN/f^2(1 - kN/f^2)[(1 - kN/f^2)r^2 - r_0^2 \cos^2 \phi_0]^{1/2} \quad . \tag{7}$$

The kernel K_1' can be computed from the model ionosphere and is therefore known. We note in (5) that δP is a linear functional of δN.

(ii) Oblique ionogram

The oblique ionogram gives the group path of ionospherically reflected signals as a function of frequency for a fixed ground distance D. The ground distance is

$$D = 2r_0^2 \cos \phi_0 \left[\int_{r_0}^{r_b} \frac{dr}{r\sqrt{r^2 - r_0^2 \cos^2 \phi_0}} + \int_{r_b}^{r_t} \frac{dr}{r\sqrt{n^2 r^2 - r_0^2 \cos^2 \phi_0}} \right] . \quad (8)$$

As the ionization profile is varied by δN, the initial elevation angle of the ray must also be varied by $\delta\phi_0$ in order to keep D fixed. Take the variation of (8) and set it to zero to produce

$$\delta\phi_0 = \left[2r_0^2 \cos \phi_0 \int_{r_b}^{r_b} \frac{-kr\delta N \, dr}{2n^2 f^2 r^2 (n^2 r^2 - r_0^2 \cos^2 \phi_0)^{1/2}} \right] \Bigg/ \frac{\partial D}{\partial \phi_0} \quad . \quad (9)$$

Inserting (9) in (4), the variation in group path of an oblique ionogram is found to be

$$\delta P = \int_{r_b}^{r_t} K_2' x \, dr \quad (10)$$

where the known kernel is

$$K_2' = K_1' \left[1 - \left(r_0^2 \cos \phi_0 / r^2 \right) (\partial P / \partial \phi_0) / (\partial D / \partial \phi_0) \right] \quad . \quad (11)$$

Comparing (5) and (10), one notes that for both kinds of oblique data, the variation in group path is given by

$$\delta P(f) = \int_{r_b}^{r_t} K'(r, f, \phi_0) x(r) \, dr \quad (12)$$

where for backscatter leading edges $K' = K_1'$, and for oblique ionograms $K' = K_2'$.

The kernel K' is referred to as the Frechet kernel [Backus and Gilbert, 1967; Green, 1975] because δP is obtained by taking the Frechet differential of (2). Our objective is to obtain the optimum x in some sense. We do this by following the Backus-Gilbert approach in the following section.

3. OPTIMIZATION

Suppose the oblique sounding data have been measured experimentally at M frequencies f_i, $i = 1,2,\ldots,M$. The corresponding group paths are P_{mi}, $i = 1,2,\ldots,M$. For the purpose of obtaining the ionization profile N(r) that produces these experimental data, we make use of our knowledge based on past experience to construct a zeroth-order approximation to the true ionization profile. This approximate profile is then used to compute M group paths P_{ci}, for M frequencies f_i, and M initial elevation angles ϕ_{oi}. The departure of the computed group path P_{ci} from the experimentally measured group path P_{mi} is y_i, i.e., $y_i = P_{ci} - P_{mi}$. Then according to (12),

$$y_i(f_i) = \int_{r_b}^{r_p} K_i(r,f_i,\phi_{oi})x(r)\,dr \qquad i = 1,2,\ldots,M \tag{13}$$

where r_p is the peak height of the ionosphere. The Frechet kernel K_i is given by

$$K_i(r,f_i,\phi_{oi}) = \begin{cases} K_1'(r,f_i,\phi_{oi}) & r_b \le r \le r_{ti} \\ 0 & r_{ti} \le r \end{cases}, \tag{14}$$

for backscatter leading edges, and, for oblique ionograms,

$$K_i(r,f_i,\phi_{oi}) = \begin{cases} K_2'(r,f_i,\phi_{oi}) & r_b \le r \le r_{ti} \\ 0 & r_{ti} \le r \end{cases} \tag{15}$$

where r_{ti} is the geocentric distance of reflection for f_i. In later development K_i will be required to be square integrable. However, since both K_1' and K_2' have singularities at the point of reflection, K_i is obviously not square integrable. This difficulty can be remedied by integrating (13) by parts twice, as suggested by Johnson and Gilbert [1972], to produce

$$y_i = -H_i(r_b)x(r_b) - G_i(r_p)x'(r_p) + \int_{r_b}^{r_p} G_i(r)x''(r)\,dr \qquad (16)$$

$$i = 1,2,\ldots,M$$

where

$$x'(r) = dx/dr$$

$$x''(r) = d^2x/dr^2$$

$$H_i(r) = \int_{r_p}^{r} K_i(r',f_i,\phi_{oi})\,dr'$$

$$G_i(r) = \int_{r_b}^{r} H_i(r')\,dr' \quad . \qquad (17)$$

Now our original nonlinear inversion problem is reduced to the linear problem of (16) in which we have M data points to determine x"(r). In general, the solution to (16) is nonunique. Among this infinite number of solutions is a particular one for which

$$\frac{1}{2}\left\{x^2(r_b) + x'^2(r_p) + \int_{r_b}^{r_p} [x''(r)]^2\,dr\right\} = \text{minimum} \quad . \qquad (18)$$

This solution, if it exists, is the optimum solution we seek. It is optimum in the sense that the constraint condition (18) is satisfied and that the errors in $x(r_b)$ and $x'(r_p)$ are minimized and the profile x(r) is very smooth since the square of its cumulative curvature over the interval $[r_b,r_p]$ is minimized. Notice that we have chosen to deal with fractional departures of

electron density here as x is defined by (6). But it is possible to introduce any other reasonable weighting function on δN if so desired by modifying the definition of the Frechet kernel as done by Green [1975]. Such weighting may be desirable in cases where parts of data are known a priori more accurately than others, and hence deserve more weight.

The optimum solution can be found easily by introducing Lagrange multipliers ν_i. Carrying out the process of minimization, the solutions are found to be

$$x''(r) = \sum_{i=1}^{M} \nu_i G_i(r) \tag{19a}$$

$$x'(r_p) = -\sum_{i=1}^{M} \nu_i G_i(r_p) \tag{19b}$$

$$x(r_b) = -\sum_{i=1}^{M} \nu_i H_i(r_b) \quad . \tag{19c}$$

Insertion of (19) into (16) yields

$$y_i = \sum_{j=1}^{M} \nu_j \left[H_i(r_b)H_j(r_b) + G_i(r_p)G_j(r_p) + \int_{r_b}^{r_p} G_i(r)G_j(r)\, dr \right] . \tag{20}$$

Equation (20) can be put in a very compact form if the following vector notations are used. Let the column vectors be

$$\vec{\nu} = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_M \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} \tag{21}$$

and the symmetric $M \times M$ matrix $A = [A_{ij}]$ have an ijth element

$$A_{ij} = H_i(r_b)H_j(r_b) + G_i(r_p)G_j(r_p) + \int_{r_b}^{r_p} G_i(r)G_j(r)\,dr \quad . \tag{22}$$

Then (20) becomes simply

$$\vec{y} = A\vec{\nu} \quad . \tag{23}$$

For independent data, det $A \neq 0$, and (23) can be inverted to give $\vec{\nu}$. Knowing $\vec{\nu}$, (19a) reduces to

$$x''(r) = (A^{-1}\vec{y})\vec{G}(r) \tag{24}$$

where $\vec{G}(r)$ is a column vector whose ith element is $G_i(r)$. Equation (24), together with the boundary conditions (19b) and (19c), enables us to compute $x(r)$ by integrating (24) twice. Once $x(r) = \delta N/N$ is determined, the desired correction δN to the zeroth-order model $N(r)$ is known. Computations can then be started anew by using the new model, $N(r) + \delta N(r)$, to produce eventually a second correction. Thus an iteration procedure is established. By repeated iteration, a final optimum solution can be achieved. At each step we may check the value

$$x^2(r_b) + x'^2(r_p) + \int_{r_b}^{r_p} [x''(r)]^2\,dr = \vec{\nu}^T A\vec{\nu} \tag{25}$$

When this value is less than a certain specified small value, the iteration procedure may be terminated. The final profile $N(r)$ will be the desired optimum electron density profile for the set of data P_{mi}, $i = 1,2,\ldots,M$.

4. VERTICAL INCIDENCE

As mentioned earlier, the vertical ionogram can be inverted exactly if the ionization profile increases monotonically with height. However, for the sake of completeness, we will also work out our scheme for this particular

case. Since the vertical incidence case is easier to manage analytically, a numerical example will be given in the next section to illustrate the computational procedures involved.

For vertical incidence the initial elevation angle ϕ_0 is 90°. The once-integrated Frechet kernel $H_i(r)$ no longer exists at $r = r_{ti}$ since the singularity of K_1' at r_{ti} as given by (7) is now of higher order. Close attention must be paid in order not to allow singularities to get out of hand. This can be done by a slight modification. Taking the Frechet differential of (2) as before, except for the vertical incidence case, ds is replaced by dr and one obtains

$$y = \delta P = \int_{r_b}^{r_t} K(r)x(r)\, dr \tag{26}$$

with

$$x(r) = \delta n/(n + \delta n) \tag{27}$$

$$K(r) = -2/n \ . \tag{28}$$

Now (26) is identical in form to (13). An identical procedure can be followed by first integrating by parts twice and setting up the minimization process similar to that given in the last section to obtain x. After knowing x(r), (27) and (3) can be used to find the perturbed ionization profile required through

$$\delta N(r) = (1 - kN/f^2)[1 - (1 - x)^{-2}]f^2/k \ . \tag{29}$$

If one wants, a new model $N(r) + \delta N(r)$ can be used to replace the initial model N(r), and the computations are repeated. Thus an iteration procedure is established as before. This iteration can be terminated when the error is smaller than some specified value.

5. AN EXAMPLE

For purposes of illustration, we shall assume a parabolic layer

$$N_p(z) = N_m[1 - (z - Y)^2/Y^2] \qquad 0 < z < 2Y \tag{30}$$

as the true ionization profile. For simplicity the vertical coordinate z starts at the bottom of the ionosphere and the ionosphere has a semithickness Y. In this case the measured group path (equal to twice the group height) is

$$P_m = Y(f/f_c)\ \ell n[(f_c + f)/(f_c - f)] \tag{31}$$

where f_c is the critical frequency.

Suppose as an initial guess of the profile we assume a linear profile

$$N_\ell(z) = az \qquad 0 < z \ . \tag{32}$$

A signal of frequency f incident vertically is reflected at a height $z_t = f^2/ka$. Its corresponding group path is

$$P_c = 4z_t \ . \tag{33}$$

The Frechet kernel (28) becomes, for this initial profile,

$$K(z) = -2/(1 - z/z_t)^{1/2} \ . \tag{34}$$

The corresponding once-integrated and twice-integrated Frechet kernels can be computed easily as

$$H(z) = \int_{z_t}^{z} K(z)\ dz = 4z_t(1 - z/z_t)^{1/2} \tag{35}$$

and

$$G(z) = \int_0^z H(z)\ dz = -(8z_t^2/3)[(1 - z/z_t)^{3/2} - 1] \ . \tag{36}$$

With (36) substituted into (19a), we obtain the solution x"(z). Integrate the resulting expression twice, constrained by the boundary conditions (19b) and (19c), and we obtain

$$x(z) = -\nu(8z_t^2/3)\left[\frac{4z_t^2}{35}\left(1 - \frac{z}{z_t}\right)^{7/2} - \frac{z^2}{2} + (z_t + 1)z + \left(\frac{3}{2z_t} - \frac{4z_t^2}{35}\right)\right] \quad .(37)$$

The Lagrange multiplier ν in (37) is computed from

$$\nu = A^{-1}y \tag{38}$$

where $y = P_c - P_m$ is the difference of time delay in a linear model profile (32) from that in a true parabolic profile (30) at frequency f, and A is computed from the expression

$$A = 16z_t^2 + (64/9)z_t^4 + (16/5)z_t^5 \tag{39}$$

by using formula (22). After knowing x(z), it is then inserted in (29) to produce $\delta N(z)$. Computations can be repeated anew by using $N(z) + \delta N(z)$ as the model profile, except now the corrected model profile is not necessarily linear so that a numerical scheme may have to be developed. One possibility is to use a piecewise linear model.

Let us now adopt some numerical values. We take, for the true (parabolic) profile Y = 100 km, $N_m \approx 10^{12}$ el/m^3. Then at a frequency f = 8.693 MHz, we obtain from (31) $P_m \cong 400$ km. For the model (linear) profile, we take $a = 1.25 \times 10^{10}$ el/m^3/km, which gives $P_c \cong 300$ km at the same frequency. This gives an error in group path of $y = P_c - P_m = -100$ km. Substitute these numerical values in (37) to obtain x(z) which, when inserted in (29), gives us the perturbed $\delta N(z)$ required to reduce the error. The results are shown graphically in Figure 1. It is seen that by using data only at one frequency, the once-iterated profile $N_\ell + \delta N$ approaches the true profile N_p reasonably fast.

6. CONCLUSION

We have outlined in this paper a method whereby ionospheric profiles can be deduced from oblique sounding data. The method starts by assuming a guessed zeroth-order profile based on our past experience and the accumulated knowledge about the ionosphere. An iterative scheme has been worked out to enable us to improve the computed profiles until the errors are smaller than some specified value. As an illustration the method is applied to the vertical incidence case, which shows reasonably fast convergence. Encouraged by this success, we would like to suggest that more extensive numerical computations be carried out.

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FIGURE CAPTION

Figure 1. Illustrating convergence of the iterative procedure. The parabolic profile N_p is assumed to be the real profile. The linear profile N_ℓ is the initial guess. The one-iterated profile is $N_\ell + \delta N$.

REFERENCES

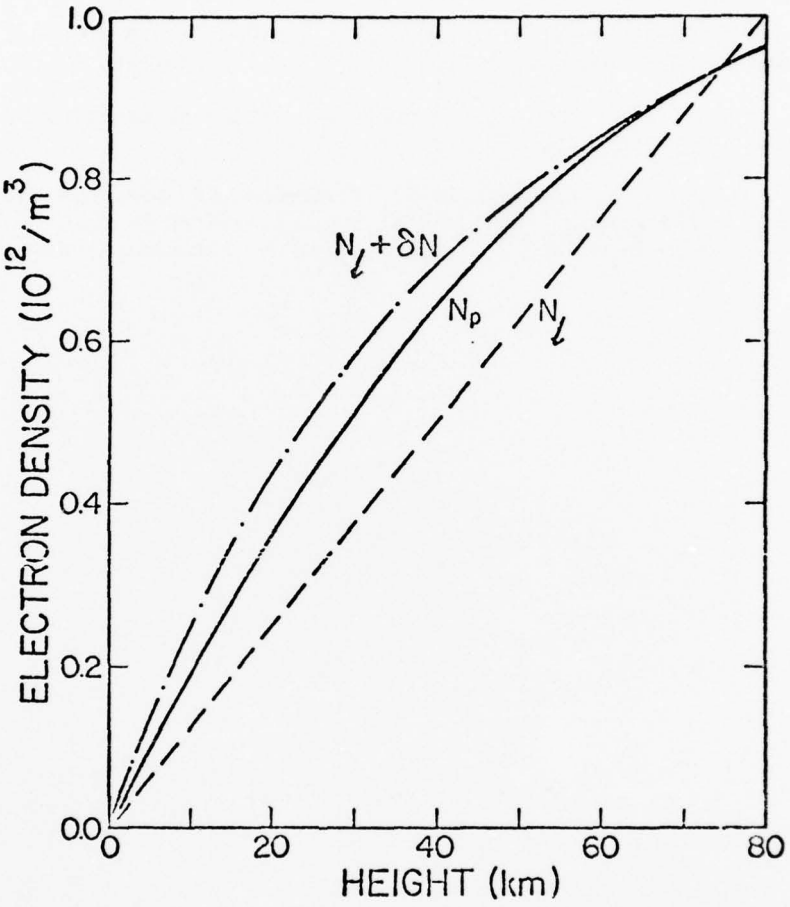
Backus, G. E. and J. F. Gilbert (1967), Numerical application of a formalism for geophysical inverse problems, Geophys. J., 13, 247-276.

Budden, K. G. (1961), Radio Waves in the Ionosphere, Cambridge University Press, New York.

Davies, K. (1965), Ionospheric radio propagation, National Bureau of Standards Monograph 80, U.S. Government Printing Office, Washington, D. C.

Green, W. R. (1975), Inversion of gravity profiles by use of a Backus-Gilbert approach, Geophysics, 40, 763-772.

Johnson, L. E. and F. Gilbert (1972), Inversion and inference for teleseismic ray data, in Methods in Computational Physics, edited by B. A. Bolt, vol. 12: Seismology, Body Waves and Sources, pp. 231-266, Academic Press, New York.


1.0
0.8
0.6
0.4
0.2
0.0
ELECTRON DENSITY ($10^{12}/m^3$)
$N_l + \delta N$
N_p
N_l
0
20
40
60
80
HEIGHT (km)

REFERENCES

Booker, H. G., "Fifty years of the ionosphere. The early years - Electromagnetic theory", J. Atmos. Terr. Phys., 36, 2113-2136, 1974.

Breit, G. and M. A. Tuve, "A test for the existence of the conducting layer", Phys. Rev., 28, 554-575, 1926.

Croft, T. A. and H. Hoogasian, "Exact ray calculation in a quasi-parabolic ionosphere with no magnetic field", Radio Sci. 3, 69-74, 1968.

Fletcher, R. and M. J. D. Powell, "A rapidly convergent descent method for minimization", Computer Journal, 6, 163-168, 1963.

Manning, L. A., "The determination of ionospheric electron distribution", Proc. IRE, 35, 1203-1207, 1947.

Rao, N. N., "Bearing deviation in HF transionospheric propagation, 1. Exact computations for some ionospheric models with no magnetic field", Radio Sci. 3, 1113-1117, 1968.

Rao, N. N., "Synthesis of three-dimensional oblique ionograms", Radio Sci., 8, 449-451, 1973.

Rao, N. N., "Inversion of sweep-frequency sky wave backscatter leading edge for quasiparabolic ionospheric layer parameters", Radio Sci., 9, 845-847, 1974.

Rao, N. N., "Analysis of discrete oblique ionogram traces in sweep-frequency sky wave high resolution backscatter", Radio Sci., 10, 149-153, 1975.

Waynick, A. H., "Fifty years of the ionosphere: The early years--Experimental", J. Atmos. Terr. Phys., 2105-2112, 1974.

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